THE STATISTICAL FOUNDATIONS OF QUANTUM MECHANICS II

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ABSTRACT

Arguing statistical foundations of quantum theory and showing the way it naturally resolves all quantum "mysterious paradoxes". In this context the great deal of attention is given to principles of quantum measurements. To facilitate a better appreciation of quantum mechanical paradigms we provide a brief essay of a quantum field theory and its natural evolution to nonrelativistic quantum mechanics, and then, to classical physics. The presentation aims at both physics students and young scientists, as well as seasoned researchers who will find the discussion of lots of interesting points. The second half contains an essay on quantum field theory and the theory of quantum measurements.

Key words: quantum mechanics, quantum field theory, de Broglie hypothesis, Schrödinger equation, wave function, statistical nature of wave function, principle of superposition, Schrödinger cat, wave-particle duality, collapse of wave function, Einstein "spooky" action at the distance, quantum non-locality, direct and indirect quantum measurements, interference and coherency, quantum dynamics of highly excited atoms (Rydberg atoms)

To the memory of M. Born and D. Blokhintsev

Part II. From quantized fields to quantum mechanics to classical physics (a short essay).

"The assumption is made that for each type of elementary particle there exists an associated field of which the particles are the quanta"
E. Fermi

Now, let us turn to the ideas of quantum field theory as a natural extension of quantum mechanics. This will allow to better understand why quantum mechanics is what it is in general, and, in particular, the origin of its technical/mathematical apparatus. Accordingly, we will briefly outline several fundamental concepts of quantum field theory that are not usually given sufficient attention in standard texts.

As was mentioned in the previous section, the wave function in ordinary quantum mechanics can be interpreted as a probabilistic field of the Schrödinger probability

amplitudes. However, this is still a classical field, i.e. it is a complex-valued function, existing at every point in configuration space, or linear momentum space, etc., depending on the representation. Advancing this field with relativity requirements, and then subjecting to a standard second quantization procedure, paves the ideological way to a consistent quantum field theory and, in particular, reproduces all technical results of quantum field theory. However, the full fledged quantum field theory aims much higher: its ambition extends to trace down the origins of all elementary particles and to constructively explain away them via excitations / quanta of a primordial vacuum field as a source of everything in existence.

The first version of a field quantization, known now as a canonical quantization, was formulated by Heisenberg and Pauli (Heisenberg, Pauli, 1929, see also Wentzel, 1947 and 2003) as early as only three years after the groundbreaking Schrödinger papers. Accordingly, an ordinary (coordinate representation) and its Fourier counterpart (momentum representation), become operators (operator-valued functions) acting on state vectors. Further details about that and much more will follow.
Reiterating once again: one should be mindful of the deep difference between classical and quantized fields. While the former are ordinary real-valued functions, solving classical equations of mathematical physics, the latter are operator-valued functions, with eigen states thereof containing particles.[1]
In a more layman parlance, quantum field theory continues along the conceptual lines of ordinary quantum mechanics: de Broglie waves and wave functions do not signify any material waves, they stand for probability amplitude distributions, appearing as waves, and deliver the statistics in the assemblages of microparticles. Similarly, quantized fields are not fields per se: they are operator-valued fields. More precisely, these operators act in the functional space of wave functions and thereby create fields of probability amplitudes, (i.e., wave functions) – exactly in the second quantization sense. Now, the key presumption of the quantum field theory is that each type of particle originates from its own specific field, and as such, is the quantum of that field.

Accordingly, at the first instances of the Universe, the initial all-permeating primordial field decayed into a great variety of sub-fields, which, in turn, produced the whole gamut of known elementary particles. (Admittedly, this is a quite an involved issue, still not fully understood, and we will not delve into it any further.).

However, the common production mechanism rests, essentially, on a picture of all-permeating elastic fabric - called a "field" - small vibrations of which around the equilibrium quantize (within a Heisenberg-Pauli quantization framework) into the eigen states whose spatial profiles are called “modes”. The latter can be perceived - and categorized - as prototypes of ordinary Schrödinger wave functions with fixed momentum in the coordinate representation. For small vibrations, these modes look like oscillations of coupled oscillators, and, accordingly, the energy of the mode looks like the energy of the oscillator. In a transition to quantum theory, this energy becomes the Hamiltonian of a harmonic oscillator with an equidistant spectrum $E = n\hbar\omega$, where n is the level number, and $\hbar\omega$ is the

[1] For that reason quantized fields are oftentimes referred to as ‘fields of quanta / particles” (Abrikosov *et al*, 1963) or, relatedly, “fields of harmonic oscillators, giving rise to elementary excitations / particles”(Polyakov, 1987).

minimum energy of a quantum, and excitation (energy) quanta are interpreted as quantum/elementary microparticles. Along the way, the level number, which determines the degree of excitation of the oscillator, gives the number of quanta populating a given level, so that the number of quanta-particles determines the mode, the wave nature of which should be considered as the source of the wave nature of the "wave" function. And even though the Schrödinger equation does not follow directly from this approach (at least, the author does not know how this could be shown), reversing the logic and considering the wave function as an operator (which corresponds to the well-known procedure of secondary quantization!), we again return to the discrete quanta that make up the energy of the mode. According to that picture, when the number of quanta in the mode - i.e. the occupation number - becomes large (for Bose particles), which corresponds to highly excited levels-states and the applicability of the semiclassical approximation, the quantized mode transforms into the usual mode of a classical field, i.e. it becomes a classical wave.
It is self-obvious that such a limit applies only to Bose particles (for example, photons), but is not applicable to Fermi quanta (for example, electrons), because of the Pauli principle. It is for this reason that we observe the classical electromagnetic field, but do not see the classical "electron" field. The latter should not be confused with a Fermi gas or a dense Fermi liquid, where the interaction of quantum particles plays a key role.

A SHORT TECHNICAL ASIDE.

