A Classical Mechanics Solution to Zeno's Arrow Paradox

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Abstract

The flying arrow paradox states that motion is impossible, as the arrow is motionless at every instant of time. Although the paradox has a mathematical solution, a physical uncertainty remains: is time continuous, or does it consist of numerous infinitesimally small intervals? Within the framework of classical mechanics, it is shown that the assumption of instantaneous rest (zero velocity at any moment) inevitably violates the laws of conservation of momentum and kinetic energy. A flying arrow cannot stop or resume its motion without external input. Consequently, the arrow cannot be "at rest" within an infinitesimally small interval of time, and/or time does not contain any "zeros" and flows continuously.

Keywords: Zeno's paradoxes, arrow paradox, conservation of momentum, conservation of energy, nature of time, classical mechanics

1. Formulation of the Paradox

Zeno's paradox according to Aristotle (Physics, 239b30): "If everything when it occupies an equal space is at rest at that instant of time, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless at that instant of time and at the next instant of time..."

Zeno's paradox according to Diogenes Laertius (Lives of Famous Philosophers, ix.72): "What is in motion moves neither in the place it is nor in one in which it is not."

2. Mathematical Aspect and the Remaining Uncertainty

Mathematically, the paradox is resolved; however, a physical uncertainty remains. It is currently unknown whether time consists of infinitesimal discrete intervals or flows continuously.

In classical kinematics, the distance traveled is expressed by the formula:

$$S = V \cdot T,$$

where V is speed and T is time.

Assuming that time does not possess any "zero" moments in its course, the speed of the arrow does not vanish and the distance traveled will be greater than zero:

$$S = V \cdot T > 0,$$

which implies that the arrow is in motion.

Speed is likewise determined by:

V = S / T,   T > 0,

which also implies motion.

Instantaneous speed is also determined by the following equation:

$V = \lim_{\Delta T \to 0} \Delta S / \Delta T > 0;$

Which also means movement.

However, if we allow that T = 0 at any instant, then multiplication by zero nullifies the distance traveled:

***S = V · 0 = 0*,**

meaning the arrow is motionless.

3. Physical Solution via the Laws of Conservation

Resolving this paradox requires clarifying whether time flows continuously or is composed of discrete infinitesimal intervals. The solution proposed here is grounded in the laws of classical mechanics.

Consider the motion of the arrow from the outset. The arrow begins its motion when the bowstring is released. Potential energy is transferred from the drawn bow, through the bowstring, into the arrow, which thereby acquires momentum (**p**) and kinetic energy ($\mathbf{E_k}$). In a realistic scenario, the acquired **p** and $\mathbf{E_k}$ are gradually dissipated into the atmosphere and ultimately transferred to a physical target upon impact.

Now suppose the arrow is "at rest" at some infinitesimally small moment during its flight. For the arrow to stop, it must transfer its momentum and kinetic energy to some physical object. An arrow cannot stop by transferring **p** and $\mathbf{E_k}$ into "nothing" -- this would violate the laws of conservation of momentum and energy in classical mechanics.

Furthermore, consider a subsequent infinitesimal moment at which the arrow is again "at rest," but now at a different position, closer to its target. For the arrow to have shifted position, it must have acquired **p** and $\mathbf{E_k}$ from some source. An arrow cannot begin motion spontaneously or acquire **p** and $\mathbf{E_k}$ from nowhere -- this, too, would violate the conservation laws.

Conclusion 1: A state of rest is impossible for a flying arrow, as it contradicts the laws of conservation of momentum and kinetic energy in classical mechanics.

Conclusion 2: If the arrow cannot be at rest at any moment during its flight, then there can be no "zeros" or pauses within time - such pauses would lead to violations of the conservation of momentum and energy. This implies that time flows continuously.

4. A Remark on Quantum Physics

Within standard quantum mechanics, time remains a continuous classical parameter (not an operator). Certain approaches to quantum gravity -- such as loop quantum gravity -- suggest discretization of time at the Planck scale (~10^-43 s), however, in my opinion, this has not yet been definitively proven and does not affect the classical argument.