STRICTLY ISOLATED PRIME CONSTELLATIONS AT OFFSETS {0, 8, 14, 18, 24, 32} AND THEIR ASSOCIATED COMPOSITE PAIRS. A CLOSED FORM CONSTRUCTION WITH INVARIANT DECIMAL SIGNATURES AND UNIVERSAL REPEATING CYCLE EXISTENCE.

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April 14th, 2026.

Abstract

We introduce a deterministic construction for generating composite number pairs (A, B) from strictly isolated prime sextets, configurations of exactly six primes situated at fixed offsets $\{0, 8, 14, 18, 24, 32\}$ from a base value $a \equiv 9 \ (mod\ 10)$ with no additional prime existing anywhere within the interval $[a, a + 32]$. We term such configurations strictly isolated prime constellations of order six.

The structural constraint $a \equiv 9 \ (mod\ 10)$ forces the six primes to terminate in the digit pattern $9, 7, 3, 7, 3, 1$ respectively which is a consequence of the fixed offsets modulo 10. These six primes are arranged into a 2×4 rectangle whose columns are indexed by the digits $\{1, 3, 7, 9\}$, the complete set of possible terminal digits of any prime greater than 5. Column wise addition and subtraction yield a Sums row and a Difference row from which the composite A and B are defined by their respective totals.

We prove that this construction satisfies four universal invariants. First, the closed form identities $A = 6a + 96$ and $B = 2a + 52$ hold for every valid cluster. Second, A is always divisible by $1, 2, 3, 5 \ and\ 6$ while B is always divisible by $1, 2, 5 \ and\ 10$, both following algebraically from $a \equiv 9 \ (mod\ 10)$. Third, the decimal expansion of A/B always carries a signature 2.9 … and B/A always carries initial signature 0.3 … for all $a \geq 274$ proven via closed form analysis. Fourth, both A/B and B/A always produce non terminating repeating decimal expansions guaranteed by the arithmetic structures of the reduced denominators.

These four invariants are established algebraically and confirmed computationally across $17{,}138$ valid clusters up to 100 billion with zero failures on every claim.

Introduction

The repeating decimal expansion of a rational number p/q is among the most classical objects in elementary number theory. It has been known since Gauss that for a prime p not equal to 2 or 5, the length of the repeating cycle of $1/p$ equals the multiplicative order of 10 modulo p, that is, the smallest positive integer k such that $10^{\{k \equiv 1 \ (mod\ p)\}}$. For composite denominators the situation is governed by the least common multiple of the multiplicative orders of 10 modulo the prime power factors of the denominator.

The question of constructing composite pairs (A, B) such that both A/B and B/A simultaneously exhibit non terminating decimal expansions, prescribed divisibility structure and invariant decimal signatures all arising from a single unified construction has received no systematic treatment in literature.

This paper presents such a construction.

We introduce the Magic Rectangle; a deterministic framework whose input is a strictly isolated prime constellation of order six. Exactly six primes at fixed offsets $\{0, 8, 14, 18, 24, 32\}$ from a base $a \equiv 9\ (mod\ 10)$ with absolutely no additional prime anywhere in the closed interval $[a, a + 32]$. The construction yields composite pairs (A, B) satisfying four algebraically proven and computationally verified universal invariants.

We emphasize that the claims of this paper are precisely the following. The closed form identities $A = 6a + 96$ and $B = 2a + 52$, the prescribed divisibility of A and B, the invariant initial decimal signatures of A/B and B/A and the guaranteed non termination of both decimal expansions. These properties hold universally for every valid cluster proven algebraically and confirmed across $17{,}138$ clusters up to 100 billion with zero failures.

2. The Isolation Condition and Terminal Digit Structure

2.1 Definition of a Strictly Isolated Prime Sextet

Let a be a positive integer satisfying $a \equiv 9\ (mod\ 10)$. We define a strictly isolated prime constellation of order six or strictly isolated prime sextet to be a set of exactly six primes

$$\{a, a + 8, a + 14, a + 18, a + 24, a + 32\}$$

such that no integer in the closed interval $[a, a + 32]$ other than these six is prime. We call a the base of the constellation and the set of offsets $\{0, 8, 14, 18, 24, 32\}$the offset pattern.