Technically the "quantization" of the Schrödinger equation goes through the transformation of Heisenberg-Pauli, namely: the wave function Ψ(x) becomes the operator-valued function $\widehat{\Psi(x)}$ *(and, correspondingly, the momentum wave function Ψ(p) -> operator* $\widehat{\Psi(\mathrm{p})}$ *). This is known in quantum mechanics as the "secondary quantization". Along this way the global harmonic oscillator in the state* $\boldsymbol{E = n\hbar\omega}$ *is replaced – as it was pointed right above – by an ensemble of local oscillators in the state n = 1 with the minimal energy* $E_1 = \hbar\omega$*. It is this setting, that explains why the quantized field can be viewed as a field of harmonic oscillators, see the previous footnote. (This simplest mechanistic picture yields quanta/particles with only zero rest mass – for example, photons, but to obtain a non-zero mass there are correction procedures, which, however, look somewhat artificial). Nowadays, there exists a much more consistent and, therefore, substantiated approach for the emergence of mass of existing elementary particles: the so-called spontaneous symmetry breaking (Cheng, Lee, 1987).*
Namely, the ancestral - and initially massless - quanta of the electroweak field (at the initial moments of the Universe life, the electromagnetic and weak interactions existed in a combined form - the electroweak field) and interacted - among other things - with the "heavy" Higgs field (the latter is characterized by a very high excitation energy, which corresponds to the extremely high temperatures at the moment of the birth of the Universe) by means of the specific potential of the "Mexican sombrero", where the stable energy minimum is shifted from the center of the sombrero to an arbitrary (spontaneous!) point of the circular groove around the center.
As a result, the electroweak quanta "acquired" mass, and the heavy Higgs quantum disappeared when the initial plasma cooled down. With a further temperature drop, the electroweak field split into a still massless electromagnetic field (with photons as quanta), and into fields with low-mass quanta (including electrons), representing modern weakly interacting elementary particles. ♦

We should point right here to a certain inconsistency of this simplified approach: the emergence of a classical field is explained by small oscillations of the field itself, i.e. it is an example of a circular logic. Therefore, it is much more reasonable and consistent to consider the original substance specifically as the quantized field, which then becomes a source of quanta, combining to form a classical field.

Let us emphasize that the consideration of the n-th level, or n-th mode, as an ensemble of n elementary energy quanta/elementary excitations (similar to phonons in an elastic medium, plasmons in a plasma, magnons in an ensemble of spins, etc.) is a critically important moment, signifying the phase transition from the ancestral field to the current state of the world. As to the reality of such a phase transition, as well as to other steps in the staircase of events that led to the world around us, this is a separate and highly non-trivial question.[2] However, as we have already noted, this relatively naive quantization was the first attempt to explain everything that exists on the basis of field concepts, and, it should be noted, quite convincing.
Since then, the quantum field theory has made a significant progress, in particular, with the help of the spontaneous symmetry breaking procedure, as just mentioned.

In addition, as was indicated above, excitation modes and their quanta arise from and in the presence of a quantum vacuum, which, having an infinite supply of energy, plays the role of a global thermostat (and the ensemble of quanta itself looks like a grand canonical ensemble, well known from statistical physics) and ensures random behavior of microparticles in quantum mechanics. It is the quantum vacuum, acting uniformly on all particles of the ensemble, that forms a coherent distribution of probability amplitudes and their wave-like interference. In other words, although quanta do not interact with each other, the quantum vacuum is the common cause that ensures the coherent distribution of quanta and the interference of the corresponding probability amplitudes. By the way, the birth of particles as quanta of wave modes, explains - as a free byproduct - why the behavior of quantum particles is described by a typical wave characteristic - amplitude (probability).

By way of analogy, vacuum acts as surface tension in water waves. The difference, however, is that in water all surface molecules interact via a surface tension “simultaneously”, which makes their motion coherent, or more precisely, ensemble coherent. In the quantum case, microparticles of the beam do not interact, their coherence arises because of their “habitat”, and, so to speak, is stretched out in time, i.e. we have a temporal coherence. One can say that the “ergodicity” implements the coherence: the consistency (let us remember: coherence is consistency) in a beam of non-interacting quanta realizes in the form of uniform behavior of individual particles of the beam, which gives the coherent probability amplitude. Thus, the quantum vacuum performs a dual function. On one hand, it is an all-permeating environment that is the source of all that exists (and this literally corresponds to the action of the birth operators in the secondary quantization procedure). On the other, the quantum vacuum plays the role of a global thermostat, in the field of which all micro-

[2] The theory of primary and secondary quantization, which have become standard tools of modern quantum theory (and which were mentioned above), may in the future turn out to be simply a convenient phenomenological approach, the detailed correspondence of which to real physics will be determined by further experiments in particle physics and cosmology.

objects “live”, and which is responsible for their non-classical/quantum behavior, in particular, for all observed coherence effects.

As a conclusion. The modes of field vibrations serve - in the sense of the Heisenberg-Pauli mechanism - as generators of ensembles of field quanta. It is not surprising and completely natural, therefore, that wave functions serve as a distribution (complex-valued) in such ensembles (Blokhintsev, 1964). All this unequivocally indicates that the ensemble of quanta/particles that arises from the quantization of the field wave modes is an alternative, but completely equivalent representation of these modes/waves. In other words, there is a correspondence of waves exactly to ensembles of particles (and not to a single particle, which was implicitly assumed by the naive wave-particle duality!), which is visible in the wave-like intensity distributions. And further, quite intuitively, quantized fields can be perceived as corresponding to wave functions acting as operators. One should, however, be cautioned against a cloudless and naïvely straightforward interpretation of this statement. Even the difficulties with the traditional interpretation of the photon wave function in configuration space and the limitations on electron fields associated with the Pauli principle give a good idea of the problems with visualizing vacuum fields. We, however, cannot go deeper into these issues here.

