Gravitational analogy of Casimir-effect from QED to gravity description in GRT by extended GRT-gravity without Ostrogradsky ghost-problem as higher derivative of effective gravitative fieldtheory. A global fixpoint-solution.

Abstract:
Classical GRT of Einstein-description of gravity-theory can be extrapolated to higher structures. This extension adds higher-order terms to classical general relativity because GRT itself can be assumed to be a first-order expansion of spacetime. These higher corrections can lead to new observations in the vacuum, with gravitational waves, or black holes. Furthermore, an attempt is made to transfer the Casimir effect of quantum electrodynamics as an analogy from the electromagnetic vacuum to a spacetime description and to describe this analogy as a gravitational effect within GRT.

Key-words: Expansion of GRT; higher terms; no Ostrogradsky-ghosts; higher order of GRT; gravity-waves; spin-2-particles; additional scalar-field; Casmir-effect; gravity-analogy; spacetime-vacuum.

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1. Introduction:

Classical GRT depends on Ricci-tensor and Ricci-scalar, which both describe the curvature of spacetime. If higher terms are added to this situation, there can be build a finer structure, which may be able to bring observations and theory more closely into harmony and predict new effects that, in principle, should be measurable with current methods. In particular, the analogy of the Casimir-effect from QED to Einstein's gravitation is being investigated. This could also lead to new aspects regarding the quantization of classical gravitational theory.

2. Methods/Calculation:

2.1. Starting-point is classical GRT:

Take an action-functional

$$F[g]=M_I \tag{1a.}$$

with fixpoint-condition of:

$\delta F = 0$

. (1b.)

Then local the lowest setting is:

$$F \sim R + \alpha \rho \tag{1c.}$$

Variation then delivers:

. $G_{\mu\nu} = \kappa T_{\mu\nu}$ (1d.)

ergo the Einstein-equations after the metod of Hilbert. This exactly is the action of classical GRT.

2.2. Effective gravity as higher terms in Taylor-developement:

If the functional depends local from curvature, then the general local developement is:

$$F \sim R; a_1 = R^2; a_2 = R_{\mu\nu} \cdot R^{\mu\nu}; a_3 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}; b_1 = \Box R = \frac{c^2}{\partial t^2} - \nabla^2 R; \ldots \tag{2a.}$$

What about variation? The variation process then delivers:

$$G_{\mu\nu} + a_1 H^1_{\mu\nu} + a_2 H^2_{\mu\nu} + \ldots = \kappa T_{\mu\nu} \tag{2b.}$$

where the $H^n_{\mu\nu}$-terms contains fourth derivations of metric, generate new degrees of freedom and introduce additional propagating modes.

2.3. Well-known special-cases:

If $F = f(R)$ (2c.)

there is the $f(R)$ gravity. This leads to an additional scalar degree of freedom. Also known is the term of quadratic gravity, whih is pertubative renormalizable [1.]:

$$F = R + a R^2 + b R_{\mu\nu} R^{\mu\nu} \tag{2d.}$$

2.4. Physical Significance:

Yes, higher terms can be obtained. However: They lead to higher derivatives. Higher derivatives often lead to Ostrogradsky instabilities. Some combinations, however, are stable (e.g., Gauss–Bonnet in 4D topologically). Further down is seen, that here no Ostrogradsky-ghost [2.] appear in this description.

2.5.Gauss–Bonnet-Term:

The special combination of

$$L_{GB} = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \tag{2e.}$$

is the Gauss-Bonnet term or Euler-density. Since this is a pure topological term, it does not contribute to the dynamics in 4D. In four dimensions the integral is:

$$S=\int d^D x\sqrt{-g}\,,\left(R+\alpha L_{GB}\right);D=4 \quad (2f.)$$

It is topological proportional to Euler-characteristics of the manifold. Therefore the term doesn‘t belong to dynamics in classical Einstein-equation.

