THE STATISTICAL FOUNDATIONS OF QUANTUM MECHANICS I

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ABSTRACT

Arguing statistical foundations of quantum theory and showing the way it naturally resolves all quantum "mysterious paradoxes". In this context the great deal of attention is given to principles of quantum measurements. To facilitate a better appreciation of quantum mechanical paradigms we provide a brief essay of a quantum field theory and its natural evolution to nonrelativistic quantum mechanics, and then, to classical physics. The presentation aims at both physics students and young scientists, as well as seasoned researchers who will find the discussion of lots of interesting points. The first half contains basics of quantum mechanics and the discussion of main quantum paradoxes

Key words: quantum mechanics, quantum field theory, de Broglie hypothesis, Schrödinger equation, wave function, statistical nature of wave function, principle of superposition, Schrödinger cat, wave-particle duality, collapse of wave function, Einstein "spooky" action at the distance, quantum non-locality, direct and indirect quantum measurements, interference and coherency, quantum dynamics of highly excited atoms (Rydberg atoms)

To the memory of M. Born and D. Blokhintsev

Preface

Since its very creation the quantum mechanics serves physics well – without any single exception! – for about of more than 100 years. However, it does so within the logic profoundly different from the one of the classical physics, that is, classical mechanics. Meanwhile, human cognition is based - to a good part - on analogies, or, stereotyping, for that matter: that is why even nowadays the quantum mechanics presents a serious challenge to a traditional physics intuition as the latter being shaped up entirely by the macroscopic world. There is no wonder then that the dawn of quantum mechanics saw a lot of paradoxes emerging via an explicit - or more so, inexplicit - usage of a macroscopic intuition in the world of microparticles. What is more surprising is that these kind of misconceptions did survive and are still quite popular despite the detailed analyses and discussions over the past 100 years. Suffices it to name just a few typical and glaring problems: the collapse of wave functions, the wave-particle duality, the spooky action at the distance (the proverbial non-locality of quantum mechanics). In addition, the mustached and striped Schrödinger Cat is well alive and still kicking in numerous publications: popular and scientific alike.

Examples of the genesis of problems arising this way are right at hand: even the fundamental principle of a superposition exhibits the whole variety of quite exotic interpretations, the leading of which - a simultaneous time-wise occurrence in superposition states - blatantly misses the statistical nature of the quantum mechanics. It is this misconception that promoted the prosperity of the legendary Schrodinger Cat, and, incidentally, resulted in conceptual challenges to quantum computing (meanwhile, even a classical diffusion clearly invalidates such an interpretation and resolves the issue trivially). Mentioned in this context as well should be "the spooky action at the distance" of Einstein, which is quite popular in terms of the quantum entanglement and the so-called quantum non-locality. We also note the "collapse" of wave function viewed as a material substance, letting alone the wave-particle duality as a plain cliché of the counter-intuitive nature of the quantum theory.

In this regard, there are following three leading motives and objectives of this work: first, to bring together under one umbrella the main "paradoxes" of 20th century quantum mechanics and explain them from a unified perspective via the simplest language possible.
Further, the second objective was to illustrate the structure of quantum mechanics as arising from the excitation of quantized vacuum fields, on one hand, and on the other -
as a passing point in the transformation to classical fields and particles in the corresponding limits.
Finally, for physics as an experimental science, the key criterion is a properly designed experiment, especially in the microworld. Therefore, a significant attention is paid to quantum measurements. In doing so, a central role is given to the now classic statistical interpretation of quantum mechanics and the reinterpretation of its key attributes on this basis of probability amplitudes / wave functions and quantum ensembles - as main manifestations of a stochastic “behavior” of microparticles, as well as the natural and direct connection of the latter with quantum field theory, on one hand, and with classical fields and macroscopic objects, on the other.
And even though the exposition style is quite loose and informal, a familiarity with at least basics of classical mechanics / classical statistical mechanics, quantum mechanics, and quantum field theory would be rather helpful.

Two comments on the work substance are in order.
First of all, the key part of the text is “talks”, not calculations. While this is unusual for the text on quantum mechanics, there is a clear reason for that. Namely, typical textbooks on quantum mechanics focus on quantum math workings, and rarely pay attention to what all this machinery is about: i.e., what is the common sense of the results and how to verify the latter experimentally.
And secondly, a clear understanding of the quantum theory foundations and its consistency is exceptionally important for modern science as a whole. Therefore the exposition revisits various important points repeatedly and at different angles, what inevitably results in duplicates and overlaps (this is especially so for the introduction vs the main text, but is the part of a main idea). When editing those are left untouched in a spirit of the key didactic principle: practice makes perfect. Thus, this text is not a polished mathematical treatise, but rather a guide for a better understanding of quantum theory.
In addition, the work contains several didactically and historically instructive asides - their text is written in italics and always ends up with symbol ♦.

It should also be emphasized that the presented material does not aim to provide a coherent and consistent exposition of all relevant issues: this would make the text unacceptably cumbersome and significantly hinder the understanding. Rather, each part is organized as a set of relatively independent essays (which can be read selectively and in different sequence), but which are nevertheless connected by a common thematic thread.

Across the parts, chapters and sections the material is distributed as follows.
The introduction provides a concise overview of the work content: the main ideas of quantum mechanics and quantum field theory, as well as the essence of quantum paradoxes, the fundamentals of the interaction of quantum systems with classical ones in quantum measurements and in highly excited states transitions. Structurally, the introduction is independent of the main text and, if desired, the latter can be omitted for the first reading.

Part I.
Section 1 traces the statistical motives in quantum theory, namely: the emergence of the Schrödinger equation and its time-oscillating solutions from classical diffusion via a transition to imaginary time (Wick rotation), and conversely, the transformation of the Schrodinger equation into the diffusion

equation with an imaginary coefficient in the evolution of highly excited states population at the boundary with the continuum (what is the exact result in the limit n -> ∞). Also discussed is the Ehrenfest extension of Newton equations to quantum mechanics.

Section 2 discusses the very principle of a superposition and its implementation in experiments of different geometries.

Section 3 discusses main quantum paradoxes: the wave function collapse and the wave-particle duality. The main focus is on the nearly 100 year old legend of wave-particle duality and its connection to various aspects of quantum mechanics in the context of its statistical interpretation. Section 4 concentrates on conservation laws, long-range correlations, and the paradox of "spooky action at a distance". We clarify the widespread misconception that has gained attention in recent decades regarding the notorious "non-locality" of quantum mechanics and Einstein's proverbial "spooky action at a distance."

Part II.
It is hard to explain the structure of quantum mechanics without at least a brief overview of the quantum field theory ideas. Therefore, Part II outlines these fundamental principles and the evolution of quantized field mechanics to wave mechanics of particle-quanta, which, subsequently , is furthering into mechanics of classical fields and classical mechanics of macroscopic objects.

Part III.
Part III outlines the fundamentals of the theory of interaction between quantum and classical objects. Section 1 contains basics of the quantum measurement theory, primarily based on the works of L.I. Mandelshtam and D.I. Blokhintsev.

Supplement deals with collisional transitions in Rydbergs from the perspective, in particular, of an imaginary diffusion.

Some results of this work were partially published in 2024 in "Quantum Fundamentals and Heuristics" at viXra:2410.0187.

As the year of 2026 marks the 100s anniversary of quantum mechanics this work is dedicated to the memory of two outstanding scholars: Max Born and Dmitry Blokhintsev, who created and developed the statistical understanding of quantum mechanics and its place in the structure of the Universe. As scientists, Born and Blokhintsev belong to different generations and social systems, but their immense contributions are genetically very much connected.

Born's role in the context of his encyclopedic influence on modern physics as the head of the Göttingen group and one of the founding fathers of quantum mechanics is well known and has been addressed may times. But his statistical postulate (which, incidentally, is often underestimated) is, in fact, the brilliant achievement of the human mind, and placing Born on par with the greatest thinkers of humanity.
D.I. Blokhintsev – besides his well-known scientific and social accomplishments – all his life long has been tirelessly explaining the role and significance of the statistical nature of quantum theory, diligently analyzing the misconceptions, underlying quantum paradoxes, thus making invaluable contributions to the statistical understanding of quantum mechanics in general, and to the theory of quantum measurements, in particular. It is to his inspiration, uniquely clarifying writings and pedagogical talent that we owe our understanding of modern quantum theory.
In particular, essential parts of his classic works (Blokhintsev, 1976, 1987, 1988) – and this is the author firm conviction – is a must for any course of quantum mechanics for physics majors, if we wish to ensure a professional mastery of quantum theory and avoid vexing and impeding “paradoxes” in the future.

It is also a great pleasure to thank L.D. Blokhintsev, I.D. Blokhintsev, V.G. Gantsevich, A.E. Khrennikov, for helpful comments, discussions and cooperation during various stages of this work in recent years. In addition, L.D. Blokhintsev read the whole text and made many useful corrections and suggestions, which contributed to an improvement of the work.
I would also like to mention with gratitude the late Moscow State University Professor D.A. Slavnov who worked a lot in recent years on foundations of quantum mechanics.

Introduction: Key ideas and main results

> *"I must take a stand with*
> *reference to the most successful*
> *physical theory of our period, viz.*
> *the statistical quantum theory"*
> ***A. Einstein***

Section 1. The bird's eye – view of particles and fields.

The purpose of this part of the introduction - the title of which is borrowed from the remarkable book of the 1960s "A guide to Feynman Diagrams in the Many-Body Problem" by R. Mattuck (Mattuck, 1965) - to outline the modern view of quantum mechanics as an intermediate step between vacuum quantized fields and classical macroscopic physics. Accordingly, it can (and should!) be read independently of the rest of the text.
However, if there are still lots of ambiguities after the initial reading, please, don't be alarmed: the author himself is not fully clear of everything, and, therefore,
revisit the introduction after the first glance at the main text.
It should be emphasized in the first place that the consistent introduction of quantum-mechanical concepts and quantities, such as, for instance, the wave function, is quite hard without a connection with quantized fields.
It is no coincidence, then, that the de Broglie wave has become a "magical" object of an unclear nature that fell off "out of nowhere", and the naïve wave-particle duality has at all gained the status of a mysterious and completely incomprehensible symbol of the new physics.
It should be noted that the founding fathers themselves were not always crystal clear of their interpretations of the new mechanics. For instance, it was Schrödinger himself, who, as if in passing, in one of his groundbreaking articles, launched the now wide spread legend of an immortal cat named after him. And the "great and terrible" Wolfgang Pauli, in his brilliant and famous at the time book "General Principles of Wave Mechanics" (Pauli, 1980, p. 33), went so far as to openly declare that the complex-valued wave function (of free particles) is only symbolic in nature, but does not carry an independent meaning, just enacting the mapping of real probabilities from the coordinate to the momentum representations and vice versa. It is no wonder then, that there emerged many other paradoxes, that have gained traction till nowadays.
And so, any concurrent student is familiar with the concept of a classical field (for instance, electrical, magnetic, gravitational, etc.) as a substance that exists everywhere and is capable to change continuously by the arbitrarily small value. In this general context, a field is understood as "something" that exists "everywhere," i.e. at all points in space.

