On the Foundation of Physics
Admissibility

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Abstract

We define admissibility as invariance under relabeling, refinement, composition, finite propagation, and closure, and show that it is necessary for well-defined comparison. Violation of admissibility produces representation dependence, non-uniqueness, or instability, rendering comparison ill-defined.

From admissibility, and from the requirement that physical content is determined through transformation rather than static configuration, relational difference and compositional structure are forced, yielding an Abelian realization and a functional constraint on admissible scalar comparisons. This constraint uniquely selects quadratic form: any admissible comparison is necessarily of the form $Q(x, y) = B(\Delta(x, y), \Delta(x, y))$.

The result is purely structural and does not assume a physical model. The quadratic invariant appears in existing physical theories as the spacetime interval in differential geometry and as scalar quantities in quantum theory. These are not independent assumptions, but realizations of a common admissible constraint.

Thus admissibility identifies a minimal and necessary condition on comparison itself: any theory admitting well-defined scalar comparison must realize quadratic invariant structure.

1 Foundations

1.1 Admissible structure

Let S be a nonempty set and $\mathcal{T}$ a group acting on S.

Definition 1 (Admissible transformations). *$\mathcal{T}$ consists of all transformations preserving content under:*

1. *relabeling invariance,*
2. *refinement equivalence,*
3. *compositional equivalence,*
4. *finite propagation agreement,*
5. *closure under admissible limits.*

1.2 Structural principles

Principle 1 (Fundamental motion). *All physical content is determined through admissible transformation. Static configurations possess no determinate content; structure is realized only through invariant relations under transformation. Motion is not an attribute of structure but its condition of existence.*

Principle 2 (Retroactive sustainability). *A description is admissible only if it remains well-defined under all admissible extensions, refinements, and recompositions of its propagation.*

Admissibility is therefore global: consistency is not evaluated along a realized history, but across all admissible transformations of that history. Any description that fails under any admissible transformation is excluded entirely and does not instantiate.

Principle 3 (Exclusion of inconsistency). *Physical structure consists only of configurations and comparisons that remain well-defined under admissible transformation. Any description assigning non-unique or representation-dependent values is excluded.*

Remark 1 (Causality as admissible residue). *By Principles 1–3, evolution is not driven by external causal force, nor by any process of selection. Principle 1 identifies physical structure with admissible transformation. Principle 2 requires that admissibility hold under all extensions, refinements, and recompositions of propagation. Principle 3 excludes any configuration or comparison that fails these conditions.*

Thus there is no mechanism by which outcomes are selected or produced. All inconsistent transformations are excluded, and what remains is not chosen, but simply not excluded. Physical structure is therefore the residue of total exclusion. Causality is not fundamental, but appears as the constraint imposed by admissibility on what remains.

1.3 Necessity

Proposition 1. *Denial of any admissibility condition produces representation dependence, refinement dependence, decomposition dependence, non-determinacy, or instability, and is therefore inadmissible.*

Theorem 1 (Necessity of admissibility). *All admissibility constraints are required for well-defined comparison.*

1.4 Intelligibility

Definition 2. *A description D is intelligible iff*

$$D(\tau x) = D(x) \quad \forall x \in S,\ \forall \tau \in \mathcal{T}.$$

Theorem 2. *Meaningful description implies intelligibility.*

1.5 Comparison

Definition 3. *A comparison is a map $Q : S \times S \to \mathbb{R}$ invariant under $\mathcal{T}$.*

2 Non-Admissible Description

2.1 Well-definedness of comparison

Proposition 2. *If admissibility fails, then comparison is not well-defined.*

Proof. A comparison must be invariant under admissible transformation. If invariance fails, then there exist representations x and Tx of the same configuration such that

$$Q(Tx, Ty) \neq Q(x, y).$$

Thus equivalent configurations are assigned distinct values. The comparison is therefore not determined solely by the underlying structure, and is not well-defined. □

2.2 Consequences of denial

Proposition 3. *If relabeling invariance fails, then comparison depends on representation.*

Proof. If relabeling invariance fails, then two equivalent descriptions of the same relational structure yield distinct comparison values. Therefore the comparison varies with representation rather than being determined by relational structure alone. Hence it is not well-defined. □