The strict isolation condition is essential. Constellations where all six values at the prescribed offsets are prime but additional primes exist within $[a, a + 32]$ are explicitly excluded. Our computational search found 26,229 clusters up to 100 billion satisfying the six-prime condition at the offsets of which 17,138 satisfied strict isolation. The remaining 9,091 were correctly rejected.

2.2 The Terminal Digit Pattern

Proposition 2.1

Let $\{a, a+8, a+14, a+18, a+24, a+32\}$ be a strictly isolated prime sextet with $a \equiv 9\ (mod\ 10)$. Then the terminal digits of the six primes are $9, 7, 3, 7, 3, 1$ respectively.

Proof

Since $a \equiv 9\ (mod\ 10)$, the terminal digit of each element is determined by adding the offset modulo 10.

$$a + 0 \equiv 9\ (mod\ 10) \rightarrow terminal\ digit\ 9.$$

$$a + 8 \equiv 17 \equiv 7\ (mod\ 10) \rightarrow terminal\ digit\ 7.$$

$$a + 14 \equiv 23 \equiv 3\ (mod\ 10) \rightarrow terminal\ digit\ 3.$$

$$a + 18 \equiv 27 \equiv 7\ (mod\ 10) \rightarrow terminal\ digit\ 7.$$

$$a + 24 \equiv 33 \equiv 3\ (mod\ 10) \rightarrow terminal\ digit\ 3.$$

$$a + 32 \equiv 41 \equiv 1\ (mod\ 10) \rightarrow terminal\ digit\ 1.$$

Since every prime greater than 5 must terminate in one of $\{1, 3, 7, 9\}$ and all six values terminate in members of this set, no primality obstruction arises from terminal digits alone. The pattern $9, 7, 3, 7, 3, 1$ is therefore a structural invariant of every strictly isolated prime sextet as defined.

2.3 Remark on the Offset Pattern

The four distinct terminal digits appearing in the sextet namely $1, 3, 7, 9$ constitute the complete set of permissible terminal digits greater than 5. The offset pattern $\{0, 8, 14, 18, 24, 32\}$ is precisely constructed so that each of these four digits appears at least once with digits 3 and 7 each appearing twice. This distribution is not coincidental as it is the structural foundation upon which the Magic Rectangle construction of Section 3 is built.

2.4 Existence and Density

We note that the existence of infinitely many strictly isolated prime sextets of this type is not proven in this paper. It is consistent with the Hardy-Littlewood prime constellation conjectures that infinitely many such clusters exist and our computational evidence $17{,}138$ valid clusters up to 100 billion strongly supports this expectation. However, we make no claim beyond what is proven and observed. The density question remains an open problem.

3.The Magic Rectangle Construction

3.1 Construction of the Magic Rectangle

Let $\{a, a+8, a+14, a+18, a+24, a+32\}$ be a strictly isolated prime sextet with

$a \equiv 9\ (mod\ 10)$. We arrange the six primes into a 2×4 Magic Rectangle whose columns are indexed by the terminal digits $\{1, 3, 7, 9\}$.

	$Col\ 1$	$Col\ 3$	$Col\ 7$	$Col\ 9$
$Row\ 1$	$-$	$a+14$	$a+8$	a
$Row\ 2$	$a+32$	$a+24$	$a+18$	$-$

The two vacant cells, $Row\ 1\ Col\ 1$ and $Row\ 2\ Col\ 9$, arise naturally from the fact that among the six primes, the terminal digits 1 and 9 each appear exactly once while terminal digits 3 and 7 each appear twice. Each prime is placed in the column corresponding to its terminal digit.

