PROOF OF COLLATZ CONJECTURE

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1. abstract

The collatz conjecture was introduced by Lothar collatz in 1937. It is also known as "3n + 1 problem". The conjecture states: Start from any positive integer, n. If n is even, divide by 2; if n is odd, multiply by 3 and add 1. Now, the conjecture says that if you keep repeating above steps, you will finally reach 1, no matter what value of n we choose. In this paper, we prove collatz conjecture using method of mathematical induction. We also use binary and trenary numbers to prove collatz conjecture.

2. Content

Let 'd' be a decimal number. we convert it to a ternary number 't'. Let t = $a_n \times 3^n + a_{n-1} \times 3^{n-1} + a_{n-2} \times 3^{n-2} + \ . \ . \ . + a_1 \times 3 + a_0$, where $a_i \geq 0$ and $a_i \leq 2$ for $i \geq 0$ and $i \leq n$. Then we have:
3×t + 1 = $3 \times (a_n \times 3^n + a_{n-1} \times 3^{n-1} + a_{n-2} \times 3^{n-2} + \ . \ . \ . + a_1 \times 3 + a_0) =$
$a_n \times 3^{n+1} + a_{n-1} \times 3^n + a_{n-2} \times 3^{n-1} + \ . \ . \ . + a_1 \times 3 \times 3 + a_0 \times 3 + 1$
and
$t/2 = (a_n \times 3^n + a_{n-1} \times 3^{n-1} + a_{n-2} \times 3^{n-2} + \ . \ . \ . + a_1 \times 3 + a_0)/2$
A ternary number is divisible by 2 if and only if the sum of its digits is even.
Assume $a_n = 1$, $a_0 = 1$ and rest all coefficients of t are zero. Then, we get:
t = $a_n \times 3^n + a_{n-1} \times 3^{n-1} + a_{n-2} \times 3^{n-2} + \ . \ . \ . + a_1 \times 3 + a_0 = 3^n + 1$
Then $t/2 = (3^n + 1)/2$
$= (3^{n-1} \times (3) + 1)/2$
$= ((3^{n-1} \times (2 + 1) + 1)/2$
$= (3^{n-1} \times (2/2 + 1/2) + 1/2)$
$= (3^{n-1} \times (1 + 1/2) + 1/2)$
$= (3^{n-2} \times (3 \times 1 + 3 \times 1/2) + 1/2)$
$= (3^{n-2} \times (3 + 3/2) + 1/2)$
$= (3^{n-2} \times (3 + 1 + 1/2) + 1/2)$
$= (3^{n-3} \times (3 \times 3 + 3 \times 1 + 3 \times 1/2) + 1/2)$
$= (3^{n-3} \times (9 + 3 + 1 + 1/2) + 1/2)$
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$= 3^{n-1} + 3^{n-2} + \ . \ . \ . + 3^2 + 3 + 1 + 1/2 + 1/2$

Date: March 2026.

$= 3^{n-1} + 3^{n-2} + . . . + 3^2 + 3 + 1 + 2/2$
$= 3^{n-1} + 3^{n-2} + . . . + 3^2 + 3 + 1 + 1$
$= 3^{n-1} + 3^{n-2} + . . . + 3^2 + 3 + 2$

Hence,a ternary number of form 1001001011....010 when divided by 2 leaves the pattern 01120001202...120 . If a ternary number contains 2's in it, then, 2 can be decomposed as: 2 = 1 + 1

Therefore, any ternary number 21001020 when divided by 2 leave the pattern as 10112010.

3. Pattern

The pattern that ternary number leaves when divided by 2 is as shown below.
Case A: If the starting trit is one, then replace it with zero. Move to right trit
Case A1: If the trit is zero, replace it one. Then move to next trit (right trit).
Case A2: If the trit is one, replace it with 2. Then name the next trip as starting trit(right trit).
Case A3: If the trit is two, keep that trit as it is. Then move to right trit.
Case B: If the starting trit is zero, keep it zero and move to right trit and name it as starting trit.
Case C: If the starting trit is two, then replace it with one. Move to right trit and name it as starting trit.

4. Proof

According to collatz conjecture, if given an integer 'n', if it is odd, multiply it with '3' and add '1'. If 'n' is even, divide 'n' by '2'. The conjecture states that repeating above cycles on any given positive integer, we will finally reach '1'.
consider a ternary number 2.
2/2 = 1
Collatz conjecture holds true on 2. Let us check 10. 10 is odd (ternary number), so, apply collatz conjecture steps, we get:
(10)×10 + 1 = 100 + 1 = 101 = 101/2 =012 = 012×10 + 1 = 121 = 121/2 = 022 = 022/2 = 11 =11/2 = 02 = 02/2 = 01
Let collatz conjecture holds true upto '222...220'. Then ,let next number which occurs after '222...220' be '222...221'. Let 222...221 be of k trits. Let A indicates '3×n + 1' step and B indicates 'n/2' step. Convert 'k' to binary form and maximize it. Now we have two cases:
Case 1: k is odd.

If k is odd, 111...111 is the biggest odd number that can be represented in binary form. Anyway the sum of the trits of 'n' (222...221) cannot be even. So, applying collatz conjecture steps, we get:
22222...2221 (k trits)(odd)(A) = 22222...22211 (k + 1 trits)(even)(B) = 11111...11102 (k + 1 trits)(even)(B) = 02020...20201 (k trits)(odd) = m
$m < n$
Hence, collatz conjecture satisfied.
Case 2: k is even.'111111...110' is the biggest even number that can be represented in binary form.
If k is even, again the sum of the trits of 'n' (222...221) cannot be even. So, applying collatz conjecture steps, we get:
22222...2221 (k trits)(even)(A) = 22222...22211 (k + 1 trits)(odd)(B) = 11111...11102 (k + 1 trits)(odd)(A) = 11111...111021 (k + 2 trits)(even)(B) = 020202...02020122 (k + 1 trits)(odd)(A) = 020202...020201221 (k + 2 trits)(even)(B) = 010101...010100222 (k + 2 trits)(even)
Now k + 2 = 1000...000 , then k - 2 = 1111...100 .
Thus, 'k - 2' is divisible by 2. Therefore, 101010...1010 is divisible by 2 and 0222 is also divisible by 2. Hence, 010101...010100222 is divisible by 2.
010101...010100222 (k + 2 trits)(even)(B) = 00120012...001200111 (k + 1 trits)(odd)
Now, k + 1 = 1111...111 . k - 3 = 1111...1100.
Thus, 'k - 3' is divisible by 4. Also, 'k - 3' is a odd multiple of 4.
Thus, in ternary number 00120012...001200111, the pattern '1200' occurs odd number of times. Adding the total of pattern '1200' to remaining '111', it becomes even. Hence, 00120012...001200111 is divisible by 2.
00120012...001200111 (k + 1 trits)(odd)(B) = 000211...202 (k trits)(even) = m
$m < n$
Hence, collatz conjecture satisfied.

5. Conclusion

Therefore, collatz conjecture is true for every integer. By the method of mathematical induction, we reached the conclusion that collatz conjecture is true for all positive integers.

References

Wikipedia

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