Proposition 4. *If refinement equivalence fails, then comparison depends on descriptive granularity.*

Proof. If refinement equivalence fails, then admissible subdivisions of a configuration produce distinct comparison values. Thus the comparison varies with the level of description rather than the underlying structure. Hence it is not well-defined. □

Proposition 5. *If compositional equivalence fails, then comparison depends on intermediate decomposition.*

Proof. If compositional equivalence fails, then there exist x, y, z such that distinct admissible decompositions through different intermediate configurations yield distinct values of $Q(x, z)$. Thus the comparison depends on the chosen decomposition rather than the total relation between x and z. Hence it is not well-defined. □

Proposition 6. *If finite propagation agreement fails, then comparison depends on inaccessible structure.*

Proof. If two descriptions agree on all finite propagation-accessible structure but yield distinct comparison values, then the comparison depends on structure that is not accessible to admissible evaluation. Thus it is not determined by physically accessible information. Hence it is not well-defined. □

Proposition 7. *If closure under admissible limits fails, then admissibility is not stable under refinement.*

Proof. If a sequence of admissible refinements converges to a configuration outside the admissible class, then admissibility is not preserved under admissible approximation. Thus comparison fails to stabilize under refinement and is not well-defined. □

2.3 Indeterminacy

Theorem 3. *Failure of admissibility yields non-unique comparison.*

Proof. In each case above, comparison assigns distinct values to equivalent configurations or yields values that vary with representation, refinement, or decomposition. Thus comparison is not uniquely determined by structure. Hence it is not uniquely defined. □

2.4 Necessity

Theorem 4. *Admissibility is necessary for well-defined comparison.*

Proof. Denial of any admissibility condition introduces dependence on representation, refinement, decomposition, or inaccessible structure, or yields non-uniqueness. In each case, comparison is no longer determined solely by admissible structure. Therefore admissibility is necessary for well-defined comparison. □

3 Relational Structure

3.1 Relational difference

Proposition 8. *Any invariant comparison $Q : S \times S \to \mathbb{R}$ is determined by a relational difference*

$$\Delta : S \times S \to \mathcal{D},$$

such that $Q(x, y)$ depends only on $\Delta(x, y)$ for all $x, y \in S$.

Remark 2. *The comparison does not depend on the representations of x and y, but only on their relational difference $\Delta(x, y)$. No structure is assumed on $\mathcal{D}$.*

3.2 Composition

Proposition 9. *Compositional consistency requires that relational differences admit a composition law such that*

$$\Delta(x, z) = \Delta(x, y) \circ \Delta(y, z), \quad \forall x, y, z \in S.$$

Proposition 10. *Consistency of transformation forces the existence of an identity element $e \in \mathcal{D}$, inverses, and associativity of $\circ$. Thus $(\mathcal{D}, \circ)$ is a group.*

Proposition 11. *If there exist $u, v \in \mathcal{D}$ such that $u \circ v \neq v \circ u$, then there exist $x, y, z \in S$ for which distinct admissible decompositions yield distinct values of $\Delta(x, z)$, and hence distinct values of $Q(x, z)$. This violates compositional equivalence. Therefore admissibility forces commutativity.*

Proposition 12 (Linear realization)**.** *Any Abelian relational structure $(\mathcal{D}, \circ)$ admitting a nontrivial real-valued invariant comparison admits a representation in a real vector space V such that*

$$\Delta(x, z) = \Delta(x, y) + \Delta(y, z), \quad \forall x, y, z \in S.$$

Remark 3. *Additivity is not assumed but forced: any invariant scalar comparison on an Abelian relational structure requires a linear realization.*

4 Quadratic Structure

4.1 Constraint

Proposition 13. *Decomposition-independence of comparison under admissible composition requires*

$$f(v+w)+f(v-w)=2f(v)+2f(w), \quad \forall v,w\in V.$$

Remark 4. *This functional equation is the unique condition ensuring invariance of comparison under all admissible decompositions of $v+w$ into v and w, and equivalently under symmetric perturbations.*

4.2 Inevitability

Theorem 5. *Any $f: V \to \mathbb{R}$ satisfying the constraint is quadratic:*

$$f(v)=B(v,v), \quad \forall v\in V,$$

for a unique symmetric bilinear form B.