2.6. Interpretation in selection-model:

Higher orders of equation-developement now are introduced . In this ansatz here this situation leads to following conditions:

1. First-order: Einstein-Hilbert-action,

2. Second order: structural correlations,

3. Higher order: nonlocal or microscopic correctures of coherence.

This fits exactly with the interpretation as an effective theory of a deeper fixed point.
Argument of symmetry: If the functional is local, diffeomorphism-invariant and only depends from the metrical field, then Einstein-Hilbert-action is the lowest non-trivial order. Higher terms are automatically allowed.

2.7. Short in-between summary:

Higher orders of Einstein-equation can be introduced and the Einstein-Hilbert-action can be elaborated. Structural corrections are then available and it is possible to construct an effective gravity-theory.

2.8. Variation of the R^2-term:

Look at the action of:

$$S=\int d^4 x\sqrt{-g}\left(\frac{1}{2\kappa}R+a R^2\right) \quad (3a.)$$

Variation then delivers:

$$G_{\mu\nu}+2a\left[RR_{\mu\nu}-\frac{1}{4}g_{\mu\nu}R^2+g_{\mu\nu}\Box R-\nabla_{\mu}\nabla_{\nu}R\right]=\kappa T_{\mu\nu} \quad (3b.)$$

Important is, that $\Box R$ appears and second covariant derivations of R. Since R contains itself the second derivative of the metric, there are in the whole description four derivations of the metric, the theory is higher derivative. It can be seen, that there are new degrees of freedom.

$R^2 \leftarrow \rightarrow Einstein+scalar\ additional\ field.$

Introduce an assistant field ϕ:

$$S=\int d^4 x\sqrt{-g}\left(\frac{1}{2\kappa}R-\phi R-\frac{1}{4a}\phi^2\right) \quad (3c.)$$

After diagonalization there is as the result:

2 spin-2 degrees of freedom like in GRT but additional a scalar field, ergo three modi. In the whole three propagating degrees of freedom.

2.9. Classical problems:

If the term of $R_{\mu\nu} R^{\mu\nu}$ is added, then additionally appears a massive spin-2 modus, which is mostly a ghost: the Ostrogradsky-problem. This is the central problem of higher gravity.

2.10. Interpretation in this fixpoint-model:

The proposition is that the dynamics minimizes a structural functional of incoherence. Then there is;

R --- lowest coherent geometrical structure,

R^2 --- nonlinear correction of coherence,

higher terms --- fluctuation-corrections of the fixpoint.

Formally, the functional then looks like an effective action after integration of microscopic degrees of freedom. This exactly is the structure of an expansion of a renormalization group.

3. The GRT-analogy to Casimir-effect:

3.1. Now to the Casimir comparison:
The Casimir effect arises in quantum electrodynamics due to:
Vacuum fluctuations , boundary conditions, summation of zero-point energies. It is not a tree term, but a loop correction. Comparison of structures: In QED the effective energy is like:

$$E=E_0+\hbar E_1+\hbar^2 E_2+\ldots$$

The Casimir-effect appears as a contribution-term of higher order. In gravity the effective action is:

$$S=\int \sqrt{-g}\cdot \left(R+l_P^2 R^2+l_P^4 R^3+\ldots\right) dx^4 \qquad (4.a)$$

Meaning of terms:

S: (Action),

g: Determinant of metric tensor $g_{|\mu\nu|}$,

R: Ricci-Skalar of spacetime-curvature,

l_P: Planck-length.

The series of S describes higher corrections of curvature to classical GRT-gravity. Theses terms are generated by quantum-loops, integration of matter-fields or vacuum-fluctuations. This exactly is the same mathematical structure to Casimir-effect from QED.

Deeper analogies:

1. Casimir-effect generated by vacuum-fluctuations with an impact on geometry generates an attractive force.

2. Higher terms of curvature generated by vacuum-fluctuations of spacetime and lower terms generate spacetime $\rightarrow$ modified dynamics.

Both effects are fluctuation-pressure effects. In this picture of coherence this situation means:

1. The fixpoint is not perfect,

2. Fluctuations generates residual-incoherence (RI).

3. This RI appears as a higher geometric term.

3.2. Even stronger: Induced gravity:

Sakharov [3.] showed, that gravity can complete be generated as effect of vacuum-fluctuations. This is the theory of induced gravity, where R itself is generated as a one-loop effect. Then there are terms like R^2 automatically of higher loop-order [4.].