Accordingly, in the “pedestrian” parlance, the concept of a quantized field corresponds to a fundamental substance, that 1) is omnipresent everywhere in its lowest state, hence the name – field, and, 2) is getting excited via finite energy portions (energy quanta) localized in space.

Mathematically, the quantized field corresponds to an operator-valued function, creating and destroying field quanta at any point of physical space.[1] It is precisely these features of quanta that afford to identify them with elementary particles.

In other words and more detail, the mechanism of a quantized field excitation looks like as follows. To begin with, the excitation of this vacuum substrate occurs in a discrete manner (which permits to think of it in terms of a harmonic oscillator - Polyakov, 1999), and, secondly, the spatial extent of the emerging object is assumed limited, which, at least in principle, is suitable as a model for the emergence of elementary particles as field quanta.

And then, the accumulation of quanta leads to an emergence of “classical fields” - a macroscopic imprint of the quantized field, that simultaneously explains why quantum particles are described by complex amplitudes, which are traditionally used as a field description.

Putting it loosely, the quantized fields may be thought of as fields only in a limited sense. That is, it is assumed that quantized fields exist as a functionally all-pervading and continuous substance, but only for energies below a certain threshold, known as a vacuum energy: this is an assumption (and it is indeed a hypothesis), as vacuum fields are not observed directly, and their existence is inferred only from indirect manifestations.

Above that same threshold, quantum (quantized) fields, firstly, materialize (“coagulate”) into clearly localized objects – quanta (quantum microparticles, i.e. elementary particles), and secondly, the energy of the latter, changes in a discrete (“quantized”) manner.

And along the way, the dynamics of quanta is random, in a sense – even “more random”, than for classical statistical mechanics and classical random process (we’ll have more to say about that down below).

In the meantime, a popular analogy likens the quantized fields right below the vacuum boundary to a boiling liquid where the bursting on a surface steam bubbles correspond to continuously appearing and disappearing virtual electron-positron pairs and other fluctuation formations.

This way there emerges the picture of a vacuum excitation via the continuous process of a chain of direct and reverse phase transitions between a spatially “distributed” quantized field substance and relatively localized elementary quantum particles.

And those quanta that have left the vacuum forever remain extremely sensitive to the impact of the latter, and, thus, move according to the Quantum Mechanics, that was discovered 100 years ago. We owe this latter - without any pathetic exaggeration - to the enduring achievement of the founding fathers, the giants of the 20th century - Planck, Einstein, Bohr, de Broglie, Born, Heisenberg, Jordan, Schrödinger, Pauli, Dirac, von Neumann, and their no less talented followers: Landau, Feynman, Bohm, Anderson, Gell-Mann, Wu, Lee and Yang, Higgs, and so many others – just to mention all the remarkable scientists would take the whole new separate book.

These names are the anthem to the spirit, mind, and talent of the greatest of mortals who paved the way for a humanity into an eye-opening future.

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[1] According to Dirac's terminology, that operator-valued function is a q – number, which creates quanta of a certain field. As a result we get a complex-valued distribution of these quanta, which is known as a Schrödinger wave function, and which is expressed in terms of “ordinary” c- numbers.

And now, after this emotional outburst, let's get back to the matter at hand and summarize the picture presented.

The genesis of the universe around us, therefore, in the most general terms, appears as follows. On its way from the vacuum field at the dawn of time to the diverse agglomerations of the macroscopic world, Nature mixes corpuscular and wave aspects in so-called quantum statistical ensembles: the initial excitation of the mainly unobservable vacuum field occurs through particle-like portions of energy (quanta), which, in turn, form statistical ensembles and demonstrate wave manifestations using coherent generalized distributions, i.e. wave functions.

It is precisely this combination of individual and collective properties of microobjects that should be understood as wave-particle duality, that is, the wave-particle duality should be perceived as two sides of one and the same reality, but at different levels: at the individual level the Nature is particle-like and random, but randomness accumulates and organizes itself into wave fields.

Let us emphasize one more time: the wave-particle duality - if to use this terminology at all – is the duality of not microparticles, but of microevents. Individual events - such as, for instance, imprints on the photoplate – appear unambiguously discrete in energy and localized in space, i.e. exhibit the corpuscular character, but their groups / collectives are organized as wave structures. The naïve duality, however, confines both of these mutually exclusive aspects to the framework of particles only, and this is that creates a problem.

It is also important to understand the following. An ensemble of quanta is a very specific formation, dramatically different from that of classical particles. Without going into technical details here, it can be roughly visualized as a jelly cloud in the space of random realizations of a quantum system, possessing a remarkable plasticity.

Namely, when quantum particles come into contact with classical measuring instruments, there emerge “condensations” in the cloud of realizations, which are then organized into structures that form complete sets. These structures, which are easily recognizable as Hamiltonian eigen functions, depend on the type of measurements and correspond to classical measuring instruments. For instance, for measurements of a linear momentum, these eigen functions are the de Broglie plane waves, while for an angular momentum - they are spherical functions and the Legendre polynomials. Subsequently, quanta (say, massless ones) accumulate to form classical fields (for example, electromagnetic ones), and the wave functions of massive quanta-particles roll up into tubes around the trajectories of classical objects. Some additional details of this picture will be illustrated and discussed down below. The mathematical theory of such structures is given in the functional analysis of Hilbert spaces, named after their inventor and promoter, the outstanding mathematician D. Hilbert from Göttingen. The applications of that analysis to the quantum theory was developed about thirty years later by another genius – P. Dirac, in his immortal masterpiece “The principles of quantum mechanics”. More details about the key technical aspects of quantum ensembles will be given down below, in the context of the wave-particle duality and quantum measurements.

The author is quite cognizant of the difficulties of perceiving these concepts by the classical mind. Here, we have a complete justification of the famous Landau dictum: the theoretical physics has learned to understand what a classical mind cannot even imagine.

Section 2. A brief preliminary overview of quantum paradoxes

This work discusses five basic myths-paradoxes of quantum theory.
The **first** – and foremost: the wave particle duality - was briefly touched upon just above, therefore here we will make just a few, mostly historical, remarks, to revisit the issue later, in the main text.
In 1923, about 100 years back, Louie de Broglie, reversing the trail, initially blazed by Planck and Einstein for massless photons, hypothesized a certain "wave-like" structure for massive microparticles. More precisely, de Broglie suggested that the combination of wave and corpuscular aspects for light (massless quanta - photons) is, in fact, also characteristic of massive particles, in particular, electrons. As he did not clarify more specifically what exactly this combination was, there emerged an implicit identification of a plane wave with literally each individual electron, and not with an ensemble of them. (It is precisely the way that the naive understanding of wave-particle duality was born, the latter being destined to become the main conceptual difficulty of quantum theory). Shortly after, by efforts of the Göttingen group (Born, Heisenberg, Jordan – matrix mechanics) and Schrödinger (wave mechanics), the Wave/ Quantum mechanics was created and began its triumphant journey into all aspects of our life. It should be noted here that the introduction of mutually exclusive corpuscular and wave properties under the umbrella of a single object - in the spirit of Sir A. Eddington's "wavicles" in 1928's "The Nature of the Physical World" (Eddington, 1928) - was the main inconsistency of the early quantum theory, which, however, has survived to these days. The obvious intuitive discomfort of the naïve wave-particle duality arose from the apparent connotation between the distributed nature of a wave and spatially localized character of a particle / corpuscle. Not the least, it became a reason that Quantum Mechanics gained a reputation of the most counter-intuitive and confusing theory of our time.
The initial formulation of it – with slight variations here and there – declared that, depending on circumstances, microparticles exhibited either corpuscular or wave-like behavior, even though no one knew what a wave-like behavior of a one sinlge particle was supposed to mean. More specifically, even if the diffraction manifestations (mainly the detection of particle traces on the screen in the area of the geometric shadow) do not yet lead to an immediate contradiction, then the interference of spatially localized objects/corpuscles is very hard to imagine. Under the pressure of such obvious difficulties, the formulation of duality began to evolve, for example, to that: quantum particles are neither waves nor classical particles, they are something else. This view is found even in the classic work of D. Bohm (Bohm, 1951), from where it has migrated to many other works and textbooks. Granted, this already is certainly better than just a naïve duality, and has at least an approximate heuristic meaning in the framework of the semiclassical approximation, however, 1) still, is based solely on the appeal to a mysterious "something else" that encloses mutually exclusive corpuscular and wave properties in one single"package", and, most importantly, 2) does not clarify yet, how this "something" leads to experimentally observed localized imprints, and, consisting of them, the interference patterns. Meanwhile, the issue resolves quite trivially by the concept of individual vs collective manifestations, depending on experimental conditions, which has already been mentioned above and will be discussed in more detail later. Let us emphasize one more time: the naïve wave-particle duality, i.e. enclosing of both mutually exclusive features in the one single entity (something "neither a particle nor a wave"), is fundamentally wrong, but as a rough approximation - this approximation is known as the semiclassical approach - can be used in the transitional zone between the macro- and microworlds. This approximation is applicable when the situation,

although no longer entirely classical, is not yet fully quantum. In this case, the well-localized particles spread out slightly, preserving the basic classical core (which is enclosed in tubes around the classical trajectories), and acquire weak but infinitely extending tails, permitting to think about the interference of particles rather than amplitudes. It is in this case, that the erroneous, but conventionally established terminology applies, and each microparticle has not only corpuscular, but also wave properties. And, in general, everything semiclassical is applicable, including the famous interference of a photon with itself in Dirac's "Principles".

The **second** and logically next paradox, but one that has not received its own name, is the interpretation of the superposition principle in quantum mechanics. The interpretation of superposition in quantum mechanics, in fact, reflects the statistical nature of wave mechanics. Ignoring this most important, but essentially elementary, fact, leads to many misconceptions.