Remark 5 (Motion and the necessity of quadratic structure)**.** *Configurations are not static objects but admissible transformations. Comparison therefore relates not states, but transformations.*

For $x, y \in S$, admissible comparison depends on their relational difference $\Delta(x,y)$, which composes under admissible concatenation. Decomposition of a transformation into intermediate segments must not alter the comparison value.

Thus admissible comparison must be invariant under all decompositions:

$$\Delta(x,z)=\Delta(x,y)+\Delta(y,z), \quad \forall x,y,z\in S,$$

and the value assigned to $\Delta(x,z)$ must be independent of the chosen decomposition.

Linear functions fail this requirement: under decomposition they introduce dependence on intermediate segmentation and therefore violate refinement or compositional equivalence. More generally, any non-quadratic function fails to remain invariant under admissible decomposition.

The quadratic form is the minimal structure invariant under such decomposition. It uniquely preserves comparison under concatenation and refinement of transformation.

Thus quadratic structure is not assumed, but forced: it is the unique form of any admissible scalar comparison of transformation.

This necessity accounts for the appearance of quadratic invariants in both relativistic and quantum theories, where physically meaningful quantities arise as invariants under admissible decomposition of transformation.

Proposition 14. *No nonzero linear function satisfies the constraint.*

Principle 4 (Exclusion)**.** *Non-admissible comparison laws are undefined and excluded.*

5 Exclusion of Escapes

The functional constraint uniquely selects quadratic forms. We now show that any apparent alternative violates admissibility.

Any admissible comparison must satisfy

$$f(v + w) + f(v - w) = 2f(v) + 2f(w), \quad \forall v, w \in V. \tag{1}$$

Non-invariant comparison. If Q is not invariant under admissible transformations, then equivalent configurations are assigned distinct values. Comparison depends on representation rather than relational structure and is not well-defined. This violates Principle 1.

Refinement-dependent comparison. If Q varies under admissible refinement, then comparison depends on descriptive granularity. Thus it is not determined by underlying structure and fails refinement equivalence and closure (Principles 1 and 2).

Decomposition-dependent comparison. If $Q(x, z)$ depends on intermediate configurations y, then distinct admissible decompositions yield distinct values. Comparison is not determined by the total relation between x and z, violating compositional equivalence (Principle 1).

Non-quadratic forms. Any function not satisfying the functional constraint fails decomposition-independence. Linear functions are excluded by Proposition 14. Any higher-order function introduces dependence on refinement or composition. Thus non-quadratic comparison is non-admissible.

Non-commutative structure. If relational composition is non-commutative, then distinct admissible decompositions yield distinct relational differences and hence distinct comparison values. This violates compositional equivalence (Principle 1).

Unbounded refinement. If arbitrarily fine refinement produces new independent comparison values, then comparison depends on level of description or fails to stabilize under admissible limits. This violates refinement equivalence and closure (Principles 1 and 2).

Proof. By Theorem 5, any admissible comparison satisfies the functional constraint and is therefore quadratic. Conversely, each alternative above introduces dependence on representation, refinement, decomposition, or non-invariant structure. In each case, comparison is not determined solely by admissible structure. Hence no alternative satisfies admissibility. □

Conclusion. Every deviation from quadratic invariant comparison violates at least one admissibility condition. Quadratic form is therefore the unique scalar comparison compatible with admissibility.

Theorem 6 (Uniqueness of Admissible Structure)**.** *Let S be a nonempty set admitting an admissible relational comparison $Q : S \times S \to \mathbb{R}$ satisfying invariance, refinement stability, composition consistency, finite propagation, and closure.*

Then any admissible realization of Q is equivalent, up to admissible transformation, to a quadratic form induced by a symmetric bilinear structure.

Moreover, any two admissible realizations are structurally equivalent.