PART III. THE THEORY OF QUANTUM MEASUREMENTS

*“By spatially separating beams with different momenta **p the**, diffraction grating suppresses the interference between them”*
D. Blokhintsev

Introduction

The main challenge of measurements for systems in the state of a superposition is the close spatial overlap of superposition components, which prevents a meaningful detection. And even if the specific quantum interference is suppressed, there still remains the usual overlap, analogous to the classical diffusion equation.
The issue is finally resolved by means of spatial separation of the superposition components, so that as a result $I \psi_1 + \psi_2 I^2 \approx I \psi_1 I^2 + I \psi_2 I^2$. With that in mind the initial beam is passed through a so-called analyzer, which performs the desired separation (for linear momenta this is achieved by a diffraction grating, for spins - by an external magnetic field, etc.).
One of the key aspects of the analyzer's operation is its reversibility. To understand this, suffices it to remember that the analyzer is described by the Schrödinger equation with a deterministic potential, the solutions of which are reversible. Experimentally, this can be achieved using either conventional mirrors, or magnetic mirrors and lenses, etc.
Next, the analyzer directs each of the beams to “its own” detector, the operation of latter is being irreversible, and, consequently, destroying the coherence of the beams (and with it, obviously, the interference).

The paragraph §1 presents the technique of direct measurements, while the second - indirect measurements, the idea of which goes back to von Neumann and is explained in detail by L.I. Mandelshtam and the works of D.I. Blokhintsev (Blokhintsev, 1968 and 1988). Then, the §3, discusses the general issues of wave scattering by many scatterers (as in a 2-slit experiment or when exciting a photographic plate). Specifically, the problem of scattering of microparticles with a fixed linear momentum on two (or more) scatterers is considered. In this setup, as will become clear, this problem applies not only to the working of a photographic plate, but also to scattering by many slits. Conceptually, this paragraph presents the ideas, resting on detailed calculations in (Blokhintsev , 1968, 1988). Specific calculations and the practice of quantum mechanical measurements provide more insight into quantum mechanics than all the lengthy discussions on this subject in many deservedly famous books combined. For that reason we will pay an increased attention to this and related issues. But first, a short preamble summarizing the essentialities.

There are two possible types of measurements: either the original beam is split into "sleeves" based on the values of the variable being measured, and each "sleeve" is equipped with its own detector, or there is only one detector, but it reacts differently to the superposition components of the beam. In both cases, the initially coherent superposition corresponding to different values being measured turns into an incoherent mixture after the detector(s) are triggered. In the language of wave functions, this means the emergence of incoherent pre-factors before the superposition components of the wave function (the details of this will be discussed below), and in terms of the density matrix this corresponds to the disappearance of the off-diagonal elements of the initially coherent density matrix and its transformation, thereby, into the diagonal density matrix of an incoherent mixture. Let us note here that in both cases, conclusions about a quantum object are made based on the reaction of a classical detector object.

Direct (static) measurements.
In this case, the measured beam passes through a diffraction grating (e.g., momentum measurement), or through a magnetic field (e.g., spin measurement) – (see Fig. III.1)

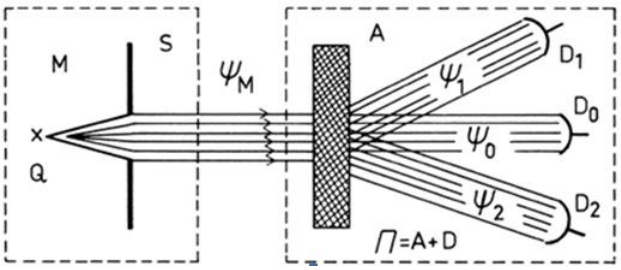

Fig. III.1 Conceptual scheme of direct measurements

Here the first box denotes the preparation part of the equipment, and the second Π = A + D denotes the actual measuring part (for more details, see (Blokhintsev, 1988)).
As a result, the beam splits into several weakly overlapping "sleeves" / arms, largely suppressing both the interference in intensity (cross terms in the density matrix), as well as the spatial overlap of diagonal terms, which makes it possible to identify detector readings

with states in the original ensemble. This way, the greater the divergence of the arms, the less interference and overlap in the output intensity. This, in fact, determines the accuracy of the measurement. As has been just noted, each "sleeve" contains its own detector, the operation of which marks the end of the reversible period of the beam evolution (according to the Schrödinger equation) and the emergence of irreversibility because of interaction with the macroscopic detector. This part of the measuring equipment (between the beam preparation part and the detectors, designated A in the figure), which can include external static fields, reflective surfaces-mirrors, etc., and which splits the original beam into arms, is called an analyzer. Let us emphasize once again: the analyzer is the key part of the measuring device, allowing the readings of each detector to be assigned to a specific component of the square modulus of the full wave function (ignoring the relative phase!) and facilitating the transformation of the coherent wave function immediately before the detectors into an incoherent mixture of superposition components of the wave function (with suppressed off-diagonal terms and non-overlapping diagonal terms in the density matrix) immediately after the detector. Similar motives are also found in another paradigm - indirect measurements – which is our next topic.

Indirect (dynamic) measurements.

This approach is used when direct measurements are difficult. The measured beam (system I) is scattered on a classical target (system II). As a result, because of the classical nature of the target, the total wave function (system (I + II)) decomposes into asymptotically (with respect to the collision time) non-interfering terms, which suppresses both the off-diagonal terms and the overlap of diagonal ones in the intensity. Along the way, the classic target serves as a kind of "arrow" of the measuring device.

Both methods are rich in physical intuition and extremely instructive didactically: we highly recommend them to the reader.

§1. Examples of direct (static) measurements.

Measurement of spin (the Stern-Gerlach experiment).