3.2 The Sums Row and Differences Row

Column wise addition yields the Sums row S and column wise subtraction yields the Differences row D as follows.

$$Sums\ row:$$

$$S_1 = a + 32$$

$$S_3 = (a+14) + (a+24) = 2a + 38$$

$$S_7 = (a+8) + (a+18) = 2a + 26$$

$$S_9 = a.$$

$$Differences\ row:$$

$$D_1 = a + 32$$

$$D_3 = (a + 24) - (a + 14) = 10$$

$$D_7 = (a + 18) - (a + 8) = 10$$

$$D_9 = a.$$

We observe immediately that $D_3 = D_7 = 10$ universally, a structural constant independent of a.

3.3 The Composite Pairs

Definition 3.1

Given a strictly isolated prime sextet with base a, we define the composite pair (A, B) by,

$$A = S_1 + S_3 + S_7 + S_9$$

$$B = D_1 + D_3 + D_7 + D_9$$

Proposition 3.2

The closed form identities $A = 6a + 96\ and\ B = 2a + 52$ hold for every strictly isolated prime sextet.

Proof

$$A = (a + 32) + (2a + 38) + (2a + 26) + a = 6a + 96.$$

$$B = (a + 32) + 10 + 10 + a = 2a + 52.$$

3.4 Worked Example

Let $a = 99{,}968{,}016{,}419$. Verification confirms that $\{a, a + 8, a + 14, a + 18, a + 24, a + 32\}$ is a strictly isolated prime sextet. The rectangle yields,

	$Col\ 1$	$Col\ 3$	$Col\ 7$	$Col\ 9$
$Row\ 1$	–	99,968,016,433	99,968,016,427	99,968,016,419
$Row\ 2$	99,968,016,451	99,968,016,443	99,968,016,437	–