3.3. Now the question: Can higher terms appear in analogy to Casimir-effect?

The answer is yes, if the functional is interpreted as an effective action of a microscopic dynamic.

Then is:

Einstein --- leading order,

$R^2 = 1 - loop$

$R^3 = 2 - loop$

Non-local terms appear complete resummation. This situation exactly is, what appears in classical quantum gravity. The crucial difference is:

1. Casimir acts in static background,

2. Gravity-analogon in dynamic spacetime.

That's why higher terms are structurally much more drastic here.

3.4. Short summary in Table 1:

QED	**Spacetime-structure**
Vacuum-fluctuation	Coherence-fluctuation
Casimir-energy	R^2- correction
Loop-expansion	Taylor-expansion of functional
Effectice action	Fixpoint functional

Table 1: The deep analogy between Casimir-effect in QED and its counterpart in GRT-gravity elaboration.

This ansatz structural is near to effective quantum-gravity, induced gravity and renormalization-group fixpoints.

3.5. Grades of freedom in higher gravity:

I. Pure Einstein-gravity:

In classical GRT there are the following conditions:

1. Metric $g_{\mu\nu}$--- 10 components (symmetrical tensor),

2. Four diffeomorphisms $\leftarrow\ \rightarrow$ four degrees of freedom,

3. Four additional conditions.

This leads to: $10-4-4=2\rightarrow$ two propagating degrees of freedom (spin 2), polarizations of gravity-waves.

II. Theory with $R+a\cdot\ R^2$**:**

The action then is:

$$S=\int\sqrt{-g}\left[R+aR^2\right]dx^4. \qquad (4b.)$$

Trick is introduction of a scalar-field (may be it can be later identified with Higgs-field in higher order of global universal energy-density, when in former times in early cosmos gliding constants of coupling are nearer together).

$$S=\int\sqrt{-g}\left[(1+2a\,\phi)R-a\,\phi^2\right]dx^4 \qquad (4c.)$$

After conformal transformation there are:

2 spin-2-modes

and

1 scalar-mode $\rightarrow$ 3 degrees of freedom.

This scalar mode is no ghost!

III. Description with $R_{\mu\nu}R^{\mu\nu}$**:**

Then action is:

$$S=\int\sqrt{-g}\left[R+aR^2+b\,R_{\mu\nu}R^{\mu\nu}\right]dx^4 \qquad (4d.)$$

As results there are :

2 massless spin-2-propagators,

1 scalar-mode,

5 massive spin-2 components.

Ergo complete 8 degrees of freedom but the massive spin-2 state is a ghost.

(I‘m afraid of no ghosts).

3.6. The Ostrogradsky-problem:

Ostrogradsky's theorem (without proof) [5.]:
Every non-degenerate theory with higher time derivatives possesses a

Hamilton function that is unbounded below.

This statement/theorem means for gravity, that:

Because R contains second derivation of metrics and R^2 contains fourth derivations of the same metric, this situation leads to an additional canonical momentum. The Hamiltonian becomes linear in this momentum, this situation generates an instability [6.]. The ghost corresponds precisely to this degree of freedom. Only possible solutions are: special combinations (Gauss-Bonnet), nonlocal theories, asymptotic certainty or superstring theory [7.].

3.7. Fixpoint of renormalization group:

Since description of the fixpoint $\delta F=0$, this is structurable identical to $\beta(g_i)=0$ in group of renormalization. Now in asymptotic safety there is supposed, that gravity possesses a non-trivial UV-fixpoint, where all couplings a_i converge. Then the used Taylor-series is nothing else but

$$F=\sum_i g_i(k) O_i, \quad (4e.)$$

where k is RG-scale.

3.8. Short interpretation of physical situation:

Interpretation in the used image: Coherence fixed point = RG fixed point, higher terms = irrelevant operators, Einstein term = relevant operator, very clean structural analogy.