The **third** paradox, that directly follows from the misinterpretation of the superposition principle - is the Schrödinger's Cat paradox - was, ironically, launched, as if in passing, by the founding father himself, Schrödinger, in one of his seminal papers as "a very strange possibility". It is based on the just mentioned incorrect interpretation of superposition in mathematical statistics in general, and the quantum principle of superposition - in particular. The longevity of this myth is due not so much to our affection for furry friends, but rather to the elementary inertia of human thinking.

The **fourth** paradox was the notorious "collapse" of the wave function, which stems from ignoring the probabilistic nature of the wave function, and was often attributed (although not entirely true) to the reputation of the great N. Bohr and his Copenhagen school. And even though the “Copenhagen interpretation” was never formulated as an “official” view, its impact was so profound, that even in 1990s quite reputable scientific journals were publishing papers of renown scholars who modeled the statistics of wave function “collapses” (for instance, according to the Fokker-Planck diffusion).

The **fifth** addition to the glorious gallery of alleged quantum paradoxes became the Einstein so-called "quantum nonlocality" or "mysterious action at a distance." Historically, this myth emerged from the influential paper of Einstein, Podolsky, and Rosen (EPR) in 1935 (Einstein et al, 1935), and gained a particular resonance and popularity in connection with the Bell hidden variable hypothesis and the role of quantum entanglement in the development of quantum computers.

And even though all of the above mysteries were adequately resolved away and long gone, the story of quantum paradoxes is extremely instructive in showing how persistent the stereotypes and prejudices can be, if not addressed timely and exhaustively. This work outgrew from the realization of this very fact.
In fairness to current physics students, the bulk of the extant quantum literature and its wisdom is so huge and formidable, that there is no chance to follow it and digest in a reasonable and timely manner by new entrants.
And yet, the harm inflicted by above paradoxes should not be taken lightly and /or underestimated. They confused and baffled generations of scholars in their student years, and had the disarming impact on a confident reasoning in Quantum Mechanics and

related sciences. It is then highly desirable to produce periodic reviews of existing and emerging “paradoxes” and promote transparent logical standards and sane judgments.

Section 3. The theory of quantum measurements.

The classical dynamic variables can be ascribed to quantum microparticles only after the latter contact the classical objects. Thus, by means of conservation laws, quantum measurements project quantum objects into the world of classical instruments. Thus, quantum measurements, through conservation laws, project objects of the quantum world into the world of classical instruments. This is not the creation of an object, as Jordan's famous statement is often and incorrectly interpreted, but precisely the attribution of classical parameters to quantum objects, where the former do not exhaust in principle the natural representation of the latter: suffices it to recall the noncommutativity of coordinates and linear momenta. Therefore, with regard to quantum particles/quanta, classical dynamic variables should be used with a certain amount of caution, inasmuch as the latter potentially hinder the undistorted perception of quantum objects. The understanding of this is of an especial importance.

In this regard, the superposition principle clearly hints the strategy for quantum measurements: it is necessary to suppress interference in the measured ensemble/beam, as well as the cross terms in the intensity of the scattered beam. Typically it is accomplished by a spatial separation of beams, corresponding to diagonal terms in the density matrix, i.e suppressing their spatial overlap. For ease of understanding and simplicity of an exposition, a general overview and characteristics are preambulated first and then the main approaches are briefly outlined, and only after that the latter are illustrated with specific examples having exceptional general and methodological significance. For all further details we refer the reader to a seminal book of Bohm (Bohm, 1951), classical works of Blokhintsev (Blokhintsev, 1964, 1968, 1988), as well as to the groundbreaking paper of London and Bauer (London, Bauer, 1932, 1982). This section is of an especial interest for experimentalists.

Supplement. This supplement shows that the dynamics of semiclassical multilevel systems is, in fact, very close to a “diffusion” over energy levels with an imaginary coefficient, i.e., again, a diffusion with an imaginary coefficient is the key mathematical aspect of quantum mechanics. Besides, there exists the direct analytical tractability of multilevel semiclassical systems, which is of an especial convenience for various practical applications.

Part 1. Quantum mechanical foundations and its main proverbial “paradoxes”.

Section 1. The statistical framework of quantum mechanics

> *“So if one asks what is the main feature of quantum mechanics, I feel inclined now to say that it is not noncommutative algebra. It is the existence of probability amplitudes which underly all atomic processes”*
>
> ***P. Dirac***

Strictly speaking, the Schrödinger equation cannot be derived from general classical principles. It is a brilliant conjecture, stemming from the no less brilliant hypothesis of Lois de Broglie in 1923. Namely, just about little more than 100 years ago, reversing the Plank and Einstein conjecture of the "quantum"/ portionwise nature of the electromagnetic radiation, de Broglie postulated the existence of a certain wave structure, corresponding in some manner to particles of the non zero rest mass (like electrons, for instance). That wave – the so called de Broglie wave - reads as $\exp[(i(kx-\omega t)]$, where the wave vector $k = p/\hbar$, and $\hbar$ is a famous Plank's constant, symbolizing a fundamentally new physical meaning, and having the dimension of classical action. (It became clear after the groundbreaking work of Schrödinger that the de Broglie wave was nothing but the wave function of particles with certain linear momentum $p = \hbar k$ in the coordinate representation). Suffices it now to differentiate the de Broglie wave over time and twice - over the spatial coordinates, to obtain (with the help of non-relativistic energy – momentum relation $E = p^2/2m$) what is known since 1926 as the Schrödinger equation (see below).

In a more general sense, a conceptual answer is that the Schrödinger equation represents a new physical meaning - manifested by the Planck constant – and therefore, does not stem from the purely classical Hamilton – Jacoby equations without fundamentally non-classical ideas / assumptions. On the contrary, the classical mechanics readily follows from the quantum in the limit of $\hbar \to 0$, i.e. the classical mechanics is, in principle, is derivable from the quantum, but not vice versa. Yet, there exists many ways to argue ("derive") the Schrödinger equation via more or less plausible classical considerations.

One of those ways, shedding light onto conceptual nature of the Schrödinger equation, will be given below. Namely, the Schrödinger equation is a linear parabolic PDE, of the same type as the equation of classical diffusion (or, for that matter, the Langevin equation), providing the distribution of particles, travelling under the influence of random forces (i.e. Brownian motion). In this regard, it has been long noticed, that the analytic extension t ->it (the so called Wick's rotation) and the replacement of the coefficient of classical diffusion by a suitable quantum expression, leads exactly to the Schrödinger equation. Speaking somewhat loosely, this manifests a specific "diffusive" nature of quantum mechanics and the Schrödinger equation.

Technically, Wick's rotation approach, outlined above, works as follows. Namely, replacing t -> it in the classical diffusion equation

$$C'_t(x,t) = D_{cl}C''_{xx}(x,t) \qquad (I.1.1)$$

where D_{cl} – the classical diffusion coefficient, and setting the quantum diffusion coefficient $D = D_Q = \hbar/(2m)$, where m – the mass of a microparticle, we obtain the Schrödinger equation for a beam of free particles:

$$\Psi'_t(x,t) = i\, D_Q\Psi''_{xx}(x,t) \ \text{ or } \ i\hbar\, \Psi'_t(x,t) = -\,[\hbar^2/(2m)]\, \Psi''_{xx}(x,t) \qquad (I.1.2)$$

(Please, note, that for infinitesimally small $\hbar$ and, respectfully, very large masses, the quantum diffusion vanishes, as it is supposed to be for consistency). And accordingly, the probability of classical diffusion (i.e. the classical Green function as the solution of the equation (I.1.1.) for $C(x.0) = \delta(x)$)

$$G_{cl}(y, t; x ,0) = (4\pi Dt)^{1/2}\exp[-(y-x)^2/ (4Dt)] \qquad (I.1.3)$$

becomes a complex probability amplitude (quantum Green function – the solution of the equation (I.1.2) for Ψ (x, 0) = δ(x))

$$G_Q(y, t; x ,0) = (4\pi D_Q it)^{1/2}\exp[i(y-x)^2/ (4D_Q t)] \qquad (I.1.4)$$

In the same vein, the classical composition law for probabilities P(y, t' ← x, t) = ∫ P(y, t' ← z, T) dz P(z, T ← x, t) changes to composition of quantum probability amplitudes (the Green function G is a synonym for the Feynman amplitude K) G(y, t' x, t) = ∫ G(y, t' ← z, T) dz G(z, T ← x, t). Also, the replacement t -> it converts the Wiener real path integral measure into the complex-valued Feynman path integral measure (for more technicalities on this, please, see e.g. Roepstorf, 1994 and Nagasawa, 2000).

This short synopsis clearly hints at the stochastic nature of quantum mechanics and indicates that a wave function plays the role of a probability amplitude distribution in an ensemble of quantum realizations, simply in a quantum ensemble (von Neumann, 1932; Blokhintsev, 1964, 1968; Ballentine, 1970).

This is an unequivocal indication of the close connection between the randomness in the classical diffusion with the quantum randomness in the microworld. And this connection is still awaits for a proper understanding. In other words, it looks quite natural to perceive the quantum randomness as a some kind of the "diffusion" induced by certain specific factors. What are those factors more specifically? Conceptually and most likely - as it follows from the developments in quantum field theory – they result from an impact of quantum vacuum, unobservable classically and containing no quanta. We, therefore, come to a fundamental hypothesis: all peculiarities of quantum mechanics, and the Schrödinger equation in particular, is a manifestation of the lowest quantum state impact on quantal behavior. Quite fine and famous effects of that type are the well known Lamb shift, the Casimir-Lifshitz force, etc.[1]

The arising this way viewpoint - that the path to quantum mechanics most consistent logically and simple technically and emphasizing its statistical nature - stems from the certain analogy with classical diffusion, the latter analytically extended to the domain of imaginary time and producing the "diffusive" equation for elementary quanta – the Schrödinger equation.

Recapping the foregoing in a little more detail: according to the Born stochastic interpretation of wave functions, a randomness of a quantum motion (i.e. stochastic

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[1] An interaction of field quanta with vacuum should not be taken as some kind of an artificial exotics unknown so far and brought up only in the context of the picture above. For instance, in the traditional renormalization theory the observed mass of a particle (e.g., an electron) results exactly from an interaction of the bare "core"with vacuum via "dressing" of the core by a fur of virtual particle-antiparticle pairs (so called Feynman polarization loop diagram).

diffusion of a quantum particle) results from an impact of the quantum vacuum. A short reminder: quantum vacuum – the lowest energy state of any system, i.e. without any particles / field quanta, underlies the state of any quantum system, yet, having an infinite supply of energy, and does have experimental manifestations, mentioned just above. Quantum vacuum appears in this context as a global thermostat / heat bath, acting uniformly on all members of a quantum ensemble / the beam of particles, and forming thereby the "coherent" – and fully deterministic - wave function of an amplitude distribution, coming from the deterministic Schrödinger equation. It is in this capacity, that vacuum serves as a common source of a wavelike intensity distribution and thereby alleviating the need for the naïve wave-particle duality.