The spin measurement along the Stern-Gerlach protocol is described in detail in the classic book of Bohm (Bohm, 1951/1989), therefore here we will touch upon only key points. The procedure begins with a beam of atoms with spin, prepared in a spin superposition state, passing between the poles of a magnet, creating a non-uniform magnetic field.

As a result, the magnetic field gradient imparts to atoms the velocities according to their spins - i.e. velocities of atoms are opposite if their spins are opposite – and, accordingly, facilitates the beam splitting along the way to the photoscreen. All that is explained on Fig. III.1.1

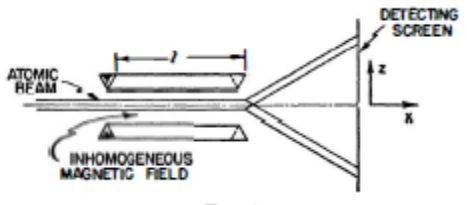

Fig. III.1.1 The Stern-Gerlach experiment (from Bohm, 1989)

In what follows the original notations from the Bohm book are used.
Accordingly, at the entrance to a gap between magnets, the total wave function is the product of the superposition of functions with opposite spins v_+ and v_- by the atomic wave function $f_0(z)$, common to both spin components, i.e.

$$\Psi = (v_+ + v_-)f_0(z) \qquad \text{(III.1.1)}$$

where z – is the vertical coordinate of an atom in the beam. Since the interaction with a non-uniform magnetic field depends on the spin direction, the Schrödinger equations for the components of the initial superposition differ, and, as a result, at the exit from the gap, the components of the superposition of spin functions acquire different atomic wave functions as weighting factors, i.e. spin interference is disrupted because of the mixing with atomic wave functions:

$$\Psi = v_+ \cdot f_+(z) + v_- \cdot f_-(z) \qquad \text{(III.1.2)}$$

where $f_+(z) \sim \exp(-i\mu z H_0' t/\hbar)$, $f_-(z) \sim \exp(i\mu z H_0' t/\hbar)$ and H_0' – the gradient of magnetic field in the z-direction , and μ – magnetic permeability. Note, that since these coefficients rapidly oscillate with z, this eliminates the standard interference terms integrated over z (all the details of the calculations are contained in (Bohm, 1951/1989)). At the same time, at the exit from the gap, the beam atoms with opposite spins acquire oppositely directed velocities, so that during the inertial flight from magnets to the photoscreen, the original wave packet splits into two clearly separated components with a distance between the maxima of the order of $\Delta z \sim 2\mu H_0' \Delta t/m$, where Δt – flight time between the magnets. Thus, at each point of the screen (i.e., at each z) suppressed is not only the phase, i.e. the typical wave interference, but also an ordinary amplitude overlap of beams with opposite spins, (i.e. this is a spatial separation of beams by an analyzer!) and f+f–*(z on the screen) -> 0, which is what allows for spin measurements. In other words, the coherent superposition of wave functions converts into their noninterferring mixture. In terms of density matrix in a coordinate form this means the disappearance of 1) non-diagonal spin elements of the density matrix as well as 2) an overlap of diagonal terms at a given z. Let us note here that the occurrence of rapidly oscillating phases (in fact, random!) when flying through magnets is very typical for interaction with a classical device, but this is not enough yet for the desired measurement, i.e. the analyzer must also "separate" the diagonal intensity components by z. This, in turn, is achieved by an inertial propagation of the beam from the magnets to the screen. Only after that are the intensity components “separated” by sufficient distances, which allows them to be identified with the original components of the superposition.

Linear momenta measurements (the scattering by a diffraction grating).

As opposed to spin measurements, the measurements of a linear momentum occur by directly suppressing the amplitude overlap, similar to how this occurs in the operation of optical diffraction gratings. Namely, light waves of longer length/smaller wave vectors are deflected more strongly by the diffraction grating, and therefore, an electromagnetic (light) packet (in this case, incoherent) is decomposed into weakly overlapping beams with different wave vectors/wavelengths. As an illustration, we present a spectrogram taken

from (Jenkins and White, 1976), where non-overlapping maxima of the same orders, arising from different wavelengths, are clearly visible:

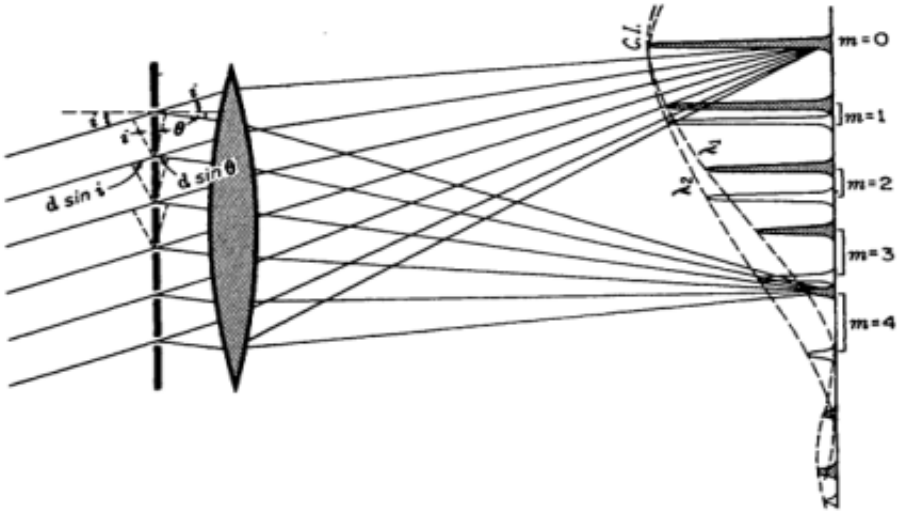

Fig III.1.2 The classical diffraction scheme (from Jenkins, White, 1976)).