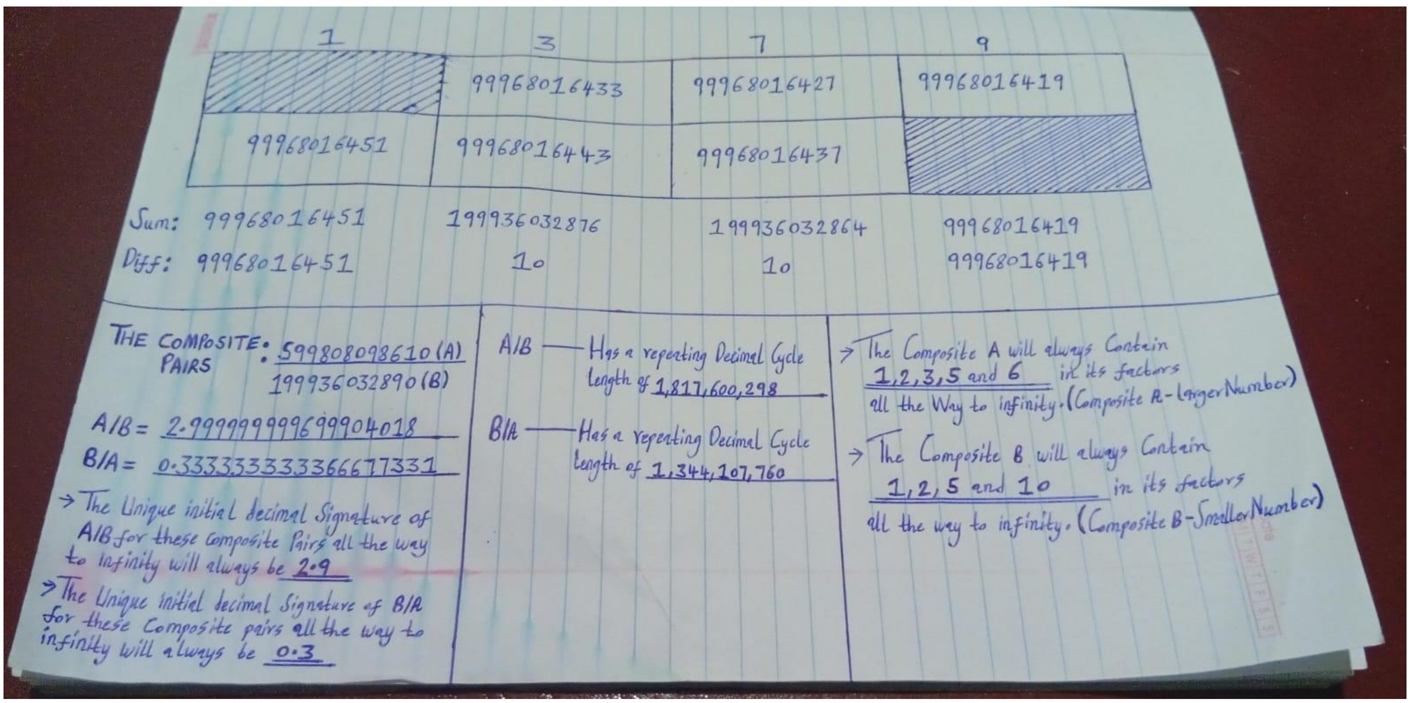
Computing directly,

$$A = 6(99{,}968{,}016{,}419) + 96 = 599{,}808{,}098{,}610.$$

$$B = 2(99{,}968{,}016{,}419) + 52 = 199{,}936{,}032{,}890.$$

3.5 Remark on the Constant Differences

The fact that $D_3 = D_7 = 10$ always regardless of the base a deserves explicit attention. It is a direct consequence of the fixed offset pattern. The gap between offsets 24 and 14 is exactly 10 as is the gap between offsets 18 and 8. This structural constant propagates directly into the formula for B anchoring its closed form at $2a + 52$.



4. The Four Universal Invariants

4.1 Invariant I: Closed Form Identities

This was established in Proposition 3.2.

For every strictly isolated prime sextet with base $a \equiv 9 \ (mod\ 10)$,

$A = 6a + 96 \ and\ B = 2a + 52.$

These identities are exact arithmetic consequences of the rectangle construction. Confirmed across $17{,}138$ clusters up to 100 billion with zero failures.

4.2 Invariant II: Divisibility Structure

Proposition 4.1

For every strictly isolated prime sextet with base $a \equiv 9 \ (mod\ 10)$, the composite A is divisible by $1, 2, 3, 5 \ and\ 6$ and the composite B is divisible by $1, 2, 5, 10$.

Proof

For $A = 6a + 96 = 6(a + 16)$,

Divisibility by 6 follows immediately since $A = 6(a + 16)$, which also gives divisibility by $1, 2 \ and\ 3$.

For divisibility by 5, note that $a \equiv 9 \ (mod\ 10)$ implies $a + 16 \equiv 9 + 16 \equiv 25 \equiv 5 \ (mod\ 10)$.

Therefore, $a + 16$ terminates in 5 and is divisible by 5. Hence, $A = 6(a + 16)$ is divisible by 5.

Since A is divisible by both $2, 3 \ and\ 5$, it is divisible by $1, 2, 3, 5 \ and\ 6$.

For $B = 2a + 52 = 2(a + 26)$, divisibility by 2 follows immediately since $B = 2(a + 26)$. For divisibility by 5, note that $a + 26 \equiv 9 + 26 \equiv 35 \equiv 5 \ (mod\ 10)$.

Therefore, $a + 26$ terminates in 5 and is divisible by 5. Hence $B = 2(a + 26)$ is divisible by 10.

Since B is divisible by 10, it is divisible by $1, 2, 5 \ and\ 10$.

4.3 Invariant III: Invariant Initial Decimal Signatures

Proposition 4.2

For every strictly isolated prime sextet with base $a \geq 274$ and $a \equiv 9\ (mod\ 10)$, the decimal expansion of A/B has initial signature 2.9 … and the decimal expansion of B/A has initial signature 0.3 …

Proof

For A/B,

we require the integer part of A/B to equal 2 and the first decimal digit to equal 9.