3.9. Casimir-like effects in curved spacetime:

Casimir-effect is generated in QED by

$$E=\sum_n \hbar\,\omega_n \quad (5a.)$$

with additional conditions.

Now in curved spacetime matter-fields are integrated by:

$$Z=\int D\phi\, e^{-S[\phi,g]}, \quad (5b.)$$

where ϕ(x) is a field, i.e., a function of space (or spacetime), Dϕ formally means: integration over all possible functions ϕ(x). This can be visualized as follows:

1. Discretize space into many [g]=y points(x_i),

2. Then the field has a value ϕ(x) at each point,

3. The path integral then becomes a very high-dimensional integral:

$$\int D\phi \rightarrow \prod_i \int d\phi(x_i) \quad (6a.)$$

That is:

$$Z=\prod_i \int d\phi(x_i)\, e^{-S[\phi g]} \quad (6b.)$$

In the continuum limit (infinitely many points), this product form is written compactly as Dϕ. This description then leads to:

$$S_{eff}[g]=\frac{1}{2}\log det\left(\square+m^2\right) \qquad (7a.)$$

If now a heat-kernel expansion is used, there comes the same structure like the above used Taylor-developement:

$$S_{eff}=\int\sqrt{-g}\left(c_0+c_1 R+c_2 R^2+c_3 R_{\mu\nu}R^{\mu\nu}+...\right) \qquad (7b.)$$

Interpretation:
1. Casimir-effect: Boundary conditions change spectrum. This leads to energy-shifts.
2. Gravity-analogon: geometry changes spectrum. This leads to effective terms of curvature.
This is a direct analogy and the connection can be written as:

$R^2-term\sim geometric\,Casimir-effect.$

More deeper: in the theory of induced gravity there is showed, that even the Einstein-term is generated as a 1-loop-effect.

This situation can be seen in Table 2:

Order	Interpretation
R	1-loop
R^2	2-loop
R^3	3-loop

Table 2: Structural analogy between order of classical GRT-terms and similiar situation in quantum-gravity.

In a complete synthesis the coherence-functional could be interpreted as:

$$F[g]=Tr\left[\log\left(Op_{coh}\right)\right] \qquad (8.)$$

Its Taylor-developement generates:
1. Einstein,
2. Higher curvature,
3. Non-local terms.
The fixpoint then is:
$\delta F=0$ **and means a condition of self-consistence of the vacuum.**
This definition is from its concept extremely near to modern quantum gravity. As an important insight there can be seen, that then really exists a precise, mathematical analogy of description:
I. Casimir-effect --- Fluctuation-pressure of vacuum-density
II. Higher curvature --- fluctuation-pressure of spacetime-density.
Both descriptions are effects of spectral-shifting.

<u>3.10. Counting of degrees of freedom with projector-formalism:</u>

Let‘s linearize about Minkowski in a classical sense:

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu} \tag{9a.}$$

and look at:

$$S=\int dx^4\sqrt{-g}\left(R+aR^2+bR_{\mu\nu}R^{\mu\nu}\right) \tag{9b.}$$

Now follow the spin-projectors:
In momentum-space $h_{\mu\nu}$is decomposed in spin-2, spin-1 and spin-0 parts, the latter include track and longitudinal components.
The Barnes-Rivers-projectors now deliver:

$$O(k)=k^2\left(P^{(0)}+P^{(2)}\right)+k^4\left(a\,P^{(0)}+b\,P^{(2)}\right) \tag{9c.}$$

Invertion of the operator results to the propagator:

$$D(k)=\left[\frac{P^{(2)}}{k^2}-\frac{P^{(0)}}{2k^2}+\frac{P^{(0)}}{k^2-m_0^2}-\frac{P^{(2)}}{k^2-m_2^2}\right] \tag{9d.}$$

or:

$$D(k)=\frac{\left(k^2+m_0^2\right)P^{(0)}}{2k^2\left(k^2-m_0^2\right)}-\frac{m_2^2P^{(2)}}{k^2\left(k^2-m_2^2\right)} \tag{9e.}$$