Quite similarly, consider a wave that is harmonic along the x-axis of propagation and uniform (flat!) in the perpendicular y-direction: thus we have a plane harmonic wave exp[(i/$\hbar$)px], corresponding to a plane wave with the momentum p. Let us now form a coherent wave packet of plane waves with close momenta p(i), $\sum_i^n \exp[(\mathrm{i}/\hbar)\mathrm{p}(i)\mathrm{x}]$,: it obviously does not have a definite linear momentum, and measuring the momenta of individual components is difficult precisely because of the strong superposition of waves corresponding to adjacent momenta p(i). However, as in optical diffraction, after passing through a narrow slit Δy, each pulse of a packet receives an addition Δp(i) in the direction perpendicular to x, i.e. y, the larger the smaller the initial pulse p(i). The latter circumstance is elementary clear: faster particles (i.e. with the greater momentum) spend less time in the potential zone and, therefore, are less deflected.

This leads to a more formal wave-mechanical explanation: as the velocity (i.e., the wave vector) increases, the original wave function rapidly oscillates, so that the scattering amplitude is concentrated in a narrow cone with an opening angle around the initial wave vector inversely proportional to the initial velocity (for example, average) in the incident beam Δθ ~ 1/v, which gives the desired result: the spatial separation of beams with close momenta. The separation of pulses by the analyzer (in the case at hand, the slit) when scattered on a set of slits (grating) becomes especially clear compared to a single slit: the first main maxima (m=1) in the scattering of each pulse gets narrow sharply, approximately as 1 / N. As a result, the wave scattered by the grating is a superposition of very spatially narrow beams, each of which corresponds to a specific momentum in the original packet. In other words, the diffraction grating, as expected of a spectral analyzer, decomposes the original packet into very weakly overlapping beams - eigenstates, i.e. suppresses the spatial overlap of pulse eigenstates, which is exactly what required for measuring the mometum. Now, even though a significant separation is expected of the scattered beams in accordance with diffraction theory, it may not be sufficient for reliable measurement of momenta: residual overlap may, generally speaking, prevent reliable momenta measurement.

To improve the separation of scattered beams, various structural changes to the grating are possible, which, in essence, change the scattering potential. This is a very rich and rather subtle engineering topic, but we cannot go into further details here. Quite similarly, it is possible to organize the measurement of other dynamic variables: for example, polarization, etc.

§2. Examples of indirect (dynamic) measurements.

As noted above, if direct measurements are inconvenient or even impossible, then indirect measurements are used, the methodology of which traces back to the ideas of von Neumann (von Neumann, 1955). Accordingly, the quantum system (“light”) is scattered by the classical (“heavy”) one, and since the reaction (recoil!) of the classical system is different for different components of the quantum superposition, the classical system effectively proves an "indicator-pointer" to the states of superposition (in the Blokhintsev terminology, becomes an "arrow" of the measuring device). This approach still uses some essential aspects of the direct measurement technique, in particular, the suppression of quantum mechanical interference and the usual amplitude overlap during the scattering process. The key idea is that when a beam of light particles (such as electrons) in a superposition state collides with a heavy particle (which is almost classical), then the corresponding classical component of the wave function effectively destroys the full wave function components overlap, that is, the talk essentially is about translating - if to use musical analogies - into the language of wave mechanics the elementary classical problem of the reflection of a light particle from a heavy one. Let us illustrate the main ideas of such an approach via a simple model proposed and calculated by Blokhintsev (Blokhintsev, 1988).

Namely, a heavy classical ball, located in a hole on the flat top of a relatively high hill (so that the profile of the hill is a trapezoid, see Fig. III.2.1), is introduced into the field of a standing wave of light particles, the beam of which is in a superposition state in momentum due to reflections from the mirrors at the ends of the device. The collision of particles with the ball pushes it out of the hole to the right or left, depending on the direction of the momentum of the light particle, and ultimately results in the ball rolling down the hill.

Thus, 1) the ball movement on the top of a hill serves as an analyzer, 2) rolling down the hill - as a detector, and the ball itself proves as the kind of an arrow (of a classical device), by the deflection of which one can infer the electron's momentum immediately before the collision.

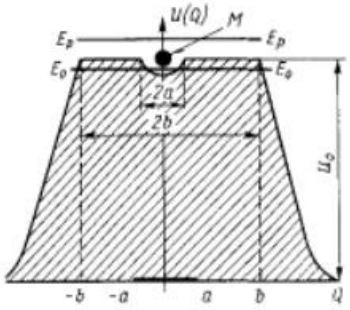

Fig III.2.1 The energy profile of hill with a ball (from Blokhintsev, 1988).

Despite the extreme simplicity of the model, its consistent implementation proves as not as simple, and therefore, instead of a complete solution, Blokhintsev gives the first order of perturbation theory. Since even this one requires quite lengthy calculations that could obscure the essence of the matter, we instead present qualitative considerations, illustrating the physics of the issue and reproducing the more rigorous result.

The qualitative idea is as follows. Before the scattering of the standing wave on the ball, the wave function of the system in the initial state is simply the product of the standing wave of the "light" component and the wave of the stationary state (as, for example, in a well) of the "heavy" component. After scattering (i.e., in the final state), in the first order in the perturbation, the result can be written immediately, without formally solving the Schrödinger equation: waves are transformed into a reflected plane wave of the "light" component and a slow plane wave of the "heavy" component. And further, in first order in perturbation, the interaction links the plane waves of the colliding components into a product of non-overlapping plane waves of light and heavy particles $\exp[(i/\hbar)p_l x]\exp[(i/\hbar)P_h Q]$ F(x, Q), (here x and Q, and p and P – coordinates and linear momenta of light quantum and heavy classical particles, respectively). Specifically, according to the conservation of momentum, the factor F(x, Q) in the wave function does not disappear only if at the coordinates of $x_{light} > 0$, then the coordinates of $Q_{heavy} < 0$, and vice versa. Mathematically, this corresponds to the contribution of the stationary phase point - see below, i.e. when the light and heavy components fly apart in opposite directions, in full accordance with the conservation of momentum. This is what disrupts the interference of the wave function components after the collision. In terms of the coordinate density matrix, this corresponds to the disappearance of its off-diagonal elements (for technical details, see Blokhintsev, 1988).