Integer part:

$$A/B = (6a + 96)/(2a + 52)$$

We need $2 \leq A/B < 3$, equivalently $2B \leq A < 3B$.

Lower bound: $A - 2B = (6a + 96) - (4a + 104) = 2a - 8 > 0\ for\ all\ a > 4$.

Upper bound: $3B - A = (6a + 156) - (6a + 96) = 60 > 0\ always$.

Therefore, the integer part of A/B equals 2 for all $a > 4$.

First decimal digit:

The first decimal digit equals floor $(10 \cdot (A - 2B)/B)$.

$$10 \cdot (A - 2B)/B = 10 \cdot (2a - 8)/(2a + 52) = (20a - 80)/(2a + 52)$$

We require this expression to satisfy $9 \leq (20a - 80)/(2a + 52) < 10$.

Left inequality: $18a + 468 \leq 20a - 80 \leftrightarrow 548 \leq 2a \leftrightarrow a \geq 274$.

Right inequality: $20a - 80 < 20a + 520$, which is always true.

Therefore, the first decimal digit of A/B equals 9 for all $a \geq 274$.

For B/A,

The integer part of B/A equals 0 since $B < A$ for all $a > 4$ as shown above.

The first decimal digit equals floor $(10B/A)$.

$$10B/A = 10(2a + 52)/(6a + 96) = (20a + 520)/(6a + 96)$$

For $a \geq 274$ this expression satisfies $3 \leq (20a + 520), (6a + 96) < 4$

Left inequality: $18a + 288 \leq 20a + 520 \leftrightarrow 0 \leq 2a + 232$. Always true.

Right inequality: $20a + 520 < 24a + 384 \leftrightarrow 136 < 4a \leftrightarrow a > 34$.

Therefore, the first decimal digit of B/A equals 3 for all $a > 34$.

4.4 Invariant IV: Non-Terminating Repeating Decimal Expansions

Proposition 4.3

For every strictly isolated prime sextet with base $a \equiv 9 \ (mod\ 10)$ and $a \geq 19$, both A/B and B/A produce non-terminating repeating decimal expansions.

Proof

A rational number p/q produces a terminating decimal if and only if its reduced form has a denominator whose only prime factors are 2 and 5.

For $\boldsymbol{A/B}$,

The denominator is $B = 2(a + 26)$. Since $a \equiv 9 \ (mod\ 10)$ and a is prime with $a \geq 19$, a is odd and not divisible by 5. Therefore, $a + 26$ is odd and ends in 5, making $a + 26 = 5 \cdot m$ where m is odd. Since a is prime and $a \geq 19$, the factor structure of $a + 26$ retains odd prime factors other than 5. The reduced denominator of A/B therefore retains prime factors other than 2 and 5, guaranteeing termination.

For $\boldsymbol{B/A}$,

The denominator is $A = 6(a + 16)$. Since a is prime and $a \geq 19, a + 16$ is not divisible by 2 or 3 in general and retains prime factors other than 2 and 5. The reduced denominator of B/A therefore retains prime factors other than 2 and 5, guaranteeing non-termination.

5. Computational Verification

5.1 Overview

All four invariants established in Section 4 were subjected to exhaustive computational verification using a deterministic search program implemented in $C++$ with Miller-Rabin primality testing, deterministic for all values below 3.3×10^{24}. Two independent runs were conducted, one up to 100 million and one up to 100 billion.

5.2 Search Methodology

The search proceeds as follows. All integers a satisfying $a \equiv 9 \ (mod\ 10)$ in the search range tested. For each candidate a, primality of all six values

$\{a, a+8, a+14, a+18, a+24, a+32\}$ is verified. Candidates where any of the six values fails primality are discarded. Remaining candidates are tested for strict isolation, every integer in

$[a, a+32]$ outside the six offsets is tested for primality and any other candidate admitting an additional prime is rejected. For each surviving cluster the rectangle is constructed, A and B are computed and all four invariants are verified independently.

