Nonlocal one-loop form factors of the spectral action with Standard Model content

David Alfyorov*

We compute the complete nonlocal one-loop form factors $F_1(\Box/\Lambda^2)$ and $F_2(\Box/\Lambda^2, \xi)$ of the curvature-squared sector of the spectral action $S = \mathrm{Tr}\ f(D^2/\Lambda^2)$ for the full Standard Model particle content: 4 real scalars (Higgs), 45/2 Dirac-equivalent fermions (3 generations), and 12 gauge bosons ($\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$). Using the Barvinsky–Vilkovisky covariant perturbation theory and the Codello–Zanusso diagrammatic heat kernel, we derive closed-form results for each spin sector (0, 1/2, 1) in the $\{C^2, R^2\}$ Weyl basis and assemble the Standard Model totals. The local limits, determined by standard heat kernel coefficients [1, 2], yield $\alpha_C = 13/120$ for the Weyl-squared coefficient and $\alpha_R(\xi) = 2(\xi - 1/6)^2$ for the R^2 coefficient, where ξ is the Higgs non-minimal coupling. Both form factors are shown to be entire functions of $\Box/\Lambda^2$, ensuring that the one-loop effective action introduces no additional propagator poles beyond those of the classical theory. We derive the c_1/c_2 ratio in the $\{R^2, R^2_{\mu\nu}\}$ basis, the scalar graviton decoupling condition at conformal coupling $\xi = 1/6$, and the UV asymptotic behavior. The form factors yield a modified Newtonian potential with calculable effective masses $m_2 = \Lambda\sqrt{60/13}$ and $m_0 = \Lambda/\sqrt{6(\xi - 1/6)^2}$, connecting the spectral action framework to solar-system phenomenology. All results are verified by independent multi-precision numerical evaluation.

I. INTRODUCTION

The spectral action principle [3, 4] provides a geometric origin for both gravitational and gauge interactions: the bosonic action is given by $S = \mathrm{Tr}\ f(D^2/\Lambda^2)$, where D is the Dirac operator of a noncommutative geometry, Λ is a cutoff scale, and f is a positive even function. At the classical level, the heat kernel expansion of this trace reproduces the Einstein–Hilbert action, cosmological constant, and Yang–Mills terms from the first few Seeley–DeWitt coefficients [5, 6].

Beyond the leading Seeley–DeWitt approximation, the spectral action generates *nonlocal* curvature-squared terms characterized by momentum-dependent form factors $F_1(\Box/\Lambda^2)$ and $F_2(\Box/\Lambda^2)$. These form factors encode the full one-loop quantum gravitational effective action at $\mathcal{O}(\mathcal{R}^2)$ and determine the modified graviton propagator, the spectrum of massive gravitational modes, and the UV behavior of the theory.

The Barvinsky–Vilkovisky (BV) covariant perturbation theory [7, 8] provides the general framework for computing such form factors for any generalized Laplacian $\Delta = -(g^{\mu\nu}\nabla_\mu\nabla_\nu + E)$ on a vector bundle over a curved manifold. The Codello–Zanusso (CZ) diagrammatic heat kernel technique [9] gives explicit expressions for the five independent form factors $(f_{\mathrm{Ric}}, f_R, f_{RU}, f_U, f_\Omega)$ in terms of a universal master function $\varphi(x)$. The Seeley–DeWitt coefficients and their role in spectral geometry are reviewed in [1, 2, 10–12].

Previous work has computed form factors for individual spin sectors [8, 9] or for simplified particle content [13]. In this paper, we present the first complete computation with the full Standard Model (SM) spectrum: 4 real scalars (the Higgs doublet), $N_f = 45$ Weyl fermions (equivalently $N_D = 45/2$ Dirac fermions), and 12 gauge bosons of the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge group. The counting convention follows Codello, Percacci, and Rahmede (CPR) [13].

The main results are:

- Closed-form expressions for $h_C^{(s)}(x)$ and $h_R^{(s)}(x)$ for $s = 0, 1/2, 1$ in the Weyl basis $\{C^2, R^2\}$, with correct ghost subtraction for spin-1.
- The SM totals $\alpha_C = 13/120$ (fixed by the SM content) and $\alpha_R(\xi) = 2(\xi - 1/6)^2$ (depending only on the Higgs non-minimal coupling ξ).
- The proof that F_1 and F_2 are entire functions of $z = \Box/\Lambda^2$, guaranteeing the absence of new poles beyond those inherited from the propagator structure.
- The c_1/c_2 ratio, scalar graviton decoupling at $\xi = 1/6$, and UV asymptotics.