To avoid the impression that the author simply adjusted the reasoning to suit the desired result, a more consistent analysis follows.

Namely, using the Green functions technique, the scattered wave can be represented - in the spirit of an integral Lippmann-Schwinger equation, but for time-dependent wave functions - as a convolution of the full desired wave with the unperturbed Green's function of the system (light particle + ball), which, in turn, is merely a product of free Green functions of light and heavy components:

$$\Psi_{scat}(X, t)=\int G_{\mathbf{system}}(X-X', t-t')V(X')\Psi_{total}(X', t')dX'dt', \qquad \text{(III.2.1)}$$

$$G_{system}(x,Q,t) = G_{light}(x, t)G_{heavy}(Q,t)$$

where X – the cumulative notation for coordinates of light and heavy components (x, Q), and V(X') – the potential of the component interaction. For simplicity, a one-dimensional problem is considered here. Clearly, the main contribution to this integral comes from the total phase of the Green's function, and the latter is simply the sum of the phases of the components, so that the stationary point of the total phase is located near the center of gravity of the classical ball (M>>m).

For certainty, we illustrate further calculations in the first order in the perturbation and set the total wave function equal to the incident wave. Then

$$\Psi_{scat}(X) = \int G_{\mathbf{system}}(X-X', t-t')\, V(X')\, \Psi_{initial}(X',t')dX' \qquad \text{(III.2.2)}$$

where the interaction is taken as delta-function, i.e. $V \sim \delta(x-Q)$. Further, the initial wave function is the product of the superposition of plane waves of the light particle by the initial wave function of the heavy ball $\Phi_0(Q)$, similar to (III.2.1)

$$\Psi_{initial}(x,Q,t') = 2^{-1/2}\{(\exp[-(i/\hbar)p_0x] + \exp[+(i/\hbar)p_0x]\}\,\Phi_0(Q) \qquad \text{(III.2.3)}$$

where x and Q are the coordinates of light and heavy particles, respectively. Thus, for both momenta $\pm p_0$ the scattered wave can be written uniformly as

$$\Psi_{scat}^{\pm}(x, Q) \sim \int dx' \frac{d\tau}{\tau} \exp\{i[m\frac{(x-x')^2}{2\hbar\tau} + M\frac{(Q-x')^2}{2\hbar\tau}]\}\exp[\pm(i/\hbar)p_0x']\Phi_0(x')\exp(-i\frac{E_0}{\hbar}\tau) \qquad \text{(III.2.4)}$$

Where $\tau = t-t'$, m and M – are masses of quantum particle and heavy ball, respectively, $\pm p_0$ – momenta of quantum particle before the collision, E_0- the total energy of the ball in the hole before the collision. where $\tau = t-t'$, m and M – masses of light quantum particle and heavy classical ball, respectively, $\pm p_0$ – the momentum of a quantum particle before a colllision, E_0- the energy of the ball in a hole. (All additional technical details in formula (III.2.4) are contained in (Blokhintsev, 1988)).

Further, the time integration over $d\tau$ with the weight $1/\tau$ leads to a relatively slow factor $\ln \tau$, and, since at small τ the phase of the exponential rapidly oscillates, then, within the accepted accuracy, the functional behavior of the phase permits to write the result of integration over τ simply as the phase at the upper limit t.

After this, the spatial integration can be performed exactly, but we will not do this, and for clarity will simply use the stationary phase method. Namely, the stationary phase point z follows from differentiating the phase of the oscillating exponential, and, upon neglecting the phase of the heavy ball as concentrated around the initial pit, looks like this:

$$-\frac{m}{\hbar t}(x-z) - \frac{M}{\hbar t}(Q-z) \pm \frac{p_0}{\hbar} \approx 0 \text{ или, при } m \ll M, \quad z \pm \frac{p_0}{M}t \approx \frac{m}{M}x + Q \qquad \text{(III.2.5)}$$

and readily demonstrates the conservation of momentum when a light particle is reflected from a heavy ball. This immediately implies the decomposition of the total system wave function (light particle + ball) into a sum of non-interfering components.

$$\Psi_{scat}(x, Q) \sim \exp[-(i/\hbar)px]\exp[(i/\hbar)PQ]F_+ + (\exp[(i/\hbar)px]\exp[-(i/\hbar)PQ]F_- \qquad \text{(III.2.6)}$$

where p and P – momenta of light and heavy components and F_+ и F_- are factors of Heaviside type, i.e. $F_+ \neq 0$, only ,if $Q > 0$, and $F_- \neq 0$ only, if $Q<0$, so that, $F_+(Q)\,F_-^*(Q) \dashrightarrow 0$, quite analogously to (III.1.2).

In other words, the result of the qualitative calculation is reproduced, as desired. (Similar arguments apply in all orders of perturbation theory, since the stationary phase point arises primarily from the unperturbed Green's function).

Thus, the destruction of interference emerges from the fact that, while the wave function of an electron beam with a certain momentum is distributed everywhere in space, the wave function of a classical ball is concentrated near the center of gravity of the ball, and the Green's function technique only reflects this trivial fact. Thus, we again arrive at what was noted above: the resulting wave function (or density matrix, for that matter) decomposes into practically non-interfering terms, which allows the classical ball to serve as a “pointer” / arrow on the scale of any macroscopic measuring instrument. Let us add in conclusion that

this method is obviously applicable to any number of superposition components, including for two- and three-dimensional cases.