Now, according to the above, the de Broglie wave is nothing but the wave function (probability amplitude) of a free particle with a linear momentum p in the q representation. It is identical to the Feynman amplitude (Feynman, Hibbs, 1965) $\exp[(i/\hbar)S(a,b)] \sim \exp(i x/ƛ)$, where S(a,b) – is the classical action with $x = a - b$, and the reduced de Broglie wavelength $ƛ = \hbar/p$. In this context the de Broglie wavelength ƛ is the scale of oscillations of the probability amplitudes, i.e. the range, where wave aspects of quantum probability amplitudes are material. The well known illustrations include classic examples from wave optics: intensity oscillations near the edge of a semi-infinite screen, the diffraction on single and double slits, and so on (more on this - in the next Sec. 2).

In addition, quite convincing arguments in favor of the diffusive nature of quantum mechanics and the Schrödinger equation ensue from the Feynman formulation of quantum mechanics (the framework of path integrals). Namely, according to the probabilistic Central Limit Theorem (CLT), the sum of many real random variables approaches the Gaussian distribution, the latter, in turn, solving the corresponding equation of a classical diffusion. If now, the following changes are made, 1) take as a random variable the classical action between initial and final point, 2) and add up not actions themselves, but rather their complex exponents $\exp[(i(S/\hbar)]$, then the sum would approach a complex Gaussian distribution, solving the diffusion equation with a complex coefficient of diffusion, or, equivalently, with an imaginary time. That latter equation is nothing but the Schrödinger equation for the wave function, which thereby becomes a sum / integral over action exponents (complex exponents, that is) on random paths from an initial to final point. In other words, the Feynman path integral, solving the Schrödinger equation, implements a "diffusive" dynamics of microparticles resulting from an impact of quantum vacuum. Such an approach clarifies the many peculiarities of quantum theory, namely: 1) the absence of deterministic trajectories of quanta (Heisenberg uncertainty principle), 2) complex probability amplitudes (instead of real probabilities themselves) and their obvious connection with the quantum field theory, 3) the correspondence of density matrix to the Gibbs distribution in the classical statistical mechanics, and so on.

For those readers who are still disturbed by the imaginary time approach, the following remarks are in order.

Parabolic PDEs may display different time-wise behavior: either exponential, or, the oscillating one - depending on the nature of the diffusion coefficient. While the former corresponds to a real diffusion (as in a classical Brownian motion), the latter corresponds to an imaginary diffusion and describes eigen quantum wave functions for certain energies. Formally, it is the imaginary coefficient of diffusion that is characteristic of the Schrödinger equation. Therefore, technically, complex quantities sneak into quantum mechanics one way or the other, and which one to prefer - is rather a matter of taste. What's more, an imaginary diffusion points to quantum field roots of quantum mechanics and its nature, radically different from the macroscopic world. It is exactly for that reason that the microworld mathematics does not "fit" the real mathematics of classical physics and that fundamental fact is heralded by the emergence of imaginary quantities.

We continue now with an aside of the imaginary diffusion and quantum theory.

An aside: an imaginary diffusion and quantum mechanics.

The imaginary diffusion coefficient in the parabolic PDE does not make the Schrödinger equation and its statistical interpretation less realistic, than classical diffusion equation. Plainly speaking, the latter provides real distribution functions in a macroscopic world, while the former - distribution of complex amplitudes for the totally different world – microworld. In that context one can trace the transition from quantum mechanics to the mechanics of the macroscopic world – the classical mechanics. Suffices it to use the quasiclassical methods, or, in fact, the method of stationary phase (steepest descent method).The reverse, however, is not true: only can we resort to more or less plausible arguments. No mistake should be made: the world of complex quantum amplitudes is not derivable from the classical world. Indeed, the real analysis is an adequate language of the macroscopic physics and, in particular, the classical mechanics. The same is true for a quantum theory as well: the imaginary unit "i" is a natural element of the quantum mechanical mathematics, and not some bizarre abnormality of the real analysis.

This terminological skew has, obviously, historical roots. In particular, the exposition of harmonic vibrations (Lissajous figures) and waves is technically easier to discuss in terms of complex amplitudes. This way, however, the complex amplitudes appear only as a suitable shortcut, and not as a natural language. Moreover, any complex number can be written as a sum of real numbers with an imaginary "I" as a coefficient. This remarkable connection of real and complex variables should not imply that the complex form of the Schrödinger equation is merely a convenient wording, and there exists a classical model of quantum motion. Such a model – despite numerous attempts over 100 years of quantum theory – has not been managed to construct, and, most likely, it is not at all possible. To the reader familiar with second quantization procedures and quantum field theory: the commutation relations of operators of coordinate and linear momentum in quantum mechanics comprise an imaginary unit "i", which is an unequivocal indication of the role that imaginary quantities play in quantum mechanics. Imaginary i and, more generally, complex amplitudes are, therefore, the natural language of quantum theory and should be taken as such. It only remains to remind the famous citation of E. Wigner about the " The Unreasonable Effectiveness of Mathematics in the Natural Sciences". ◆

Chronologically, one of the earliest and most convincing arguments in favor of statistical interpretation (besides the very Born postulate) was the Ehrenfest approach. Accodingly, the classical coordinates and linear momenta turn random, and their distributions and various statistics follow from the wave function. In particular, and quite importantly, coordinates and conjugated linear momenta complement each other via the Heisenberg uncertainty relation, while the very Ehrenfest equations - i.e. the Newton equations in the quantum domain - signify the relations between the ensemble average coordinates and momenta. The most illuminating aspect here is that the random dynamic variables exist not by themselves, but rather constitute manifolds, characteristic of a probabilistic approach and known as ensembles. This precludes ascribing the wave function to an individual particle only, and make it the "distribution" in the whole quantum coherent ensemble. Signatures of the diffusion-like behavior are found across various quantum systems. For instance, in highly excited states (say, atomic) the Schrödinger equation can appear as an imaginary diffusion (i.e. takes an explicitly diffusive form with an imaginary diffusion coefficient, without resorting to an imaginary time extension; see below, the Supplement). Indeed, Rydberg states are, in fact, semiclassical states, i.e. states, whose quantum description utilizes a lot of classical features. This means, in particular, that the distribution of dynamic variables proves close to those in ensembles of classical trajectories, stemming from Hamilton-Jacoby equations (see, for instance, Landau and Lifshitz, 1974 and Fain, 1972), and energy spectra are almost equidistant – very similar to those of a harmonic oscillator. It turns out, along the way, that the transition amplitudes between those states (induced, for example, by collisions in plasmas) can be written as fourier-components of a one single generation function, which, in turn, emerges from collision-wise corrections to the classical action in the Hamilton-Jacoby equation. As a result, the transitions in such a semiclassical system follow a diffusion in the energy space with an imaginary diffusion coefficient (more details on that are given below, in the Supplement - in an essay "Imaginary diffusion in Rydbergs" and the corresponding literature).

As a conclusion. Emphasizing one more time: the statistical interpretation of quantum mechanics is in no way just a verbalization of its mathematical engine. The critical element of this interpretation , which is also a feature, distinguishing it from the Copenhagen interpretation, is a concept of an ensemble, ascribing the wave function not to a single microparticle, but rather to the whole multitude of microparticles emerging in the process of a measurement, and playing, therefore, a fundamental role in a probabilistic framework for random variables. This, in turn, drastically changes the interpretation of a superposition for random variables as compared to a traditional one for deterministic phenomena. As a result, lots of quantum mechanical "paradoxes" get resolved and disappear quite instantaneously, first and foremost, the Schrödinger Cat paradox (for details, see the next section).
We turn now to the Principle of Superposition in quantum mechanics.

Section 2. The Principle of a Superposition.

> *"The things, that interfere in quantum mechanics*
> *- are not particles. They are probability amplitudes*
> *of certain events."*
> ***R. Glauber***

Of all linear PDEs of mathematical physics obeying the principle of a superposition, it is the Schrödinger equation that has always been and still is the source of endless mysteries and paradoxes, stemming from improper interpretations. Suffices it to "Google" a request on "Quantum superposition" to obtain: "Quantum superposition is a fundamental principle of quantum mechanics where a physical system, such as an electron or photon, exists in multiple states, locations, or configurations **simultaneously**". (With regard to other versions and associated problems we refer the reader to classic works of D.I. Blokhintsev: 1964, 1968, 1988).

In reality, however, and in a nut-shell, 1) what interferes in quantum mechanics is the probability amplitudes, and not particles themselves(!), and 2) this interference is by no means a signature of any kind of "simultaneity", but rather a consequence of the overlap of probability amplitudes.

First of all, we remind the reader of the epochal Tonomura 2-slit experiments for electron diffraction (Tonomura *et al*, 1989), which showed a pronounced interference pattern as a build-up result at the long temporal exposures, while for short exposures only random / irregular spots were observed.

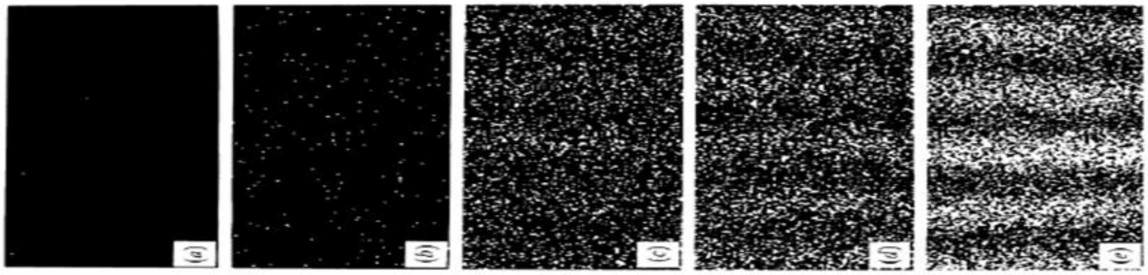

Fig . I.2.1 The cumulative pattern generated by the "self-interference" of electrons sent one by one through a two-slit interferometer. Number of electrons: a) 10, b) 100, c) 3000, d) 20 000, e) 70 000 (from Tonomura *et al* (1989))

Inasmuch as the results similar to Tonomura et al. for electrons, were known also for photons since Taylor experiments in 1904 (Taylor, 1904), the build-up nature of diffraction patterns becomes evident for massive and massless microparticles alike, and lends a direct support to an interference of probability amplitudes, rather than interference of particles.

