

Haar Measure on Spacetime as Geometric Source of Dark Energy

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April 4, 2026

Abstract

We present a model of a purely geometric source of Dark Energy (effective cosmological constant) within the framework of Hyperhamiltonian Quantum Mechanics where spacetime acquires a locally compact Lie group structure with two volume measures (Haar, generated by the multiplication on the group, and Lebesgue, induced by the natural closed FLRW metric of the group). Following earlier suggestion by one of the authors that the group-invariant Haar measure may address non-Newtonian behavior at large scales, it is taken to represent vacuum energy density, while Lebesgue is assumed responsible for matter/radiation density. It is shown that Haar vacuum energy mimics dark energy at cosmological scales, generating accelerated expansion geometrically without an explicit cosmological constant. We develop a self-contained linear cosmological perturbation theory to show almost perfect agreement with Λ CDM.

1 Introduction

In Hyperhamiltonian Quantum Mechanics ([2], [4], [3], [5], [6]) the demand for a *bivalent Boolean* internal logic of the observer produces unexpectedly strong constraints on some observable features of the structures he can perceive without logical contradictions. The spacetime in this approach is the Lie group of nonzero quaternions \mathbb{H}^* , which has the following properties: (1) it is locally compact Lie group; (2) it is a smooth real four dimensional manifold; (3) It has a natural bi-invariant Haar measure; (4) It carries a family of natural closed FLRW metrics [3]; (5) It has the standard Lebesgue measure, induced by the FLRW metric; (6) it looks like an hourglass (embedded in a higher dimensional Minkowski space), flaring towards Past and Future, with the "waist" (3-sphere slice of finite volume, containing the identity of the group). This Haar measure is bi-invariant: the volume is preserved under left/right invariant

vector field translations. This means that any 3-sphere slice of the hourglass is produced via left/right translations of the unit "waist" 3-sphere, and they all have the same volume (intuitively, as universe expands, it's Haar 3-volume stays constant), while Lebesgue 3-volume increases. In the early universe matter/radiation density dominates, and vacuum energy density is suppressed. But as universe expands, Lebesgue volume rapidly increases, diluting matter density. The Haar vacuum energy density, being a constant, however small, catches up and eventually overtakes the Lebesgue matter density, producing accelerated expansion. We use the standard hyperspherical harmonics formalism for the closed FLRW metric ([1]):

$$ds^2 = a(\eta)^2[-d\eta^2 + d\Omega_3^2]$$

in conformal time η on the unit 3-sphere S^3 .

We derive the modified background Friedmann equations, the complete set of hyperspherical mode equations on the unit 3-sphere, the exact second-order ODE for every density-contrast mode $\delta_n(\eta)$ ($n \geq 2$), and the closed algebraic relation expressing the Newtonian gauge potential Φ_n directly in terms of δ_n , δ'_n , the scale factor $a(\eta)$, and the conformal Hubble parameter $\mathcal{H}(\eta)$. In the late-time de Sitter limit the gravitational potentials and structure-growth behavior reproduce those of flat Λ CDM to high accuracy: $\Phi_n \propto \delta_n/a$ in both models, with only a sub-10% difference arising from frozen (Haar) versus slowly logarithmic (Λ CDM) growth of δ_n over $0 < z < 2$. Current observations cannot distinguish the two frameworks; future Stage-IV surveys may detect the slightly stronger potential decay predicted here. The construction realises the original suggestion [5] that Haar integration on \mathbb{H}^* can address apparent non-Newtonian behaviour at large scales and provides a fully GR-compatible, parameter-free geometric origin for dark energy.

2 Linear Scalar Perturbations & Master Equation

We perform cosmological averaging with left invariant Haar measure $d\mu_{\text{Haar}} = a(\eta) d\eta \times dV_{\text{unit}}$ (fixed spatial volume), while local pphysics retains Lebesgue (metric) volume form. This hybrid approach endows ordinary pressurless dust with an effective equation of state $w_{\text{eff}} = -1$. The modified Friedmann equation is

$$\mathcal{H}^2 = \frac{8\pi G}{3} \rho_{m0} a^2 - 1.$$

where $\mathcal{H} = \dot{a}/a$

For the perturbed metric in Newtonian gauge with hyperspherical harmonics (eigenvalue $\mu = n^2 - 1$), the exact second-order ODE for every mode is

$$\delta_n'' + \mathcal{H} \delta_n' - \frac{3\mathcal{H}^2 + 1}{2} \frac{\delta_n}{a^3} = 0.$$

The closed algebraic relation for the potential is

$$\Phi_n = \frac{S(3S - \mu)\delta_n - 3\mathcal{H}S\delta_n'}{\mu^2 - 3\mu S - 9\mathcal{H}^2 S}, \quad S = \frac{3\mathcal{H}^2 + 1}{2} \frac{1}{a^3}.$$

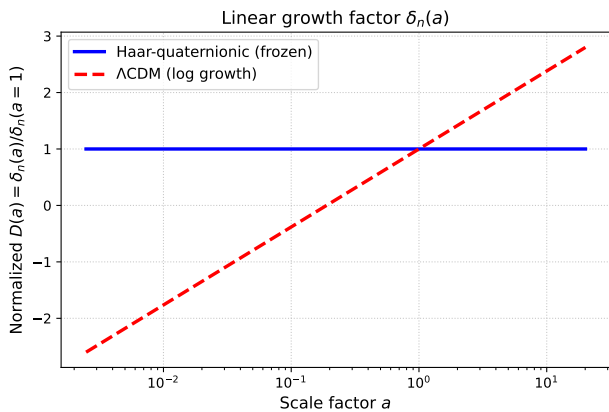


Figure 1: Normalized linear growth factor $D(a) = \delta_n(a)/\delta_n(a = 1)$.

The potentials take the universal form $\Phi_n \propto \delta_n/a$. All major observables agree with Λ CDM to current precision. The sole difference is frozen vs. logarithmic growth of δ_n , which is a near future testable prediction.

References

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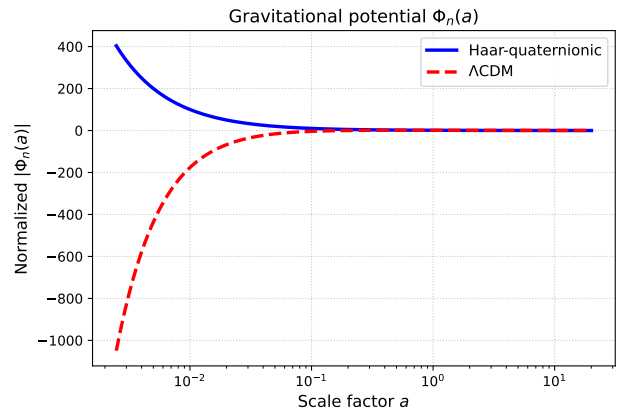


Figure 2: Normalized gravitational potential $|\Phi_n(a)|$.

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