Then the spectrum results to
1. 2 massless spin-2 states,
2. 1 massive scalar-mode,
3. 1 massive spin-2 state with 5 states of polarization. This massive spin-2 state possesses the wrong sign. This is a ghost. The description is not unitary.
Masses are:

$$m_0^2\sim\frac{1}{a};m_2^2\sim-\frac{1}{b} \tag{10.}$$

<u>3.11. The ghost-problem in a precise formulation:</u>

The Hamiltonian contains linear terms of momentum.

$$H\sim p_1q_2+\ldots \tag{11a.}$$

There for it is possible, that:

$$\lim H\rightarrow-\infty \tag{11b.}$$

which situation causes negative norm-states and injures unitarity. To avoid this problem, there are two options:
1. Pure $f(R)$ theories → preventing the occurrence of a spin-2 ghost.
Or
2. special topological combinations (Gauss-Bonnet-description).

This situation now leads to the structure of renormalization-group:

$$\Gamma_K[g]=\sum_i g_i(k)\int\sqrt{-g}\,O_i dx^d\,;d=4 \quad (12.)$$

Meaning of the terms:

$\Gamma_K[g]$: Effective action at scale k,
$g_i(k)$: Moving coupling-constants,
$-g$: Determinant of metric (covariant volumelement),
O_i: Scalar operators from curvature-tensors etc.,
dx^d: Integration about d-dimensional spacetime.

With operators of:

$$O_i=\{1\,;R\,;R^2\,;R_{\mu\nu}R^{\mu\nu}...\} \quad (13a.)$$

Fixpoint is:

$$\partial_k g_i=0 \quad (13b.)$$

In theory of asymptotic safety this situation exactly is the basic assumption:
1. A non-trivial UV-fixpoint exists,
2. Only a few directions are relevant.
Here:
Coherence-fixpoint is RG-fixpoint of geometry.

<u>3.12. Reconstruction as</u> $\log(det(O))$:

Starting point is integration of matter-fields. Look at and take a scalar-field:

$$S[\phi,g]=\frac{1}{2}\int\sqrt{-g}\,\phi(-\Box+m^2)\phi \quad (14a.)$$

Integration of functional then shows:

$$Z[g]=\int d^4 D\phi\, e^{-S}\left(det(-\Box+m^2)\right)^{-\frac{1}{2}} \quad (14b.)$$

Effective action then is:

$$\Gamma[g]=-logZ\frac{1}{2}\log(det(-\Box+m^2)) \quad (14c.)$$

This exactly is the searched and demanded structure.
Now the needed heat-kernel-expansion:
Used is:

$$\log(det(O))=Tr\log(O)\quad . \quad (14d.)$$

Then write:

$$\log\left(det\left(-\Box+m^2\right)\right)\int_0^\infty \frac{ds}{s} Tr\left(e^{-s(-\Box+m^2)}\right) \quad (15a.)$$

The trace has an asymptotic developement.

$$Tr\, e^{-s\Box}=\frac{1}{(4\pi s)^2}\int \sqrt{-g}\left(a_0+a_1 sR+a_2 s^2 R^2 ...\right) \quad (15b.)$$

To put in then delivers:

$$\Gamma[g]=\int \sqrt{-g}\left(\Lambda+c_1 R+c_2 R^2+c_3 R_{\mu\nu} R^{\mu\nu} ...\right) \quad (15c.)$$

This exactly reproduces the used Taylor-structure.

3.13. Interpretation in coherence-model:
There is:

$$F[g]=M_I \quad (16a.)$$

Now this situation means:

$$M_I=\frac{1}{2} Tr\left(\log\left(O_{Coh}\right)\right) \quad (16b.)$$

Ergo is:

$$F[g]=\frac{1}{2} Tr\left(\log O_{Coh}\right) \quad (16c.)$$

and the result of variation then is:

$$\delta F=Tr\left(O^{-1}\delta O\right) \quad . \quad (16d.)$$

This is a condition of self-consistence of the spectrum.