For an additional and deeper insight, consider now a classical analogue of the Tonomura set-up, namely: a vertical one-dimensional classical diffusion from two well separated sources of identical substances (depicted as small circles with short arrows up and down on Fig I.2.2) with the drift (wind) pushing to the right in the horizontal direction.

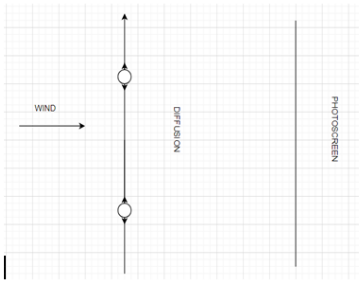

Fig. I. 2.2 2-slit experiment: classical analogue

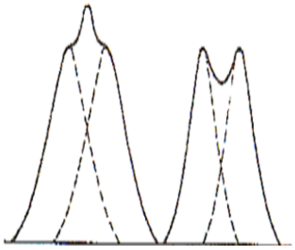

Fig I.2.3 (a,b)Density on the screen: a) close sources, b) distant sources

At the beginning of the experiment (short exposures!) the spots on the screen will be totally random / chaotic, and not showing any systematic tendency – exactly as in the classic Tonomura quantum case. But as time progresses, the accumulation of spots will show more and more evidence of the smooth overlaid distributions (Fig. I.2.3, a and b). The only difference from the original Tonomura set-up is the clear oscillatory interference pattern in the quantum case. We therefore come to a conclusion, that it is the short exposure / low-intensity limit that helps reveal the true source of interference: and that is the interwinding of random outcomes preventing from tracing back the origins of individual events. We emphasize that this effect exists in both classical and quantum cases alike, with the exception that quantum case is further embellished with intensity oscillations due to interference of complex-valued probability amplitudes (i.e. $|a +b|^2$ instead of $|a|^2 + |b|^2$).

To paraphrase: quantum superposition is simply an epitomized overlay of probability amplitudes, i.e. a trivial distribution overlap, plus the oscillations, ensuing from the complex nature of overlapping amplitudes. This clearly demonstrates that intensity “waves’ is nothing, but a build-up of elementary events into assemblages, distributed in a wave-like fashion. And there is no whatsoever interference of particles situating simultaneously around same spatial locations. This concludes the paradox resolution and staves off any proverbial mysteries arising from “simultaneity” in the quantum interference. It is exactly the reason why quantum measurements need the separation of “beams” as much as possible – compatible with an experimental equipment – to suppress the overlap of the superposition components (for more on this see the part on “Quantum measurements”).

Now, the well known from classical optics diffraction patterns demonstrate the result of the superposition of quantum amplitudes in the limit of long exposure in terms of wave-like structures, to wit: the diffraction on the edge of a semi-infinite opaque screen (Sömmerfeld, 1954) and corresponding intensity oscillations, the diffraction on the single slit (the reminder: any slit is a combination of two edges, and, depending on the distance between edges in comparison to the de Broglie ƛ, the intensity pattern on the screen transforms from almost geometrical optics to a typical interferential wave distribution), the archetypal 2-slit diffraction, etc. The heuristic significance of these examples is in that they illustrate how the de Broglie scale ƛ interplays with other geometrical scales of experimental set-ups in classical optics (Figs. I.2.4-I.2.6).

In particular, on Fig. I.2.4: for a semi-plane diffraction there is only one scale – the de Broglie wave-length ƛ, affecting the profile.

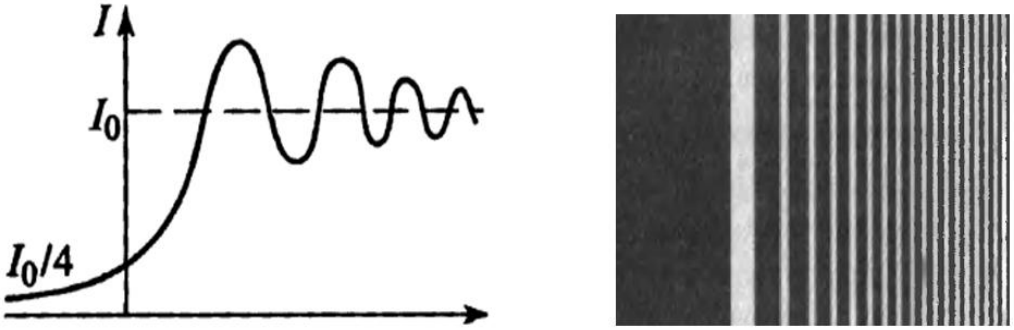

Рис. I.2.4 The diffraction on the edge of a semi-infinite plane

Fig. I.2.5: two semi-planes at some distance D, forming thereby a slit of width D. The pattern depends on two scales, ƛ and D, a) D >> ƛ – geometrical optics, b) D > ƛ – intermediate case, c) D < ƛ – wave optics.

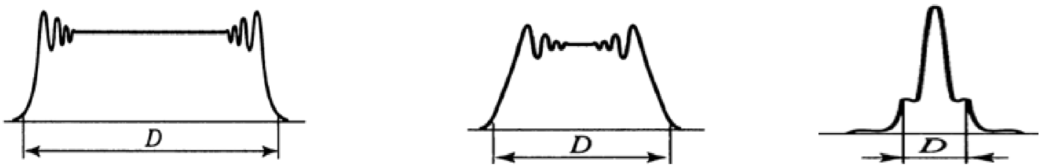

Рис. I.2.5 The diffraction on a slit: а) D >> ƛ, б) D > ƛ, в) D < ƛ

For the diffraction on two identical slits, Fig.I.2.6. – two identical slits, and, therefore, the pattern depends now on 3 scales: ƛ, D, and B – distance between slits. All of the above patterns arise from an interference of the incident plane wave exp(ikx) with a wave, scattered off the obstacle. However – and this is of a key importance – for a sufficiently short exposures (how sufficiently – depends on the intensity!) all experiments would show only random spots, with practically NO traces of any regularity.

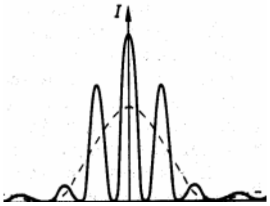

Fig. I.2.6 2-slit diffraction (solid line, dashed line - one slit intensity), the case of D < B << ƛ;

Accordingly, the interference pictures on Figs. I.2.4 – I.2.6 become visible only after sufficiently long observations (long exposures, that is). This clearly indicates, that waves in quantum mechanics (i.e. wave functions!) are but congregations of random elementary events, with distribution thereof modulated by wave profiles, and points in favor of the Statistical Interpretation of quantum mechanics and wave functions as an Ensemble Measure, and not a measure related to an individual particle (Blokhintsev, 1964, 1968, Ballentine, 1970). And further, in the context of the Born postulate, probability amplitudes (wave functions, that is) bridge, so to say, wave features of the ensemble with a behavior of individual particles, to wit: an amplitude is clearly a wave attribute, but applies to a probability, related to detecting an individual particle in an ensemble of similar particles.

In conclusion, let's make a few comments on the transition from classical to quantum mechanics. In this regard, a deeper technical grasp of the diffraction wave profiles can be obtained via a so-called quasi-classical approximation, which aims to smooth the sharp transition from classical trajectories to quantal interference of probability amplitudes by treating the system as a combination of classical and quantum features. Specifically, the infinitesimally thin classical trajectories "swell" into Feynman "tubes" around them. And accordingly, the semiclassical wave function becomes $A(x)\exp[(i/\hbar)S_{cl}(x)]$, where $S_{cl}(x)$ is a classical action along the classical trajectory at the centre of the tube, and $A(x)$ is a tube envelope (a transverse tube size is of the order of the de Broglie wavelength ƛ) to preserve a normalization of probability (Migdal, 1977). Thus, a tube becomes a microparticle channel, so that 1) the particle spatial spread appears is about ƛ, and, therefore, 2) quantum interference still appears as the interference of particles. Clearly, this is only a rather far-fetched make-shift, catering to reconcile the quantum interference with classical mechanics, and to make the feeling about quantum interference more palatable for the classical intuition. However, it is a surrogate of quite limited applicability, and, when used without a proper care, leads to well known non-sensical statements, such as "in a 2-slits diffraction single particle passes through both slits and interferes with itself" or "on a semi-transparent mirror particle splits into two pieces, which later on interfere in the interferometer legs" and so on. Especially pronounced these problems prove in interference of extremely faint beams. In this context, any phrases like "photons can / cannot interfere with each other or even with itself" or "one - or two - photon interference" is nothing but a bizarre rudiment of a classical thinking, a make-shift for dragging classical logic and symbols into quantum world, making no whatsoever sense for a consistent quantum framework.[1]
And even though it indeed can create - for a time being - some illusory comfort of understanding, it is helpful only so much and only up to the point. Relatedly, the well known Dirac dictum "photon can interfere only with itself and with nothing else" (Dirac, 1958) should be taken exclusively as that type of a pseudo-classical language and in this context only.

It is convenient in this context to discuss right here the so-called Schrödinger's Cat paradox, which stems from a nearly century-old misconception regarding the content of the superposition principle, just mentioned above.

The Schrodinger Cat paradox

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[1] The same applies to the ambiguous statement of Dirac (Dirac, 1955): "The traditional logic believes that the classical object cannot be simultaneously at two different locations. Quantum theory teaches us that this statement is not always true." An obvious misunderstanding is in the tacit carryover of the superposition principle interpretation from deterministic quantities to random variables.

The famous thought experiment with the Schrödinger Cat (named so after the founding father, who was the first to discuss it and qualified it as a quite strange development) runs as follows.

The cat is placed into a chamber equipped with the radioactive substance (i.e. the substance, capable of a radioactive decay). The chamber, in turn, is connected to the tube with a poison. If the substance undergoes the decay event, the latter activates the poison, which - sorry for the brutality – kills the poor cat. However, if the decay does not happen – the cat is well alive and kicking. Therefore, quite analogously to the wave function of a radioactive atom that is the superposition of a wholesome atomic wave function just prior to the decay and right after it, the wave function of the cat is also the superposition of dead and alive cat (once again, apologies for an unpleasant example). That concludes the paradox: the cat is dead and alive simultaneously. Question arises: where and when the error has been committed?