The structure of the paper is as follows. Section II establishes the conventions and reviews the BV/CZ formalism. Sections III–V derive the form factors for each spin sector. Section VI assembles the SM totals and derives their properties. Section VII discusses the entire-function nature, UV behavior, and the effective graviton masses. Section VIII compares with existing results and discusses implications. We conclude in Sec. IX.

Throughout this paper we use the Euclidean signature $(+,+,+,+)$, natural units $c = \hbar = 1$, and the generalized Laplacian convention $\Delta = -(g^{\mu\nu}\nabla_\mu\nabla_\nu + E)$ of Barvinsky and Vilkovisky [7]. The Lorentzian continuation to $(-,+,+,+)$ signature is discussed in Sec. VIII. A summary of the principal results is given in Table I.

* davidich.alfyorov@gmail.com

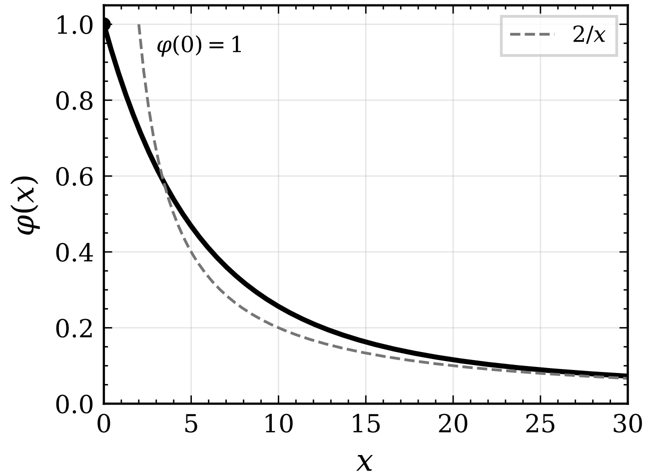

FIG. 1. The master function $\varphi(x)$, where $x = \Box/\Lambda^2$. Starting from $\varphi(0) = 1$, the function decays monotonically and approaches the asymptotic form $2/x$ (dashed) for $x \gg 1$.

II. SETUP AND CONVENTIONS

A. Generalized Laplacian and spectral action

We consider a closed (compact, boundaryless) Riemannian spin 4-manifold (M, g). The spectral action at $\mathcal{O}(\mathcal{R}^2)$ in the Weyl basis is

$$S_{\rm spec}\Big|_{\mathcal{R}^2} = \int_M \mathrm{d}^4x\,\sqrt{g}\,\big[F_1(z)\,C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + F_2(z,\xi)\,R^2\big]\,, \tag{1}$$

where $z \equiv \Box/\Lambda^2$ and Λ is the spectral cutoff scale. For a generalized Laplacian $\Delta = -(g^{\mu\nu}\nabla_\mu\nabla_\nu + E)$ acting on a vector bundle of rank d_V (so that $\mathrm{tr}\,\mathbf{1} = d_V$), the CZ endomorphism is $\mathbf{U} = -E$ and the bundle curvature is $\Omega_{\mu\nu} = [\nabla_\mu, \nabla_\nu]$.

B. Master function

All form factors are built from the universal master function

$$\varphi(x) = \int_0^1 \mathrm{d}\xi\, e^{-\xi(1-\xi)x}\,, \tag{2}$$

with the closed form

$$\varphi(x) = e^{-x/4}\sqrt{\frac{\pi}{x}}\;\mathrm{erfi}\left(\frac{\sqrt{x}}{2}\right)\,. \tag{3}$$

Key properties: $\varphi(0) = 1$, $\varphi'(0) = -1/6$, and Taylor coefficients $a_n = (-1)^n\, n!/(2n+1)!$, so that $\varphi(x) = \sum_{n=0}^{\infty} a_n\, x^n$ with infinite radius of convergence. In particular, φ is an entire function of order 1 (Fig. 1).

C. CZ form factors

The five CZ form factors [9] are:

$$f_{\rm Ric}(x) = \frac{1}{6x} + \frac{\varphi - 1}{x^2}\,, \tag{4a}$$

$$f_R(x) = \frac{\varphi}{32} + \frac{\varphi}{8x} - \frac{7}{48x} - \frac{\varphi - 1}{8x^2}\,, \tag{4b}$$

$$f_{RU}(x) = -\frac{\varphi}{4} - \frac{\varphi - 1}{2x}\,, \tag{4c}$$

$$f_U(x) = \frac{\varphi}{2}\,, \tag{4d}$$

$$f_\Omega(x) = -\frac{\varphi - 1}{2x}\,. \tag{4e}$$

Their local limits are $f_{\rm Ric}(0) = 1/60$, $f_R(0) = 1/120$, $f_{RU}(0) = -1/6$, $f_U(0) = 1/2$, $f_\Omega(0) = 1/12$.