Corollary 1 (Exclusion of Non-Admissible Structure)**.** *Any relational system not equivalent to this structure violates admissibility and therefore fails to define a well-formed system.*

Remark 6. *This result is not specific to physical theory. It applies to any system admitting well-defined invariant comparison. Apparent alternatives either reduce to this structure under admissible transformation or fail to instantiate as coherent systems.*

6 Physical Realization

Remark 7 (Configurations as transformation)**.** *Configurations $x \in S$ are not static states but admissible transformations. A particle is identified with its worldline or admissible trajectory. Relational difference $\Delta(x, y)$ therefore compares transformations, and composition corresponds to their concatenation.*

Remark 8 (Scope of admissible comparison)**.** *Admissible comparison concerns invariant scalar quantities. In quantum theory, noncommutativity arises at the level of operators, while admissible comparison applies to scalar quantities such as expectation values and probabilities, which admit commutative composition. The derivation does not require continuity: discrete systems admit the same quadratic scalar structure at the level of comparison.*

Proposition 15. *$\Delta(x, y)$ represents invariant physical separation for all $x, y \in S$.*

Proposition 16. *Quadratic comparison induces metric structure:*

$$Q(x, y) \mapsto ds^2 = g_{\mu\nu} dx^\mu dx^\nu.$$

Remark 9. *Differentiable structure is required only for coordinate realization.*

Connection to established theories

Any admissible comparison takes the form

$$Q(x, y) = B(\Delta(x, y), \Delta(x, y)), \quad \forall x, y \in S, \tag{2}$$

for a symmetric bilinear form B.

This is the minimal invariant scalar structure appearing in existing physical theories.

In differential geometry, invariant comparison of nearby configurations is given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{3}$$

the spacetime interval [1].

In quantum theory, physically meaningful quantities such as probabilities and expectation values are quadratic in the state representation [2].

Thus both general relativity and quantum theory realize the same structural necessity: their physically meaningful scalar quantities are quadratic invariants.

This correspondence is not assumed. It follows from admissibility. The present framework therefore derives the common scalar structure underlying both theories. Subsequent development of their full dynamical and geometric structure proceeds within these established frameworks.

Assumptions and Scope

The results apply to any nonempty set S and relational difference map

$$\Delta : S \times S \to D$$

admitting a nontrivial admissible scalar comparison Q.

Admissibility is defined strictly as invariance under relabeling, refinement, composition, finite propagation, and closure. Any structure failing these conditions yields representation dependence, non-uniqueness, or instability, and is excluded.

Degenerate cases—where no nontrivial invariant comparison exists—are vacuous and contain no admissible relational content.

Closure under admissible limits is defined structurally: sequences of admissible refinements stabilize within the admissible class. In discrete systems, this stabilization is combinatorial; in continuous systems, it corresponds to convergence within admissible equivalence classes. No external topology is assumed.

Any Abelian relational structure admitting a nontrivial invariant scalar comparison necessarily admits a linear realization in a real vector space. This realization is not an additional assumption, but a consequence of invariant scalar definability.

Within this domain, admissible comparison is fully constrained. The functional condition imposed by compositional invariance uniquely determines quadratic form. Thus any admissible scalar comparison must take the form

$$Q(x, y) = B(\Delta(x, y), \Delta(x, y)),$$

for a symmetric bilinear form B.

Any alternative either fails admissibility or reduces to the quadratic case under admissible equivalence. Quantities not admitting such representation are excluded by Principle 4 and do not correspond to physically meaningful comparison.

7 Conclusion

Admissibility was defined as invariance under relabeling, refinement, composition, finite propagation, and closure. Violation of these conditions yields representation dependence, non-uniqueness, or instability, rendering comparison ill-defined.

From these constraints, relational difference and compositional structure were forced, yielding an Abelian realization and a functional constraint on admissible comparison. This constraint uniquely selects quadratic form.

All alternatives were shown to violate admissibility. Quadratic structure is therefore the unique scalar comparison compatible with well-defined description.

This structure appears in existing physical theories as the spacetime interval in differential geometry and as scalar quantities in quantum theory. These are not independent assumptions, but realizations of a common admissible constraint.

The result is model-independent: any theory admitting well-defined scalar comparison must realize quadratic invariant structure. The present framework therefore identifies a minimal structural condition from which the scalar core of both general relativity and quantum theory follows.

References

[1] A. Einstein, *The Foundation of the General Theory of Relativity*, Annalen der Physik, 49 (1916), 769–822.

[2] P. A. M. Dirac, *The Principles of Quantum Mechanics*, Oxford University Press, 1930.