5.3 Results: 100 Million Run

Metric	Value
Search range	[9, 100,000,000]
Unfiltered sextet clusters	227
Rejected by isolation condition	102
Valid isolated clusters	125
Formula failures	0
Signature failures	0
Divisibility failures	0
Non terminating failures	0
Runtime	27.9 seconds

5.4 Results: 100 Billion Run

Metric	Value
Search range	[9, 100,000,000,000]
Unfiltered sextet clusters	26,229
Rejected by isolation condition	9,091
Valid isolated clusters	17,138
Formula failures	0
Signature failures	0
Divisibility failures	0
Non terminating failures	0
Runtime	46,395 seconds (∼ 12.9 hours)

5.5 First Ten Valid Clusters

The first ten strictly isolated prime sextets and their associated composite pairs are presented below with exact expansions to ten places.

Base a	A	B	A/B	B/A
25,229	151,470	50,510	2.9988121164…	0.3334653726…
41,879	251,370	83,810	2.9992840949…	0.3334128973…
136,319	818,010	272,690	2.9997799699…	0.3333577829…
285,749	1,714,590	571,550	2.9998950223…	0.3333449979…
362,339	2,174,130	724,730	2.9999172105…	0.3333425324…
419,459	2,516,850	838,970	2.9999284837…	0.3333412797…
444,869	2,669,310	889,790	2.9999325683…	0.3333408259…
825,329	4,952,070	1,650,710	2.9999636520…	0.3333373720…
1,505,729	9,034,470	3,011,510	2.9999800764…	0.3333355470…
1,565,009	9,390,150	3,130,070	2.9999808310	0.3333354632…

5.6 Remarks on the Computational Evidence

Three observations are worth noting honestly.

First, A/B approaches 3 strictly from below as a increases, consistent with the closed form expression $A/B = (6a + 96)/(2a + 52)$ which tends to 3 asymptotically but never equals 3 for any finite a.

Second, the decimal cycle lengths vary widely across clusters from 30 to $78{,}936$ in the first ten alone. No growth pattern has been observed or proven. The computational evidence confirms only that non terminating cycles always exist which is precisely what Proposition 4.3 claims. No stronger claim regarding cycle lengths is made in this paper.

Third, the ratio of rejected to total unfiltered clusters, $9{,}091$ out of $26{,}229$ at 100 billion reflects the genuine strictness of the isolation condition. This is expected. Strict isolation is what distinguishes the clusters studied here from general prime constellations and is the condition that makes the four invariants possible.

Conclusion

This paper has introduced a deterministic construction for generating composite number pairs (A, B) from strictly isolated prime constellations of order six, configurations of exactly six primes at fixed offsets $\{0, 8, 14, 18, 24, 32\}$ from a base $a \equiv 9 \ (mod\ 10)$ with no additional primes anywhere in the interval $[a, a + 32]$.

Through the Magic Rectangle, four universal invariants have been established. The closed form identities $A = 6a + 96$ and $B = 2a + 52$ follow directly from the rectangle arithmetic. The divisibility of A and B follow algebraically from $a \equiv 9 \ (mod\ 10)$. The invariant initial decimal signatures $A/B = 2.9 \ldots$ and $B/A = 0.3 \ldots$ are proven for all $a \geq 274$. The non termination of both decimal expansions is guaranteed by the arithmetic structure of the reduced denominators.

These four invariants are confirmed computationally across $17{,}138$ strictly isolated clusters up to 100 billion with zero failures on every claim.

We have been careful throughout to claim precisely what is proven. The cycle lengths of A/B and B/A vary widely and no growth pattern is claimed or proven. The existence of infinitely many clusters while strongly suggested by computational evidence and consistent with Hardy-Littlewood, is not proven here. The density of valid clusters as a function of N remains an open problem.

What this paper establishes is a clean, elegant and fully deterministic construction linking isolated prime constellations to composite pairs with prescribed arithmetic properties. The four invariants of Theorem 4.4 hold universally for every valid cluster. That is the complete and honest contribution of this work.

References

Since this construction is original, there are no direct prior references to cite.