D. Weyl basis assembly

For any spin sector, the $\{C^2, R^2\}$ form factors are obtained from the CZ form factors via the BV assembly rules. The details depend on the endomorphism $\mathbf{U}$, the bundle curvature $\Omega_{\mu\nu}$, and the trace identities specific to each spin [8, 14]. We write the result as

$$F_i^{(s)}(z) = \frac{h_i^{(s)}(z)}{16\pi^2}\,, \quad i = C, R\,, \tag{5}$$

where $s = 0, 1/2, 1$ labels the spin and $h_C^{(s)}, h_R^{(s)}$ are the reduced form factors. The local limits $\beta_W^{(s)} \equiv h_C^{(s)}(0)$ and $\beta_R^{(s)} \equiv h_R^{(s)}(0)$ are the one-loop β-function coefficients of the C^2 and R^2 operators.

III. SPIN-0: REAL SCALAR

For a real scalar field with non-minimal coupling $\xi R\phi^2/2$, the generalized Laplacian has $\mathrm{tr}\,\mathbf{1} = 1$, $\mathbf{U} = \xi R$, and $\Omega_{\mu\nu} = 0$ (trivial bundle). The BV assembly gives

$$h_C^{(0)}(x) = \frac{1}{12x} + \frac{\varphi - 1}{2x^2}\,, \tag{6a}$$

$$h_R^{(0)}(x,\xi) = f_{R,\rm bis}(x) + \xi\, f_{RU}(x) + \xi^2\, f_U(x)\,, \tag{6b}$$

where $f_{R,\rm bis} = \frac{1}{3}f_{\rm Ric} + f_R$. The local limits are

$$\beta_W^{(0)} = \frac{1}{120}\,, \qquad \beta_R^{(0)}(\xi) = \frac{1}{2}\left(\xi - \frac{1}{6}\right)^2\,. \tag{7}$$

The individual spin-sector form factors are plotted in Figs. 2 and 3. The Weyl coefficient $\beta_W^{(0)}$ is independent of ξ (the Weyl tensor is traceless, so the R-proportional endomorphism decouples from C^2). The R^2 coefficient vanishes at conformal coupling $\xi = 1/6$, reflecting the conformal invariance of the massless conformally-coupled scalar in $d = 4$.

IV. SPIN-1/2: DIRAC FERMION

For a massless 4-component Dirac fermion, the squared Dirac operator $D^2 = -(\nabla^*\nabla - R/4)$ gives $\mathbf{U} = R/4$ (from the Lichnerowicz formula) and $\mathrm{tr}(\Omega_{\mu\nu}\Omega^{\mu\nu}) = -\frac{1}{2}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ with $\mathrm{tr}\,\mathbf{1} = 4$. The reduced form factors are

$$h_C^{(1/2)}(x) = \frac{3\varphi - 1}{6x} + \frac{2(\varphi - 1)}{x^2}\,, \tag{8a}$$

$$h_R^{(1/2)}(x) = \frac{3\varphi + 2}{36x} + \frac{5(\varphi - 1)}{6x^2}\,, \tag{8b}$$

with local limits

$$\beta_W^{(1/2)} = -\frac{1}{20}\,, \qquad \beta_R^{(1/2)} = 0\,. \tag{9}$$

The negative sign of $\beta_W^{(1/2)}$ is a geometric consequence of the spinor bundle: the curvature contribution f_Ω (from $\mathrm{tr}\,\Omega_{\mu\nu}\Omega^{\mu\nu} = -\frac{1}{2}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$) outweighs the Ricci contribution f_{Ric} in the Weyl-squared projection, yielding $h_C^{(1/2)}(0) = 2\,f_{\mathrm{Ric}}(0) - f_\Omega(0) = 2/60 - 1/12 = -1/20$. Note that some references quote the absolute value $|\beta_W^{(1/2)}| = 1/20$, with the sign tracked separately via a $(-1)^{2s}$ factor [13]; in our convention, the sign is built into $h_C^{(1/2)}$.

The vanishing of $\beta_R^{(1/2)}$ reflects the conformal invariance of the massless Dirac action in $d = 4$.

