A Measure-Theoretic Cosmology: Dark Matter Abundance and Suppressed Growth Without New Particles

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Abstract

We propose a cosmological framework in which the observable matter content arises from a measure-theoretic structure rather than additional particle species. The construction distinguishes between a metric (Lebesgue-type) measure and an invariant structural (Haar-type) measure. Their interplay leads to a finite dark-matter distribution, a visible matter component localized near a crossover scale, and a predicted abundance ratio $\Omega_{\rm DM}/\Omega_b \approx 5.4$ without introducing new particles.

At the background level, the model remains close to ΛCDM over observable redshifts. However, it predicts suppressed late-time growth of structure and enhanced decay of gravitational potentials. These effects provide observational tests in current and upcoming surveys. The framework offers a unified interpretation of dark matter and dark energy as manifestations of underlying measure structure.

1 Introduction

The ΛCDM model [1], [2], [3], [14] successfully describes a wide range of cosmological observations, yet the physical nature of dark matter and dark energy remains unknown. Despite extensive searches, no non-gravitational evidence for dark matter particles has been established. This motivates exploring alternative frameworks in which these phenomena arise from deeper structural properties of spacetime.

In this work, we consider a cosmological model in which matter content emerges from the interplay of two natural measures: a metric (Lebesgue-type) measure associated with spacetime geometry, and an invariant (Haar-type) measure associated with an underlying configuration space [10], [11], [12]. We show that this structure leads to a finite dark-matter distribution and a visible matter component localized near a crossover scale.

The model predicts the observed [14] abundance ratio

$$\frac{\Omega_{\rm DM}}{\Omega_b} \approx 5.4 \tag{1}$$

without introducing new particle species or free cosmological parameters. While the background expansion remains close to ΛCDM, the model predicts distinctive deviations in the growth of structure [16] and gravitational potentials.

2 Two-Measure Framework

We assume a standard FLRW spacetime description with comoving radial coordinate χ. In addition to the usual metric-induced measure, we introduce a second, invariant measure [29], [30] representing underlying structural degrees of freedom [4], [6], [7], [8], [9].

We define a characteristic crossover scale

$$\chi_s = \frac{r_*}{R(\eta)}, \tag{2}$$

where $R(\eta)$ is the scale factor and r_* is a fixed length scale.

The key assumption is that:

- The invariant measure dominates the extended (dark) sector,
- The metric measure governs observable matter,
- Visible matter arises in the crossover region where both measures contribute.

3 Dark Matter Distribution

We model the dark sector density [13], [15], as

$$\rho_{\rm DM}(\chi) = \frac{C}{\chi(\chi + \chi_s)^3}, \tag{3}$$

where C is a normalization constant.

The enclosed mass is finite:

$$M_{\rm DM} = \frac{2\pi C}{\chi_s}. \tag{4}$$

This avoids the logarithmic divergence present in naive shell-counting arguments and provides a natural extended halo profile.

4 Visible Matter from Observability

Define the dimensionless variable

$$x = \frac{\chi}{\chi_s}. \tag{5}$$

We introduce a visibility kernel

$$\Sigma(x) \propto \frac{x}{(1+x)^2}, \tag{6}$$

which captures the overlap between the two measures.

The baryonic density is then

$$\rho_b(\chi) = \kappa \Sigma(x) \rho_{\rm DM}(\chi), \tag{7}$$

which simplifies to

$$\rho_b(\chi) = \kappa \frac{C\chi_s}{(\chi + \chi_s)^5}. \tag{8}$$

The total baryonic mass is

$$M_b = \frac{\pi \kappa C}{3\chi_s}. \tag{9}$$

5 Abundance Ratio

The dark-to-baryonic ratio becomes

$$\frac{\Omega_{\rm DM}}{\Omega_b} = \frac{M_{\rm DM}}{M_b} = \frac{6}{\kappa}. \tag{10}$$

Estimating $\kappa = 10/9$ yields

$$\boxed{\frac{\Omega_{\rm DM}}{\Omega_b} \approx 5.4}. \tag{11}$$

This matches observational values [1] without introducing additional matter components.