§3. General aspects of interference of scattering centers

Interference manifests itself not only in the incident beam, but also in the work of scattering centers. Consider in this regard the general problem of scattering a beam of microparticles with a fixed momentum off two (or more) scatterers. This setup, as we shall see below, is applicable not only to the functioning of a photographic plate, but also to scattering on many slits, etc.. Even the Born approximation proves sufficient to demonstrate the corresponding effects. Indeed, in this approximation the scattered wave can be written via the Green function as

$$\Psi_{scat}(\mathbf{r}) = \left(\frac{m}{2\pi\hbar^2}\right)\int G(\mathbf{r}-\mathbf{r}')\, V(\mathbf{r}') \exp(\mathbf{kr})\, d^3x' \quad \text{and} \quad G(\mathbf{r}-\mathbf{r}') = \frac{1}{|\mathbf{r}-\mathbf{r}'|} e^{i\mathbf{k}|\mathbf{r}-\mathbf{r}'|}, \; k = \frac{\mathbf{p}}{\hbar} \qquad \text{(III.3.1)}$$

If the potential V is located at point Q, then it can be written as V(r-Q). In the limit of long scattered waves or in the case of concentrated / point scattering, it is permissible to take the incident wave out of the convolution integral, i.e. sum up all the terms of the scattering series locally. Then

$$\Psi_{scat}(\mathbf{r}) = \exp(\mathbf{kQ})\, F(\mathbf{k}, \mathbf{r}-\mathbf{Q}) \quad \text{and} \quad F(\mathbf{k}, \mathbf{r}-\mathbf{Q}) = \left(\frac{m}{2\pi\hbar^2}\right)\int \frac{1}{|\mathbf{r}-\mathbf{r}'|} e^{i\mathbf{k}|\mathbf{r}-\mathbf{r}'|} V(\mathbf{r}')\, d^3x' \qquad \text{(III.3.2)}$$

i.e. the scattered wave is obtained as the product of the amplitudes: the original plane wave at the location of the scatterer and the scattering amplitude itself of the incident plane wave (we omit the effects of the non-elastic scattering). For several scatterers, for instance, two, it follows then (Blokhintsev, 1988)

$$\Psi_{scat}(\mathbf{r}) = \exp(\mathbf{kQ_1})\, F(\mathbf{k}, \mathbf{r}-\mathbf{Q_1}) + \exp(\mathbf{kQ_2})\, F(\mathbf{k}, \mathbf{r}-\mathbf{Q_2}) \qquad \text{(III.3.3)}$$

Note that we assume here the scattering occurring through a simple deterministic potential, permitting the solution of the Schrödinger equation. In general, this is, of course, not the case - see below. From this it follows directly that the total amplitude on the screen behind the scatterers can be written as the sum of the products of the incident wave amplitude at the scatterer location and the scattering amplitude itself. Then two limiting cases follow. The first is when the scatterers are located quite closely, so the distances between them are much smaller than the scattering radius (which we consider merely as a distance at which the impact of a scatterer is “felt”). This is a typical case of interference at two or more slits. Then the amplitude observed on the screen will be actually proportional to the coherent sum of incident wave amplitudes on each of the scatterers, modulated by the scattering amplitude (which for practical purposes can be roughly viewed identical for all scatterers). This is where all the interference effects in the paradigmatic double-slit scattering come from (see, for example, Feynman, 1965). Another limiting case is when, on the contrary, the distances between the scatterers are much larger than the scattering radius. Then each scatterer acts, essentially, independently of the others, and the interference on a screen is effectively suppressed (this takes place, for example, when the observation screen is very close to the plane of diffusers – closer, then the scattering radius, so that the picture right behind the diffuser is determined practically by this diffuser only). The same result materializes for the excitation of atoms in the seeds of a photographic plate used to

determine where exactly the scattering of an incident wave occurred (see, for example, Blokhintsev, 1988). A similar mechanism is essentially involved in the formation of tracks in a bubble chamber (Wilson chamber), (see on this matter Blokhintsev, 1968). These considerations show how the interference of scattered waves is preserved, or, on the contrary, destroyed, depending on the ratio of the distances between the scatterers and the scattering length.

§4. Coherence vs interference

In conclusion, let's emphasize once again the following important consideration. The destruction of interference can occur via both irreversible and reversible ways. In the first case - with an increase in entropy - the loss of the coherence, takes place with the immediate disappearance of interference as a direct consequence: this, for instance, always happens in the detection process. But the interference suppression is quite possible even without a destruction of the coherence. Indeed, if the work of the analyzer - i.e., suppression of interference and separation of initially interfering beams - is carried out either via external fields or ideal/deterministic scattering, then this process is reversible. Quantitatively, if the interference suppression can be obtained from the solution of the Schrödinger equation with a deterministic potential, then, because of the reversibility of the Schrödinger equation, the analyzer function is also reversible. This was, for instance, explicitly noted by Bohm (Bohm, 1951/1989) in connection with the Stern-Gerlach effect (see, also, below). The same can be obtained in the scattering of a standing wave on a macroscopic ball in the resonator, via the reflection of the scattered waves from the resonator mirrors. Clearly, the next step – detection – is always associated with an increase of entropy, and, accordingly, is accompanied by an irreversible loss of coherence. This emphasizes the difference in the functioning of the analyzer and the detector: while the former carries out the controlled scattering of the beam (which corresponds to the deterministic potential in the Schrödinger equation), then the latter effectuates an uncontrolled exchange of quanta between the quantum object and the detector itself. In Bohm's book (Bohm, 1951/1989) this is specifically noted and emphasized (p. 120):"... **we must take into account the fact that, while an observation is taking place, there is an interaction between the particle and the observing apparatus. Thus far, Schrödinger's equation, which defines the wave function, has been derived only for a free particle. The effect of any kind of force of interaction (for example, electrical, gravitational, electromagnetic, etc.) is to modify Schrodinger's equation**."[3]

Supplement. **Imaginary diffusion in Rydbergs.**

Highly excited (Rydberg) atoms are an interesting example of the so-called mesoobjects: intermediate between micro-quantum and macro-classical ones.
Accordingly, the statistical nature of wave manifestation in Rydberg atoms materializes by means of an imaginary diffusion over energy levels. Besides, although in connection with the