V. SPIN-1: GAUGE BOSON

For a gauge boson (Proca field) in the spectral action framework, the generalized Laplacian acts on the vector bundle with $\mathrm{tr}\,\mathbf{1} = 4$, $\mathbf{U} = R_{\mu\nu}$ (Ricci tensor as endomorphism), and $\Omega_{\mu\nu} = R_{\mu\nu\rho\sigma}$ (Riemann tensor as bundle curvature) [2, 14].

The unconstrained (before ghost subtraction) form factors are [9]:

$$h_C^{(\mathrm{unc})}(x) = \frac{2}{3}f_{\mathrm{Ric}}(x) + \frac{1}{3}f_\Omega(x)\,, \tag{10a}$$

$$h_R^{(\mathrm{unc})}(x) = \frac{4}{3}f_{\mathrm{Ric}} + 4f_R + f_{RU} + \frac{1}{3}f_U - \frac{1}{3}f_\Omega\,. \tag{10b}$$

The coefficients $(4/3, 1, 1/3, -1/3)$ on $(f_{\mathrm{Ric}}, f_{RU}, f_U, f_\Omega)$ follow from the R^2 projection of the respective curvature invariants for $\mathbf{U} = R_{\mu\nu}$ and $\Omega_{\mu\nu} = R_{\mu\nu\rho\sigma}$ on the tangent bundle [8, 9]; they are verified by the local limit $\beta_R^{(1)} = 0$.

The physical spin-1 form factors require subtraction of two Faddeev–Popov ghost scalars (with $\mathbf{U} = 0$, $\Omega_{\mu\nu} = 0$):

$$h_i^{(1)}(x) = h_i^{(\mathrm{unc})}(x) - 2\,h_i^{(0)}(x)\Big|_{\xi=0}\,. \tag{11}$$

This yields

$$h_C^{(1)}(x) = \frac{\varphi}{4} + \frac{6\varphi - 5}{6x} + \frac{\varphi - 1}{x^2}\,, \tag{12a}$$

$$h_R^{(1)}(x) = -\frac{\varphi}{48} + \frac{11 - 6\varphi}{72x} + \frac{5(\varphi - 1)}{12x^2}\,, \tag{12b}$$

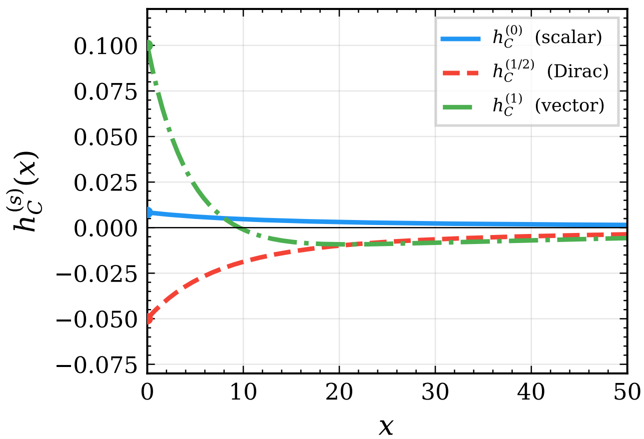

FIG. 2. Weyl-squared form factors $h_C^{(s)}(x)$ for the three spin sectors, where $x = \Box/\Lambda^2$. The local limits are $h_C^{(0)}(0) = 1/120$ (scalar, blue), $h_C^{(1/2)}(0) = -1/20$ (Dirac, red), and $h_C^{(1)}(0) = 1/10$ (vector, green). All form factors decay as $1/x$ for $x \to \infty$.

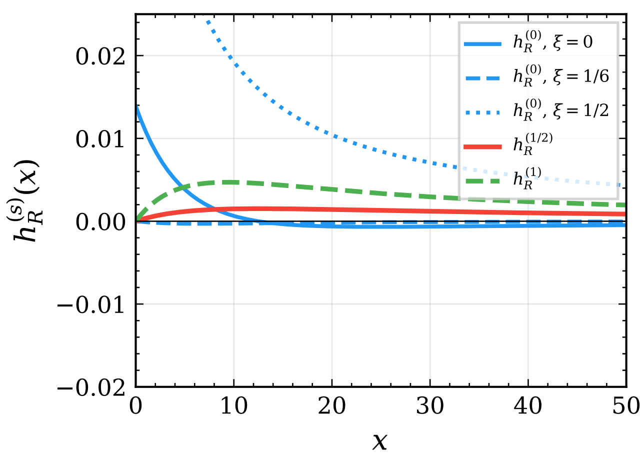

FIG. 3. R^2 form factors $h_R^{(s)}(x)$, where $x = \Box/\Lambda^2$. The scalar form factor (blue) depends on the non-minimal coupling ξ: it vanishes identically at conformal coupling $\xi = 1/6$ (dashed) and grows with $|\xi - 1/6|$. The Dirac (red) and vector (green) contributions are ξ-independent and start from $h_R(0) = 0$.