The answer is quite trivial: the error roots in the incorrect interpretation of the superposition in quantum mechanics (i.e. the superposition of random variables).

Namely, the overlap of quantum probability amplitudes has nothing to do with the time of the corresponding random events. Therefore, the interpretation of a superposition in a spirit of simultaneity of random events is only a hypothesis, which is totally ungrounded and leads to an absurd conclusion. It is the very gist of a random process that we cannot predict the value of a random variable in the very next moment, i.e. the forthcoming values are totally unpredictable. The only thing that we do expect – is the average value that can be obtained over time, but nothing more than that. Therefore, there is no whatsoever point to discuss the simultaneity of possible states of a cat based on a superposition for the wave function of a radioactive atom(or, for that matter, any other microobject wave function) , triggering the breakup of the poison tube.

As a conclusion: the principle of a superposition is valid for any linear equations, but its interpretation for random variables (and processes) is quite different as compared to deterministic ones. In deterministic processes the displacement, velocity of the object or the force acting on it, can be decomposed on the components, which act literally simultaneously. On the contrary, for random processes and variables - as it indeed happens in the classical diffusion - this simultaneity does not make sense, and all that one has is the overlap of the possible outcome range, be it in configuration (coordinate) or the linear momenta space. The random events themselves, corresponding to superposing states, can occur at any time during the observation and are not connected to each other in any way. It is that consideration that is missed in the Schrödinger Cat paradox. And nothing more than that.

Section 3. The main quantum paradoxes.

The collapse of a wave function

The collapse of a wave function – it is a misnomer, the result of an improper interpretation. Namely, if the wave function corresponds to a one single particle, then, naturally, there emerges the question of a wave function collapse, i.e. the destruction of the material structure by measurement.

In reality though, the wave function "collapse" paradox is based on a misunderstanding and arises largely from the confusion with terminology. If we abstract from minor wording variations, the essence of the matter looks as follows.

A priori, i.e. before the measurement, the wave function or ψ-field gives a field / distribution of probability amplitudes, i.e. we expect what is predicted by the Schrödinger equation. It is this meaning that Bohm put into his potentialities, comprised in the wave function (Bohm, 1951).

A posteriori, i.e. as a result of measurements, a field / distribution of the imprints of quanta / particles (and the corresponding $|\psi|^2$) is obtained, which emerges at large (asymptotically - infinite) densities of quanta, or at long (asymptotically - infinite) exposures. Clearly, if the wave function does not correspond to a material wave, i.e. it is not a material formation, but simply a probabilistic structure, then the "collapse" is a trivial and well-known from standard probability theory transition from expected *a priori* values to actually observed *a posteriori* values, so the problem disappears instantly. Any actual measurement occurs at finite densities/times and inevitably reveals a granular structure (Tonomura, 1989) and, naturally, realizes as *a posteriori* result.

Accordingly, conservation laws hold for any level of granularity. The full restoration of a phase information enclosed in the *a priori* wave function or ψ-field does require separate efforts and, strictly speaking, is impossible in the classical world.

There exists many indications that the wave function corresponds not to only one individual quantum / particle, but the whole ensemble of the latter. First, in the classical limit $\hbar$ -> 0 the wave function is simply the imaginary exponential of the classical action from the Hamilton-Jacobi equation, and the latter corresponds not to a single trajectory but to an ensemble of such trajectories. Second, the wave function phase characterizes the whole ensemble of particles, but not at all a single particle. Third, the uncertainty relation connects the coordinate dispersion to that of a linear momentum - the relation that does not exist in mechanics of a one single particle - and is a manifestation of a fundamental quantum variable: phase. The latter is a characteristic of the ensemble of realizations, and the uncertainty relation is a link existing in a group of realizations. The connection between the dispersions of coordinate and linear momentum results from the existence of the wave function as a coherent characteristic of the ensemble, "gluing" all realizations into a coherently connected collective. This is the phase coherency that is a source of a correlation between the dispersions of coordinates and that of linear momentum.

There emerges a legitimate question: what is the main function of the phase in quantum mechanics? The answer: the state before measurement (which is coherent) and after it (the incoherent mixture) differ precisely in phase. And it is this phase, that governs main aspects of the two-slit measurement.

Solutions to the Schrödinger equation are solutions determined precisely by the coherent phase, but they still need to be transformed to facilitate the measurement, i.e. to produce a spatial separation of the superposition components. This is exactly the role of analyzers in the quantum measurement technology.

Finally, it should be noted that there is a close connection between the naive wave-particle duality, the wave function "collapse" and the Copenhagen interpretation.

Indeed, if we accept that a quantum microparticle, depending on the circumstances, acts either as a particle or as a wave, i.e. that both the particle and the wave - with its main attribute, the wave function - are two sides of the same entity (like two sides of the same coin), then the wave function necessarily becomes the main quantitative characteristic of the one and the same individual particle. And this is precisely what is implicitly assumed by the Copenhagen interpretation.

Wave-particle duality

The naïve form of the wave-particle duality is closely related to the problem of wave function collapse (just discussed above). Historically, the initial formulation of the wave-particle duality stated that the individual microparticle, depending on circumstances, exhibited either corpuscular, or wave properties, i.e. behaved either as a particle, or the wave. However, it is quite hard to fathom how the spatially localized object can lead to an interference everywhere in the physical space. Under the pressure of these kind of difficulties and experimental evidence, the naïve duality concept has gradually evolved to, for instance, that one: a microparticle is neither wave, nor corpuscle, this is something else (Bohm, 1951). This already is somewhat better, but still vague and unclear, and does not fully explain the experimentally observed interference patterns. And it was only the Statistical interpretation that put the issue on the common sense footing: wave properties are characteristic of the entire beam/ensemble of particles, and not of individual particle. A single individual particle does not and certainly cannot have wave properties. And conversely, anticipating the discussion in Part II (the connection to quantum field theory): a wave considered quantum mechanically can be categorized, as usual, via an excitation level and number of field quanta, populating that level and exhibiting particle properties. This is the essence of a modern understanding of the wave-particle paradigm, replacing the ill-famed naïve wave-particle duality.
Bottomlining: wave patterns emerge only through particles, coalescing into assemblages, and, vice versa, the quantization of wave fields into its quanta produces particles. These two – seemingly independent and equally valid paradigms – become inherently related in the quantum field theory, which capitalizes on this aspect, and gives to fields the status of the origin of all particles found in the Universe.
And further: inasmuch as the wave function is defined for all points in space, it can be considered - albeit to a certain extent formally - as a complex wave field (usually called the Schrödinger field). This, in turn, serves as a convenient starting point for transitions to the ideas of quantum field theory. However, that understanding did not gain the recognition immediately and took most of the XXth century to make its way through. The point is that the classical physics uses two fundamentally different concepts of everything in existence: corpuscular-like and wave-like. However, after the revolutionary works of Planck and Enstein, they began their interpenetration, culminating in the creation of quantum mechanics (the mechanics of quanta) and the formulation of wave-particle duality. At first, however - as has been repeatedly mentioned - the principle of wave-particle duality was of a naïve character. Its key error was a so-called “primitive “ attribution, which has explained wave (i.e. global) and corpuscular (i.e. local) properties within one single microobject. However, this error turned extremely influential and contagious: the widely acclaimed Copenhagen (i.e. the Bohr school) interpretation (relates the wave function specifically to a one individual particle). And because of the Bohr overwhelming – and totally deserved! – reputation, this viewpoint exerted - for quite a while - a profound impact on the understanding of quantum mechanics, which is still found in the extant literature and remains a major day in and day out source of problems and paradoxes in quantum mechanical practices.

Understanding of the shortcomings of the naïve wave-particle duality, as has been already pointed out, stimulated its evolution: the most notable attempt was presented in the book of D. Bohm (Bohm, 1951). However, it still—and fundamentally!—encloses particle and wave properties into a single object of a special, quantum nature, which is fundamentally consistent with the Copenhagen interpretation.

In the interests of an utmost clarity, it makes sense to summarize what has been said just above: both, the naive wave – particle duality (“a microparticle is both a particle and a wave at the same time”), and the Bohm modification (“a microparticle is neither a particle nor a wave, it is some kind of third kind of matter”) are different only on the surface, but in essence they are based on the same misunderstanding, namely: either formulation attributes corpuscular and wave manifestations to a single object, like two sides of a coin, while in reality a corpuscularity relates to individual quanta properties, and “wavelikeness” is a manifestation of the quanta assemblages.

A historical aside. D. Bohm and his “Quantum theory”.

Published in the early 1950s, this book by one of the greatest theorists of the 20th century strikes with an exceptional clarity and simplicity, absence of an unnecessary mathematical formalities and irritating academicism. In terms of its impact on our understanding of quantum theory, it is quite comparable to Dirac's universally acknowledged masterpiece “The Principles of Quantum Mechanics”. However, there are important differences. While Dirac’s “Principles” is rather a mathematical bible of quantum mechanics, built in a didactive manner – from general postulates to specific applications, Bohm’s “Quantum theory” is first and foremost the book on physics foundations of quantum mechanics and constructs quantum mechanics inductively – from intuitively clear physical considerations to the Schrödinger equation, which is way more useful to beginners and more instructive methodologically. It makes sense at this point to cite Bohm directly, namely(Bohm, 1951 / 1989, p. 10):

“…In accordance with the general plan outlined above, unusual emphasis is placed (especially in Part I) on showing how the quantum theory can be developed in a natural way, starting from the previously existing classical theory and going step by step through the experimental facts and theoretical lines of reasoning which led to replacement of the classical theory by the quantum theory. In this way, one avoids the need for introducing the basic principles of quantum theory in terms of a complete set of abstract mathematical postulates, justified only by the fact that complex calculations based on these postulates happen to agree with experiment …”

Despite a respectable age of the book, its main sections are not at all outdated, and the author’s view on wave-particle duality and corresponding discussion - although bearing the imprint of its time - are still an impressive illustrations of the application of the author's concept of potential possibilities / “potentialities” to a radically new physical situation.

A short clarification on the above is in order. As it was explained in the main text, the naïve wave-particle duality pictured the quantum microparticle as a kind of symbiosis of a corpuscle and a wave in a single entity (which was quite eloquently termed by A. Eddington as a “wavicle”). In other words, the wavicle is a wave and a particle at the same time. It is self-evident that such a coalescence does contradict not only a physical intuition, but also a common sense. In this regard there emerged an “improvement”(essentially – just semantic!),

that is offered in Bohm's book, namely: the microparticle is is neither wave, nor particle. Microparticle is some new - and fundamentally non-classical! – substance, which, depending on the circumstances, exhibits either wave or corpuscular properties.
It should be emphasized nonetheless: the main shortcoming of the naive wave-particle duality (and Bohm's version in particular) was the perception of the wave and corpuscular properties of microparticles as equally valid "forms" of the same material substance/entity.

