

# A Phenomenological Harmonic Semilattice for Selected Physical Scales

Alberto Coe

## Abstract

We examine the possibility that selected physical observables may approximately organize around a discrete half-integer harmonic semilattice generated by the ratio  $3/2$ . Taking the electron rest energy as a reference scale, we consider

$$E_i \approx E_e \left(\frac{3}{2}\right)^{q_i}, \quad q_i \in \frac{1}{2}\mathbb{Z}.$$

A simple mismatch statistic suggests that the observed alignment is better than expected under a random-logarithmic null model, and comparison with several nearby rational alternatives indicates that the ratio  $3/2$  is empirically preferred within the tested class.

We also note a dimensionless length relation involving reduced Compton wavelengths,

$$\frac{\lambda_{ce}\lambda_{cp}}{e\alpha^2 L_P^2} \approx \left(\frac{3}{2}\right)^{3/2},$$

and a compact muonic relation,

$$\left(\frac{3}{2}\right)^{5/2} N_* E_\mu \approx \hbar \times 10^{52}.$$

The proposal is presented as a constrained phenomenological hypothesis rather than a fundamental theory.

## 1 Introduction

The distribution of physical scales remains one of the most persistent open structural questions in fundamental physics. This note explores the possibility that a subset of observed rest-energy and length scales may exhibit a sparse logarithmic organization generated by the ratio  $3/2$ .

## 2 Half-Integer Harmonic Semilattice

We consider the phenomenological relation

$$E_i \approx E_e \left(\frac{3}{2}\right)^{q_i}, \quad q_i \in \frac{1}{2}\mathbb{Z},$$

where  $E_e$  is the electron rest energy.

For each observed energy  $E_i$ , define

$$q_i^{\text{cont}} = \frac{\ln(E_i/E_e)}{\ln(3/2)}.$$

The nearest semilattice site is denoted

$$q_i^{\text{lat}} \in \frac{1}{2}\mathbb{Z}.$$

For the selected dataset considered here, the resulting assignments are:

Table 1: Selected spectral assignments.

Particle	$q_i^{\text{cont}}$	$q_i^{\text{lat}}$	Relative error
electron	0.000	0.0	0.00%
up quark	3.57	3.5	2.2%
down quark	5.45	5.5	1.7%
muon	13.13	13.0	5.8%
proton	18.52	18.5	1.4%
Higgs boson	30.61	30.5	4.8%
top quark	31.43	31.5	3.4%

### 3 Simple Statistical Test

Define the nearest-site mismatch

$$d_i = \min_{q \in \frac{1}{2}\mathbb{Z}} |q_i^{\text{cont}} - q|,$$

and the mean mismatch

$$D = \frac{1}{N} \sum_i d_i.$$

Excluding the electron, which is fixed by construction, the observed value is

$$D(3/2) \approx 0.0811.$$

Under a simple random-logarithmic null model, the expected mean residual is

$$\mathbb{E}[D] = 0.125,$$

with an approximate one-sided p-value

$$p \approx 0.07.$$

This is not statistically decisive, but it suggests that the observed alignment is better than expected from random logarithmic placement.

### 4 Comparison with Alternative Rational Bases

To test whether the ratio 3/2 is distinguished, we compare it against several nearby rational alternatives.

For each candidate ratio  $r$ , define

$$D(r) = \frac{1}{N} \sum_i \min_{q \in \frac{1}{2}\mathbb{Z}} \left| \frac{\ln(E_i/E_e)}{\ln r} - q \right|.$$

The tested values are:

Table 2: Comparison with alternative rational bases.

Ratio	Mean distance $D(r)$
3/2	0.0811
5/4	0.0951
8/5	0.1046
7/4	0.1070
4/3	0.1227
7/5	0.1493
5/3	0.1515

Among the tested candidates, 3/2 yields the smallest mismatch.

## 5 Independent Supporting Relations

We define

$$N_* = \frac{a_0}{e \alpha^2 L_P},$$

where  $e$  denotes Euler's number.

Two additional empirical relations are then observed:

### Length sector

$$\frac{\lambda_{ce} \lambda_{cp}}{e \alpha^2 L_P^2} \approx \left(\frac{3}{2}\right)^{3/2}.$$

### Muon sector

$$\left(\frac{3}{2}\right)^{5/2} N_* E_\mu \approx \hbar \times 10^{52}.$$

Both relations are notable because they independently reproduce the same half-integer harmonic structure.

## 6 Geometric Heuristic

A simple geometric motivation for the ratio 3/2 may be obtained from the ratio between the spatial diagonal and face diagonal of a cube:

$$\frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}.$$

If repeated transitions between effective surface-like and bulk-like modes are associated with this factor, one is naturally led to

$$\left(\frac{3}{2}\right)^q, \quad q \in \frac{1}{2}\mathbb{Z}.$$

This interpretation is heuristic only, but it provides a possible geometric intuition for the appearance of half-integer powers of 3/2.

## 7 Conclusion

We have identified a constrained phenomenological pattern in which selected physical observables appear to cluster around a half-integer harmonic semilattice generated by the ratio  $3/2$ .

The present note does not propose a dynamical theory. Its value lies instead in isolating a compact and testable numerical structure that appears across several independent quantities. Whether this reflects a deeper physical principle remains an open question.