with local limits

$$\beta_W^{(1)} = \frac{1}{10}\,, \qquad \beta_R^{(1)} = 0\,. \tag{13}$$

The result $\beta_R^{(1)} = 0$ reflects the conformal invariance of the Maxwell action in $d = 4$. The ghost count of 2 (not 1) is essential: the unconstrained vector gives $\beta_W^{(\mathrm{unc})} = 7/60$, and subtracting two minimally-coupled scalar ghosts ($2 \times 1/120$) yields $7/60 - 1/60 = 1/10$. This matches the standard Faddeev–Popov procedure [14].

VI. STANDARD MODEL ASSEMBLY

A. Particle content

The SM field content relevant for the one-loop spectral action is:

Species	Count	Notation
Real scalars (Higgs doublet)	4	$N_s = 4$
Weyl fermions (3 generations)	45	$N_f = 45$
Gauge bosons $(8+3+1)$	12	$N_v = 12$

Since the form factors of Sec. IV are defined per 4-component Dirac fermion, the effective Dirac count is $N_D = N_f/2 = 45/2$, following the convention of [13].

B. Total Weyl coefficient

The total Weyl-squared form factor is

$$\alpha_C(z) = N_s\, h_C^{(0)}(z) + N_D\, h_C^{(1/2)}(z) + N_v\, h_C^{(1)}(z)\,. \tag{14}$$

At $z = 0$:

$$\begin{aligned}\alpha_C &= 4\cdot\frac{1}{120} + \frac{45}{2}\cdot\left(-\frac{1}{20}\right) + 12\cdot\frac{1}{10} \\ &= \frac{4-135+144}{120}\,,\end{aligned} \tag{15}$$

giving the central result:

$$\boxed{\alpha_C = \frac{13}{120} \approx 0.1083\,.} \tag{16}$$

This is positive and independent of ξ. The sign structure $(+4 - 135 + 144)$ shows that the vector sector $(+144/120)$ dominates the fermionic sector $(-135/120)$ by a margin of $+9/120$, while the scalar sector contributes $+4/120$. The positivity of α_C implies that the spin-2 gravitational mode in the linearized theory is a ghost [15], a well-known feature of quadratic gravity.

C. Total R^2 coefficient

$$\alpha_R(\xi) = N_s\cdot\frac{1}{2}\left(\xi - \frac{1}{6}\right)^2 + N_D\cdot 0 + N_v\cdot 0\,, \tag{17}$$

so that

$$\boxed{\alpha_R(\xi) = 2\left(\xi - \frac{1}{6}\right)^2.} \tag{18}$$

Only the scalar sector contributes: the Dirac and Maxwell β_R vanish by conformal invariance. The result is a perfect square, hence $\alpha_R(\xi) \geq 0$ for all ξ, with $\alpha_R(1/6) = 0$ at conformal coupling.

D. Basis conversion and c_1/c_2

Using the Gauss–Bonnet identity to convert to the $\{R^2, R_{\mu\nu}^2\}$ basis ($C^2 = 2R_{\mu\nu}^2 - \frac{2}{3}R^2$ modulo the topological Euler density), one obtains $c_1 = \alpha_R - \frac{2}{3}\alpha_C$ and $c_2 = 2\alpha_C$, giving

$$\boxed{\frac{c_1}{c_2} = -\frac{1}{3} + \frac{120\left(\xi - \frac{1}{6}\right)^2}{13}\,.} \tag{19}$$

At conformal coupling $\xi = 1/6$, this reduces to $c_1/c_2 = -1/3$. At minimal coupling $\xi = 0$, one obtains $c_1/c_2 = -1/13$.

E. Scalar mode decoupling

The combination $3c_1+c_2$ controls the mass of the scalar graviton ($m_0^2 \propto \Lambda^2/(3c_1+c_2)$). A direct computation gives

$$3c_1 + c_2 = 3\alpha_R(\xi) = 6\left(\xi - \frac{1}{6}\right)^2. \tag{20}$$

The scalar mode decouples (infinite mass) if and only if $\xi = 1/6$, providing a dynamical mechanism for avoiding the scalar graviton sector through conformal coupling of the Higgs field.