Example [citation from Bohm (Bohm, 1951/1989, p. 166)]: **"... For example, it[microparticles] might turn from something resembling a wave to something resembling a particle..."**
And further, in more detail:
"**...We conclude from the above that with a single electron one cannot really investigate the interference pattern, and that the wave properties of matter can be demonstrated clearly only when there are enough electrons to yield a statistical aggregate. We may ask why the individual electron is regarded as having any wave properties at all, if it is always found to arrive at the detector in a fairly definite location just as if it were a particle. The answer is that, to explain the appearance even of a statistical interference pattern, we must ascribe to matter certain wave properties, at least while it is in the process of going through the slit system. If we assumed that the electron always acted like a particle, then we would conclude that it could go through only one slit at a time. It is difficult to see, then, how the opening of another slit which, for example, may be millions of miles away, could make it unlikely that electrons would reach certain points to which they would otherwise have a high probability of going. Such long-range action of slits on particles is certainly contrary to all of our previous experience with particles. We might try to assume various modifications of the law of force between electrons and slit systems so as to try to explain this result but, as we shall see in Sec. 11, this effort would lead to all sorts of ad hoc hypotheses, which conflict with some of the most elementary requirements for a sensible theory. On the other hand, the wave interpretation of matter explains this result, as well as a whole host of other results, in a comparatively simple and yet quantitatively correct way. Thus, we conclude that even the individual electrons seem to be able to show certain wavelike properties.**
The preceding discussion leads to the idea that an electron is neither a particle nor a wave, but is instead a third kind of object which has some, but not all, of the properties of both particles and waves. Under different circumstances, either the wave or the particle aspects of this object may manifest themselves more strongly."
Thus, although Bohm moves from the naive wave-particle duality to a kind of "centaur" - neither a corpuscle nor a wave, but something third, exhibiting either wave or corpuscular properties depending on the experimental situation – that latter view still inherits the main problem of naive duality, namely: the combination of corpuscular and wave properties in one single physical entity.
Regarding what just have been said, we emphasize one more time: an individual microparticle does not have and cannot have wave properties. The latter are the

prerogative/feature exclusively of assemblages of microparticles, where they manifest themselves via interference structures. The further citations of " Quantum theory" speak for themselves:

"Thus, the electron is capable of undergoing continual transformation from wave-like to particle-like aspect, and vice versa ". *Or (p. 139):*" **The transformations of the electron from wave-like to particle-like object, and vice versa, which are implied by the quantum theory refer to fundamental...".** *Let us emphasize that both quotes refer to a one single electron, which is completely incompatible with the concept of interference.*

A solution to this difficulty was proposed by Nikolsky in late 1930s and comprehensively developed by Blokhintsev (Blokhintsev, 1964, 1968,1988) and L. Ballentine (Ballentine, 1970, see also Glauber, 1995). Namely, the indicated problem disappears if we accept that 1) corpuscular manifestations arise as individual events with one individual particle, and wave manifestations - in collective assemblages of such individual events, and that 2) it is not the particles that interfere, in accordance with the principle of superposition, but precisely the probability amplitudes in the ensemble. The further aspects of this profound consideration are discussed in the main text.

And even though the book bears a clear imprint of its time, and, accordingly, the naïve misconceptions of those years, the main one of which, of course, is the primitive wave-particle duality, all the concepts in it are carefully thought out, and their details are organically connected and presented in a convincing manner. In a word, it is the work of a powerful mind and the hand of a Master.

It should be also noted that in addition to the key question of the relationship between the concepts of waves and particles in quantum theory, the book contains a deep analysis of the EPR (Einstein-Podolsky-Rosen) paradox, possible parallels between the structure of matter at the quantum level and at the global level of the Universe, the role of quantum principles in thought processes, and much more. Let us especially point out to the following: in his book, Bohm repeatedly emphasizes that the option of decomposing wave functions into different and complete function systems reflects the existence of different potentialities/possibilities corresponding to different measuring instruments. In other words, each of the possibilities is realized by using a specific measuring device. For instance:"… **Thus, we are led to an exceptionally fluid and dynamic concept of the nature of matter, a concept in which a given object can always escape any well-defined system of categories that may be appropriate under a given set of conditions and that, according to classical lines of reasoning, would permanently limit its behavior in a definite way."** And further:

"… **We therefore conclude that the wave function contains a description of both space-time and causal aspects of matter implicitly within it (and each description is, of course, carried to an intermediate degree of accuracy, such that the uncertainty principle is not violated).**

Consequently, to obtain a qualitative account of the nature of matter as implied by quantum theory, we must likewise retain both descriptions, each with an intermediate and flexible degree of accuracy."

Overall, written three quarters of a century ago, the "Quantum Theory" remains a treasure trove of ideas on quantum mechanics for readers of any level, and in terms of the wealth of its content it is quite on par with Dirac's "Principles", Feynman's"Quantum mechanics and path integrals", "Quantum mechanics" of Landau and Lifshitz, and "Basics of quantum mechanics" of Blokhintsev. Bohm's book demonstrates a profound intuition and *understanding, which, coupled with brilliant writing, leaves us with a remarkable work on the foundations of quantum theory.* ◆

An aside. The retrospective view of the wave-particle duality and its lessons.

The history of the naive perception of wave-particle duality serves as an instructive lesson against an excessive attention to formal technical details without sufficient understanding of the essence of the mathematical apparatus used (especially – in textbooks), what served as one of the main reasons for the spread of the renowned paradoxes.

The very fact that wave manifestations are characteristic of ensembles, and not individual particles, was recognized quite a long time ago, almost simultaneously with the creation of quantum mechanics, but has been very often missed during the 20th century.

For example, back in 1935, in lectures at Moscow State University, Yu. B. Rumer pointed out that (Rumer, 1935): "There is no analogy between the motion of a single particle and a wave. Meanwhile, very often, through the lack of caution, people talk about the wave nature of the electron, while they should be talking about the wave nature of electron beams." Quite analogously, G. Makkey noted in lectures at Harvard (Makkey, 1960, see also the epigraph to that section):" Therefore, this mathematical system incorporates both waves, as well as corpuscle properties of the electromagnetic radiation, and the paradox emerges only because of an attempt to imagine the physical picture too primitively". Finally, D.I. Blokhintsev emphasized in his classic book (Blokhintsev, 1964): "Therefore, the state of a particle with a certain wave function should be interpreted as belonging to a certain quantum ensemble".

It should be noted also, that overall, the concept of the wave-particle duality is, certainly, a figurative concept, which, as has been already pointed out in the introduction, should be understood as a combination of particle-like manifestations of individual microparticles with the wave-like nature of distributions in assemblages of quanta.

However, at first, especially right after the creation of quantum theory, these two aspects were perceived as two inseparable sides of a single entity (so to speak, "packed" into one formation), in the spirit of "wavicle" - "wave" of A. Eddington.

It is not that the strangeness and counter-intuitiveness of such a concept was not recognized (which is precisely why it acquired the status of a paradox). In the above-mentioned Bohm's book, a significant attempt was made to overcome this problem, according to which a microparticle is neither a wave nor a corpuscle, but something third, alternately exhibiting both properties.

However, such an approach, as it was repeatedly indicated above, does not explain the formation of experimentally observed interference patterns. To finally overcome the intuitive discomfort from the interference of spatially localized objects, the following view became popular: individual quanta definitely have wave properties (such as interference, for example), but to observe the latter, there must be many quanta, i.e. wave properties show up only in assemblages of microparticles. It is easy to see, though, that this too is merely a clumsy attempt at self-reassurance, an attempt to "sweep under the rug" an obvious logical contradiction.

Let us emphasize once again: the key shortcoming of the naive and all subsequent modifications of the understanding of the wave-particle duality is in the substitution of a purely statistical effect by the manifestations of mysterious pseudo-object with the important caveat that the observation of this object properties is possible only in an ensemble. The situation is quite similar to that, as if the properties of the Gibbs distribution

in classical statistics were replaced by the properties of a hypothetical individual subsystem with manifestations only in an assemblage of such subsystems.
The self-consistent resolution of this difficulty, the 100 years later of the creation of quantum mechanics, is quite obvious: corpuscular and wave properties are manifestations of not one, but different entities, namely, individual and collective, and the wave function has no meaning for an individual particle, but corresponds to the distribution of amplitudes in an ensemble of quanta.◆

Some important aspects of quantum mechanics in the context of the statistical Born postulate and the wave-particle duality.

The "behavior" of quanta / microparticles in quantum mechanics is a random process. As befits a random process - similar to the classical case - the specific realization of the random variable (coordinate, linear momentum, angular momentum, etc.) is unknown, and can be expected only with a certain probability (excluding eigenstates) until it is measured through the contact with a classical measuring instrument. But until then, all we can talk about is the probability (in a classical case), or the probability amplitude – in a quantum process. In the latter case, the role of the generalized distribution of probability amplitudes belongs to a wave function. Once more: in that context, the role of a quantum complex-valued distribution (wave function) is quite analogous to a real classical distribution with the only difference, that the former is not directly related to the classical density of particles. It is in this sense that Wheeler's colorful - but somewhat confusingly vague - expression should be understood: in quantum mechanics the phenomenon is not a phenomenon, until it is measured. Furthermore, the stochasticity of field quanta is combined with their "plasticity": the projection of a quantum state into the classical world (into the world of classical measuring instruments, that is) can be different and depends on the details of the measurement procedure. This is exactly what allows us to analyze the distribution of quanta - for example, a spatial wave packet - using various eigenstates: for instance, by linear momenta or by non-commuting with the former angular momenta. And even though there is a certain similarity here with the classical mechanics, yet the fundamental difference is obvious: quantum superposition allows for analysis by different - and, moreover, non-commuting - eigenstates (not to be confused with the simultaneity of such states!).

Therefore, **there does not exist the one-to-one correspondence between states of classical and quantum world!** The naïve wave-particle duality in various formulations arises exactly as a result of this mutual ambiguity, and as an attempt to overcome it in primitive classical terms. The projections of the quantum world into the classical one, which Bohm called "potentialities", are implemented and differ by their amplitudes, related to the usual probabilities by the Born relation $P_i = |c_i|^2$. Thus, wave functions are coherent "repositories" of amplitudes that emerge when wave functions are expanded into corresponding eigenfunctions. In this regard, as noted above, wave functions behave similarly to an analog signal in radio-technical devices (in simpler terms, radio receivers), which respond to the receiver's own oscillations at a certain frequency.