6 Effective Expansion Dynamics

Define the effective equation of state:

$$w_{\rm eff} = -1 - \frac{1}{3}\frac{d\ln\rho}{d\ln a}. \tag{12}$$

In the asymptotic regime,

$$\rho \propto a^{1/2}, \quad w \to -\frac{7}{6}. \tag{13}$$

However, over observable redshifts ($z \lesssim 2$), the model remains close to $w \approx -1$, ensuring consistency with current data.

7 Growth of Structure

The evolution of perturbations is governed schematically by

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\rm eff}\rho\delta. \tag{14}$$

In this framework, effective coupling is suppressed at late times [17], [18], [20], [21] leading to:

- Earlier freezing of growth,
- Reduced $f\sigma_8(z)$ at low redshift.

Figure 1 shows an illustrative comparison between a ΛCDM-like growth history and an HHQM-like suppressed-growth curve. The plot is intended as a schematic target for the model rather than a precision data fit.

8 Gravitational Potentials

The Newtonian potential scales [27], [28] as

$$\Phi \propto \frac{\delta}{a}. \tag{15}$$

Thus, earlier freezing of δ implies:

- Faster decay of Φ,
- Reduced weak lensing amplitude,
- Enhanced ISW effect.

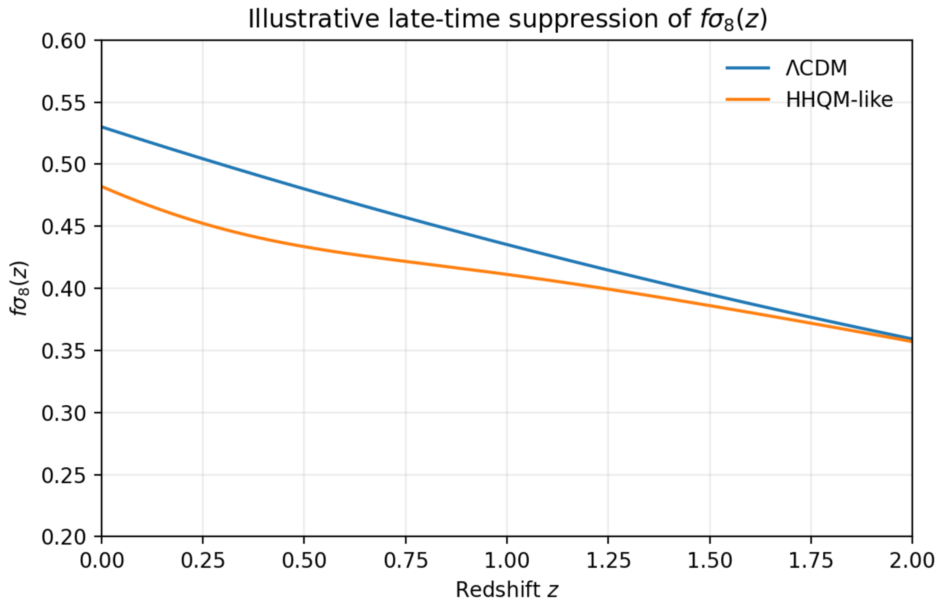

Figure 1: Illustrative comparison of $f\sigma_8(z)$ in a ΛCDM-like model and in an HHQM-like scenario with suppressed late-time growth. The qualitative signature is a few-to-ten percent reduction at low redshift, while remaining close to ΛCDM at higher redshift.

9 Observational Tests

The model predicts :

- Suppressed growth rate $f\sigma_8(z)$,
- Reduced lensing signal,
- Enhanced ISW contribution.

These effects are testable with current and upcoming surveys [22], [23], [24], [25], [26].

10 Discussion

The two-measure structure provides a unified interpretation of dark matter and dark energy as emergent phenomena. A deeper derivation may arise from an underlying geometric or algebraic framework.

11 Conclusion

We have presented a cosmological model in which matter abundance and dynamics emerge from measure structure rather than additional particle species. The framework reproduces the observed abundance ratio and predicts distinctive deviations in structure formation.

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