VII. PROPERTIES OF THE FORM FACTORS

A. Entire-function nature

Since $\varphi(z)$ is entire of order 1 [its Taylor series has infinite radius of convergence, cf. Eq. (2)], and the form factors (6a)–(12b) are rational combinations of φ and $1/z$ with all apparent singularities at $z = 0$ removable (verified by L'Hôpital's rule), each $h_C^{(s)}(z)$ and $h_R^{(s)}(z)$ is

TABLE I. Summary of principal results. All quantities are defined in the text; Λ is the spectral cutoff scale and ξ is the Higgs non-minimal coupling.

Quantity	Value	Eq.
α_C	$13/120 \approx 0.1083$	(16)
$\alpha_R(\xi)$	$2(\xi - 1/6)^2$	(18)
c_1/c_2	$-1/3 + 120(\xi-1/6)^2/13$	(19)
$3c_1 + c_2$	$6(\xi - 1/6)^2$	(20)
$\lim_{x\to\infty} x\,\alpha_C$	$-89/12$	(22)
m_2	$\Lambda\sqrt{60/13} \approx 2.148\,\Lambda$	(25)
$m_0\ (\xi = 0)$	$\Lambda\sqrt{6} \approx 2.449\,\Lambda$	(26)
$m_2/m_0\,\vert_{\xi=0}$	$\sqrt{10/13} \approx 0.877$	(27)
$V(0)/V_{\rm N}(0)$	0 (finite at origin)	(28)

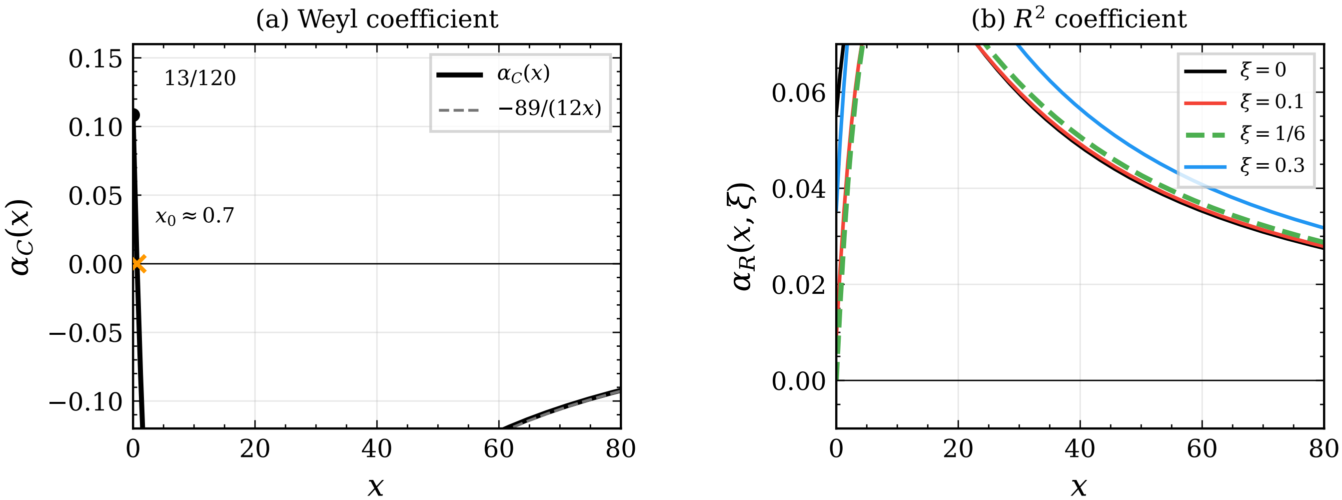

FIG. 4. Standard Model totals as functions of $x = \Box/\Lambda^2$. (a) The Weyl coefficient $\alpha_C(x)$ starts at 13/120 and crosses zero at $x_0 \approx 0.7$, approaching the asymptote $-89/(12x)$ (dashed gray) for $x \gg 1$. The sign change signals a transition in the effective Weyl-squared coupling. (b) The R^2 coefficient $\alpha_R(x, \xi)$ for several values of the Higgs non-minimal coupling ξ. At conformal coupling $\xi = 1/6$ (green dashed), α_R vanishes at $x = 0$ and remains small for all x.

entire. By linearity, the SM totals $\alpha_C(z)$ and $\alpha_R(z, \xi)$ are entire functions of z.

Physically, the spectral action form factors therefore introduce *no new poles* in the graviton propagator beyond the classical ones. Any ghost poles arise solely from the zeros of the dressed propagator $\Pi_{TT}(z) = 1 + \frac{13}{60}\, z\, \hat{F}_1(z)$, not from singularities of F_1 or F_2 themselves.