Let us emphasize one more time: unlike the trajectories of classical particles, the dynamics of quantum microparticles (photons, electrons, protons, neutrons, etc.) is controlled by complex probability amplitudes that do not vanish in any of the many slits exposed to the wave packet (this, by the way, is one of the main heuristic values of Feynman's derivation of the Schrödinger equation), and to which real probabilities are not directly applicable, but emerge only through a nonlinear transformation (quadratic!) of the amplitudes. In other words, microparticles do not fly through the slits in the screen like shot balls, but interact with it according to field equations. Therefore, the innocent question “Which hole did the quantum pass through in Young’s experiment?” is, in essence, a classical question (at best, a quasi-classical one), based on the principles of classical probability theory and using based on the principles of classical probability theory and using real probabilities with the corresponding consequences. Therefore, in the consistent quantum mechanics there is no answer to it, and, most importantly, there cannot be the one, and, hence, the interference pattern is destroyed during any attempt to determine the “passage” slit.

There is no any medieval devilry or mysticism here: such attempts to impose classical behavior on quantum objects destroy quantum mechanics itself. Putting it simply, that question does not make sense in quantum mechanics, it is, so to speak, from the other world.

Missing this fundamental aspect of microparticles was precisely the reason that provoked the long-standing legend about the intractability of the double-slit experiment (for more on this see part III, section 1).

As we shall see further in the essay on quantum field theory, it is exactly these eigen functions that correspond to the waves/modes “carved out” (i.e., separated states in quantum measurement theory, Part III) from the wave packet by the measuring equipment, in a full accordance with the resonance of the radio-technical oscillatory circuit at a certain frequency / Fourier component in the signal incident on the device. Microparticles, therefore, are spatially localized quanta of the field, populating the natural oscillation modes of this field (more details - Part III).

And right here it is appropriate to briefly return to the qualitative picture outlined in the introduction. Mathematically, a quantum ensemble - a cloud of random realizations of a system (i.e., its wave function) - is nothing else than a vector in an infinite-dimensional (i.e., Hilbert) space. The expansion of a wave function into an infinite set of eigenfunctions, analogous to the usual expansion of a vector into basis vectors, is well known from analytic geometry. Since a vector can be expanded in any basis, similarly a wave function can be expanded in an arbitrary complete system of eigenfunctions. This decomposition corresponds to the choice of measuring instruments that implement the decomposition of the original wave packet into eigenfunctions adequate to the measurement being performed. This is precisely what was meant above when using the analogy of extracting from a radio signal an oscillation corresponding to the tuning of the oscillatory circuit of a radio receiver.

It should be emphasized one more time: the wave function is quite an abstract characteristic, a complex-valued amplitude, that is NOT REDUCIBLE TO CLASSICAL CONCEPTS, but makes it possible to determine the detection result at any desired point. Its undoubtful and outstanding merit is a complete and unconditional compliance to the conservation laws.

Section 4. The conservation laws and distant correlations.

> *“There is no physical connection between*
> *two subsystems at large distances.*
> *Yet, **the conditional** probability,*

does depend on which state of the
either subsystem we select"
A. Migdal

Ironically, this was Einstein who dubbed the long-range correlations as "spooky action at a distance," which became the paradox's byword. Meanwhile, measurements in extended quantum systems (such as, e.g., coherent pairs of photons) in conjunction with conservation laws naturally lead to the concept of entanglement, but at the same time, no miracles, like "spooky action at a distance," are at all needed. One just needs to understand the role of conditional, not absolute, probabilities, the underestimation of which provokes the myth of "quantum nonlocality", which has been showing up for many years not only in popular but also in specialized literature.

In a bit more detail, consider in this context photon pairs coherently produced in a radiative double-atomic decay or PRDC (parametric down-conversion) process. (A brief reminder: a coherent pair describes both photons with a single wave function, so that the states of both photons in the pair are correlated, i.e. coherent. If the pair is, for example, in the S-state, then the possible states of the photon components are limited by the condition: the total angular momentum of the pair is always zero). Such conditions are found in well known experiments by Freedman and Clauder (Freedman, Clauder, 1972), Aspect et al (Aspect, Dalibard, Roger, 1982), etc. Similar conditions are realized in thought experiments with spins, analyzed by D. Bohm (Bohm, 1951), and the like. The correlation of data, obtained at both ends of a pair, unequivocally points to the conservation of related variables. And yet, this became a source of innumerable discussions about an alleged non-locality of quantum mechanics, routing back to the well-known EPR paper (Einstein *et al*, 1935) and the Einstein "spooky action at the distance". In the mean time, the "paradox" emerges trivially from the confusion in elementary probability theory and resolves equally so. And yet, these results became the source of countless discussions of the notorious "non-locality" of quantum mechanics, rooting back to 1935, when Einstein, Podolsky, and Rosen wrote the famous EPR article (Einstein et al, 1935), arguing their bewilderment at the results of the then new quantum mechanics.

After this introduction, let's get down to business, and first discuss measurements of scalar quantities (the generalization to vector variables is trivially simple). Consider in this regard a generic set-up for detecting, say, charges within an electron-positron pair e-p (emerging in the process, reversed to a two-photon annihilation of an electron-positron pair). Inasmuch as the pair emerges (t = 0) via the local (point-wise) interaction, the latter trivially enforces a correlation between the pair components. However, we access (measure!) the related information only later on, when the components moved away from each other (it is easy see, that this already is an implicit use of a classical logic). This is precisely the arrangement that creates an impression of "non-locality".

Quantitatively: before a measurement, the charges of components are uncertain (more precisely, random), since the products of the wave functions of the free components are not the wave function of the pair (the latter is a superposition / coherent mixture of the states $\psi^{e}_{detector1}\cdot\psi^{p}_{detector2}$ with $\psi^{p}_{detector1}\cdot\psi^{e}_{detector2}$ in a 50/50 ratio because of the interaction generating the pair at t = 0). Accordingly, it is equally possible to detect each of the components in an electronic or positronic state. But once the result at any end is obtained, the outcome at the opposite end is ascertained instantaneously (conservation induced correlation!), and regardless of the spatial separation between components. (At large distances, the electron-positron Coulomb interaction is negligible, and the correlation arises only because of the conservation of charge). It is this situation that creates an illusion of “non-locality“ and oftentimes is interpreted as the “spooky action at the distance”. However – and this is critically important for resolving the paradox - what’s missed in this consideration, is that the result is premised on the **CONDITIONAL**, not absolute probability: that is, once the outcome at one end is, say, – e (+ e), then the **CONDITIONAL** probability of getting + e (– e) at the other end, is immediately 100%. In other words, for spatially extended systems the outcome of the second measurement is always **CONDITIONAL** of the first measurement, regardless of the distance. And obviously, it is this **CONDITIONAL** probability that is always non-local in spatially extended systems, as it deals with spatially separated entities.

We can as well reverse this set-up to consider correlations between two photons (ensuing from the annihilation of an e-p pair), say, in terms of linear momenta of emerging photons: quite analogously, these correlations will again obey CONDITIONAL probabilities. The same logic readily applies to measuring other vector dynamic variables (e.g. spin, etc.) by adding a spatial orientation to arguments of conditional probabilities. In other words, it is our framework that is non-local, and not quantum mechanics. That closes the “paradox” and the whole issue of “spooky actions”.

To make things easier to understand, we will end this chapter with a short essay–aside about quantum randomness.

***An aside: about quantum randomness*.**

Note that the often repeated assertion that randomness in quantum mechanics arises from interactions with the detector is, obviously, incorrect. The interaction with the detector is responsible for the destruction of coherency, and, as a result, an irreversible destruction of an interference. But the randomness in quantum mechanics arises quite earlier. This is especially clear from the consideration of experiments with the production of two or more particles. Indeed, if a random choice between, say, two alternatives (charge + or -, spin up or down, etc.) is provided at the moment of interaction with the detector, then, in order to comply with conservation laws, information about this choice must somehow be transmitted to another detector, which can be located arbitrarily far away. And this directly leads to an infinite speed of the propagation of interaction (Einstein's "mysterious action at a distance"). Meanwhile, if the mentioned randomness constitutes the very essence of quantum stochasticity, then a random choice is realized, for example, at the moment of the birth of a couple, when the emerging quanta are infinitely close to each other, and their contact interaction naturally ensures the fulfillment of conservation laws. This means that if a certain value is measured at one end of a pair, then this value is simply one component of that pair that randomly emerged at the initial moment.

It should be bared in mind that the amplitude of the pair has a phase degree of freedom, which - before the detector is triggered - is the phase of the complex distribution function, i.e. the wave function. However, when the detector is triggered, the information contained in this phase degree of freedom is lost, which corresponds to the loss of coherence and the replacement of the coherent ensemble before interaction with the detectors by a incoherently mixed ensemble after activation of the detectors. By the way, it is also clear from here, that when a pair arise, it does not spontaneously transform into the one that *arises as a result of detection which was discussed in the the well known note by Furry in 1936, (Furry, 1936). In other words, before detection, the production products are described by a coherent wave function, and after detection - by a mixed ensemble, or a density matrix with no correlations, or diagonalized density matrix. This role of quantum randomness and its "activation" not at the moment of detection, but during the random population of the quantum state itself, is completely consistent with what happens in the first and subsequent measurements. Indeed, it is the first measurement that "reveals" the choice of quantum randomness, and all the others simply repeat it. Dirac brought up this idea even in the first editions of his epochal book. And further, the main error leading to an emergence of the mystical non-locality in quantum mechanics (Einstein's spooky action) is precisely in the incorrect attribution of the role of randomness in the measurement process. The main work of chance is done long before the detection: at the moment of formation/preparation of a beam of particles or pairs to be measured. Detection only enhances the measured degrees of freedom, cutting off the so-called "phase" degrees, i.e., transforming the "pure", coherent ensemble of states into a mechanistic mixture, an incoherent ensemble.*

***The bottom line**: detection merely makes apparent the component attributes of a pair created randomly at birth, but there is no, of course, any mysterious transmission over a distance. The randomly created components of the pair enter the classical world through a classical device, but their phase "framework", inherited from the excitation of the quantum vacuum (quantized fields) and embodied in the structure of probability amplitudes (wave function), is erased.* ♦

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