[3] This is why the work of the analyzer is fundamentally reversible, but the detector - is not. Indeed, the detector is a relatively localized object, placed in "its" beam of superposition components separated by the analyzer, and in the process of exchanging quanta between the latter and the detector, the coherent phase is irreversibly destroyed, so that the system is described by a probabilistic mixture of superposing wave functions (density matrix), each of which is funneled to "its own" detector.

latter, Rydberg atoms exhibit remarkable analytic properties. Here we are not going to delve into the nature of highly excited (almost classical) atomic levels and transitions between them in collisions with neutral or charged projectiles (for the detailed description and bibliography the readers are referred to the monograph of Beigman and Lebedev (Lebedev, Bejgman, 1999)). Our goal here is different: to show how the statistical nature of quantum mechanics manifests itself in Rydbergs, and to demonstrate the efficiency of the multilevel system analytics for the processes in Rydbergs and their basic characteristics.

As just noted above, highly excited (Rydberg) states of atoms and molecules have remarkable analytical properties. Namely, the energy spectrum of Rydberg states near the ionization boundary in a limited range of principal quantum numbers Δn is very close to the oscillatory one, i.e. to equidistant, since the energy distance between levels decreases as n^{-3}. Therefore, the migration over such levels under the impact of an external collision potential (more precisely, its main part - the dipole one) can be approximated - in a limited range of principal quantum numbers - by a "ladder" of interaction of the neighboring levels (Δk = 1) amplitudes

$$i\hbar\dot{c}_k = Ve^{-i\omega t}c_{k+1} + Ve^{i\omega t}c_{k-1}, \quad -\infty < k < \infty \tag{S.1}$$

where ω - is the transition frequency between them. Here, as just indicated, are taken into account only transitions between adjacent levels Δk = 1, corresponding to the dipole potential V(t), dominating the interaction with perturbing (charged) particles (Transitions with Δk = 2.3, etc. correspond to multipole interactions and are treated similarly).
Note that in the equidistant model, the potential V does not depend on the level number, but only on the multipolarity of the interaction (in the case at hand, it is Δk = 1). In the limit of k-> ∞ the oscillating exponents are negligible. Then, by means of the unitary transformation c_k -> $c_k e^{-2i\int V dt}$ (S.1) reduces to

$$i\hbar\dot{c}_k = Vc_{k+1} - 2Vc_k + Vc_{k-1}, \quad -\infty < k < \infty \tag{S.2}$$

which, in the limit of k-> ∞, is nothing but the equation of a diffusion in the energy space with the imaginary coefficient

$$i\hbar\dot{c} = Vc''_{\varepsilon\varepsilon}$$

where c = c(ε) is the discrete amplitude c_k, mapped from k-space to the ε – space, where ε is an energy ε of the Rydberg electron.
Thus, we again come to the diffusion view of the Schrödinger equation, as in Part I, Sec. 1. This once again confirms the diffusion-like - and therefore, the statistical - nature of the Schrödinger equation.
Moreover, it turns out that the original discrete system (S.1) can be solved in a closed analytic form by rendering the amplitudes c_k as the k-Fourier components of a certain generating function (for further details see Presnyakov, Urnov,1970)). The result proves rather simple: $c_k \sim J_k(2P_1)$, where J_k is the Bessel function of the order k, and $P_1 = |\int_{-\infty}^{\infty} dt\, V(t)^{i\omega\tau}|$ - is the module of the amplitude of the first order.

In turn, the generating function is obtained in the semiclassical approximation as a correction from collisions with neutral or charged particles to the classical Hamilton-Jacobi function (for example, in laboratory or astrophysical plasmas). More details of these and related issues are discussed in (Lebedev, Beigman, 1999).
A similar approach is applicable to collisional transitions between sublevels with angular momentum l. The only difference is that, while an infinite system of n-level energies is symmetric with respect to up-down transitions relative to the starting point, transitions between l-sublevels are limited from below by the state l = 0, i.e. the s-state (semi-infinite system!). This difference is relatively easy to eliminate. Namely, the solution of a semi-infinite system with the initial point at l = 0, i.e. in the s-state, is obtained simply by the "reflection" relative to the point k = 0 of the solution of the "infinite" system:

$$c_k \sim [J_{k-1}(2P_1) + J_{k+1}(2P_1)] \qquad (S.3)$$

where the notations are as above. In reality, though, the energy gap between l-sublevels rapidly decreases with the angular momentum l, so a more suitable model would be a semi-infinite system separated from the initial level (say, the s- or p-level) by a notable gap.
The solution to this problem is way more complex, but can be readily obtained in the first (Born) order of perturbation theory. Specifically, the transition probability is

$$|c_k|^2 \sim [k/(2P_1)J(4P_1)]^2 \, W_B(\rho,v) \qquad (S.4)$$

where W_B is a total Born probability for transitions from the level k_0 to all levels $k > k_0$ as a function of the impact parameter ρ and the projectile velocity v. An important advantage of this approximate formula is that it explicitly satisfies the unitarity condition, i.e., does preserve the total transition probability.

Instead of a conclusion

Any reasoning in wave mechanics concerning particles requires special care due to the conceptually limited applicability of the latter in quantum theory. As Glauber (1995) eloquently noted, in quantum mechanics it is not the particles that interfere, but the probability amplitudes of the corresponding events. However, in the semiclassical approximation, when a particle can be provisionally considered as still classical, but an already smeared in space object - a cloud, although concentrated mostly around its classical center of mass, both concepts - corpuscular and wave - are approximately compatible. We all are classical creatures, and absorb the corresponding worldview with our mother's milk. Therefore, one must be prepared for a critical re-evaluation of fundamental classical concepts when moving into other worlds.

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