B. UV asymptotics

For $x \to \infty$, $\varphi(x) \to 2/x + O(1/x^2)$, so that

$$x\, h_C^{(0)} \to \frac{1}{12}\,, \quad x\, h_C^{(1/2)} \to -\frac{1}{6}\,, \quad x\, h_C^{(1)} \to -\frac{1}{3}\,. \tag{21}$$

The total UV coefficient is

$$\lim_{x\to\infty} x\, \alpha_C(x) = \frac{4}{12} - \frac{45}{12} - \frac{48}{12} = -\frac{89}{12}\,. \tag{22}$$

The sign change from $\alpha_C(0) = +13/120$ to $x\, \alpha_C \to -89/12 < 0$ implies that $\alpha_C(z)$ crosses zero at a finite $z_0 \approx 0.7$ (Fig. 4a), signaling a transition in the effective sign of the Weyl-squared coupling.

C. Effective graviton masses

In the linearized theory around flat space, the dressed spin-2 and spin-0 propagators are

$$G_{TT} \propto \frac{1}{k^2\, \Pi_{TT}(k^2/\Lambda^2)}\,, \tag{23}$$

$$G_s \propto \frac{1}{k^2\, \Pi_s(k^2/\Lambda^2, \xi)}\,, \tag{24}$$

with $\Pi_{TT}(z) = 1 + (13/60)\, z\, \hat{F}_1(z)$ and $\Pi_s(z, \xi) = 1 + 6(\xi - 1/6)^2\, z\, \hat{F}_2(z, \xi)$. Expanding near $z = 0$ using $\hat{F}_1(0) = 1$:

$$m_2 = \Lambda\sqrt{\frac{60}{13}} \approx 2.148\, \Lambda\,, \tag{25}$$

and for $\xi \neq 1/6$:

$$m_0(\xi) = \frac{\Lambda}{\sqrt{6(\xi - 1/6)^2}}\,. \tag{26}$$

At minimal coupling $\xi = 0$, this gives $m_0 = \Lambda\sqrt{6} \approx 2.449\, \Lambda$. The mass ratio

$$\left.\frac{m_2}{m_0}\right|_{\xi=0} = \sqrt{\frac{10}{13}} \approx 0.877 \tag{27}$$

is fixed by the SM particle content and independent of Λ.

The modified Newtonian potential in the linearized theory [15] is

$$\frac{V(r)}{V_{\rm N}(r)} = 1 - \frac{4}{3}\, e^{-m_2 r} + \frac{1}{3}\, e^{-m_0 r}\,, \tag{28}$$

which is plotted in Fig. 5. At $r = 0$ the exponentials cancel, giving $V(0) = 0$ and a finite gravitational potential at the origin. Newton's law is recovered for $r \gg 1/\Lambda$.

VIII. DISCUSSION

A. Comparison with existing results

The local limits $(\beta_W^{(s)}, \beta_R^{(s)})$ agree with the established results of [11, 13–15], as compiled in Table II. In partic-

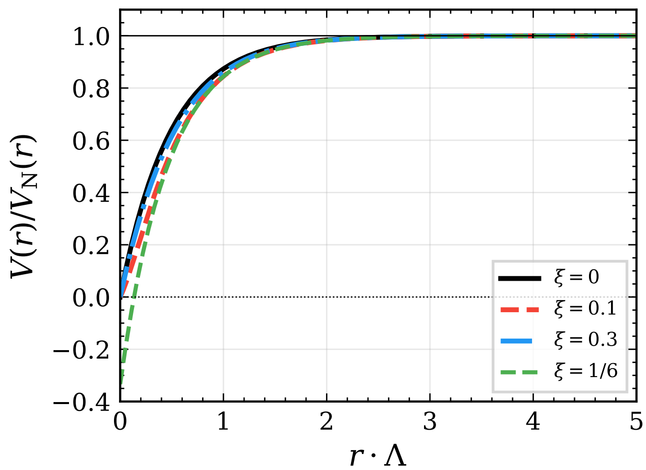

FIG. 5. Modified Newtonian potential $V(r)/V_{\rm N}(r)$ for several values of ξ, as a function of $r\Lambda$. At conformal coupling $\xi = 1/6$ (green dashed) the scalar mode decouples and the potential dips to $-1/3$ before recovering Newton's law. For all ξ, $V(0) = 0$ (finite at the origin) and $V \to V_{\rm N}$ for $r \gg 1/\Lambda$.

TABLE II. One-loop β-function coefficients $\beta_W^{(s)} \equiv h_C^{(s)}(0)$ and $\beta_R^{(s)} \equiv h_R^{(s)}(0)$ for each spin sector, compared with the literature. The sign convention for spin-1/2 includes the $(-1)^{2s}$ factor explicitly; CPR [13] quote the unsigned value $|\beta_W^{(1/2)}| = 1/20$. CZ = Codello–Zanusso [9], PT = Parker–Toms [14], S77 = Stelle [15].

Spin	$\beta_W^{(s)}$	Lit.	$\beta_R^{(s)}$	Lit.
0	$\frac{1}{120}$	CZ, PT	$\frac{1}{2}\left(\xi-\frac{1}{6}\right)^2$	PT
$\frac{1}{2}$	$-\frac{1}{20}$	CZ, CPR	0	CZ, PT
1	$\frac{1}{10}$	S77, CPR	0	CZ, PT
SM	$\frac{13}{120}$	–	$2\left(\xi-\frac{1}{6}\right)^2$	–

ular, the SM Weyl coefficient 13/120 can be decomposed as

$$\frac{13}{120} = \frac{N_s}{120} - \frac{N_D}{20} + \frac{N_v}{10}\,, \tag{29}$$

which matches the general formula of [13] with $\delta_s = 1/120$, $|\delta_f| = 1/20$ (per Dirac, unsigned in their convention), and $\delta_v = 1/10$ (per gauge boson, after ghost subtraction).

The nonlocal form factors $h_C^{(s)}(x)$ and $h_R^{(s)}(x)$ reduce to the CZ expressions [9] upon specifying the Lichnerowicz endomorphism $E = -R/4$ for the Dirac sector and the ghost count of 2 for the gauge boson sector.

B. Role of the non-minimal coupling

The parameter ξ enters exclusively through the scalar sector and affects only α_R (not α_C). This has two consequences:

1. The spin-2 graviton spectrum (Π_{TT}, ghost masses, UV behavior) is entirely ξ-independent and thus a robust consequence of the framework.

2. The scalar graviton sector is controllable: at $\xi = 1/6$, the entire R^2 sector vanishes, and the scalar mode decouples. This provides a natural resolution of the scalar ghost problem through conformal coupling.

C. Lorentzian continuation

The form factors computed here are Euclidean ($\Box_E > 0$). The Lorentzian form factors are obtained by the Wick rotation $z_E \to -z_L$, i.e. $F_i(\Box_E/\Lambda^2) \to F_i(-\Box_L/\Lambda^2)$. Since $\varphi(z)$ is entire, this continuation is well-defined for all z. The dressed propagator poles (ghost masses) are obtained by solving $\Pi_{TT}(-z_L) = 0$ and are discussed in a companion paper.

D. Implications for nonlocal gravity

The entire-function nature of the form factors connects the spectral action to the program of nonlocal (infinite-derivative) gravity [16–18], where entire functions of $\Box$ are introduced to improve UV behavior. In the spectral action, these form factors arise *derivatively* from the heat kernel, rather than being postulated. The one-loop degree of divergence is reduced from $D = 4$ (Einstein gravity) to $D = 0$ (logarithmic), consistent with the results of [15] for local quadratic gravity but with the additional structure of momentum-dependent form factors that modify the UV behavior at all scales.

IX. CONCLUSIONS

We have computed the complete nonlocal one-loop form factors of the spectral action with full Standard Model content. The central results are the SM-determined Weyl coefficient $\alpha_C = 13/120$ and the ξ-dependent R^2 coefficient $\alpha_R(\xi) = 2(\xi - 1/6)^2$, together with the closed-form expressions (6)–(12) for each spin sector.

The form factors are entire functions of $\Box/\Lambda^2$, ensuring no new propagator poles, and yield effective graviton masses $m_2 = \Lambda\sqrt{60/13}$ (spin-2) and $m_0 = \Lambda/\sqrt{6(\xi - 1/6)^2}$ (spin-0), both determined by the SM spectrum. The scalar graviton decouples at conformal coupling $\xi = 1/6$, and the UV behavior is logarithmic ($D = 0$), improving upon Einstein gravity ($D = 4$).

These results provide the foundation for a systematic study of the quantum gravitational phenomenology of the spectral action, including the modified Newtonian potential, PPN parameters, gravitational wave propagation, and unitarity structure, which will be presented in companion papers.

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DATA AVAILABILITY

The computational tools used in this work, including arbitrary-precision implementations of all form factors, are available from the author upon reasonable request.

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