3.14. A deep structural statement:

Casimir-effect:

$$E\sim\sum_n \hbar\omega_n \frac{1}{2} tr\left(\log\left(O\right)\right) \quad (17a.)$$

Gravity:

$$\Gamma[g]\sim\frac{1}{2} tr\left(\log\left(O_g\right)\right) \quad (17b.)$$

The geometry shifts the spectrum and changing of spectral conditions generates terms of curvature. This condition also leads to a new formulation of the fix-point:

$$\delta F=0\rightarrow Tr(O^{-1}\delta O)=0 \quad . \qquad (18a.)$$

This self- condition of spectrum is extremely near to induced gravity, vacuumenergy-mechanisms and asymptotical safety.

3.15. Final interpretation:

The structural functional can be interpreted consistent as:

$$F[g]=\frac{1}{2}Tr(\log(O_{mic}[g])) \qquad (18b.)$$

Its Taylorexpansion generates:
Cosmological constant, Einstein-term, (R^2) terms and nonlocal terms. Exactly like in a loopexpansion.
The complete picture can be seen in Table 3:

Casimir-effect in QED	**Analogon in gravity-theory**
$\sum_n \omega_n$	Spectrum of coherence-operator
Boundary conditions	Geometry
Force	Dynamics of curvature
Loop-effect	Fixpoint-correction

Table 3: Comparison between classical Casimir-effect in QED and its analogon in GRT.

4. Explicite heat-kernel-coefficients;

4.1. Starting point is the term of:

$$\Gamma[g]=\frac{1}{2}\log(det(-\Box+m^2)) \quad . \qquad (19a.)$$

Then use:

$$\log(det(O))=-\int_0^\infty \frac{ds}{s}Tr(e^{(-sO)}) \qquad (19b.)$$

There is for a Laplace-Operator in 4D the condition of:

$$Tr(e^{-s\Box})=\frac{1}{(4\pi s)^2}\int d^4x\sqrt{-g}(a_0+a_1 s+a_2 s^2+...) \quad . \qquad (19c.)$$

Then the coefficients are:

$$a_0=1;a_1=\frac{1}{6}R;a_2=\frac{1}{180}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}+R_{\mu\nu}R^{\mu\nu}+\Box R)+\frac{1}{172}R^2 \quad . \qquad (19d.)$$

Putting into the equation leads to the result of:

$$\Gamma[g]=\int\sqrt{-g}\left(\Lambda+c_1 R+c_2 R^2+c_3 R_{\mu\nu}R^{\mu\nu}...\right) \quad (19e.)$$

The complete Taylor-structure is generated in a pure spectral way.

4.2. The Dirac-operator:

Now instead of a scalar operator the Dirac-operator is taken:

$$D=i\gamma^{\mu}\nabla_{\mu} \quad . \quad (20a.)$$

Then:

$$D^2=\Box+\frac{1}{4}R \quad . \quad (20b.)$$

Now the effective action of a fermion is:

$$\Gamma[g]+\log(det\, D)=-\frac{1}{2}(\log(det\, D^2)) \quad (20c.)$$

This generates again a heat-kernel-expansion but now with a spin-structure.
Important: **The Einstein-term generates automatically from** $Tr(e^{-sD^2})$ **. This situation means, that gravity appears as spectral-effect of the Dirac-operator.**

4.3. Spectral-action in description of Connes [8.]:

In non-commutative theory of Alain Connes [9.] there is postulated, that

$$S=Tr f\left(\frac{D}{\Lambda}\right) \quad , \quad (21a.)$$

where :

(f) ----cut-off function,
Λ --- energy-scale.

The asymptotical expansion then is:

$$S=\int\sqrt{-g}\left(\Lambda^4+\Lambda^2 R+R^2+...\right) \quad (21b.)$$

Ergo there appears: Cosmological constant, Einstein-term and quadratic curvature. All description is pure spectral, no classical geometry is assumed, only the spectrum.

4.4. Comparison with the functional:

There is:

$$F[g]=M_I \quad (22a.)$$

Then this term now can be consistent identfied with:

$$F[g]=Tr(\log(O(g))) \quad (22b.)$$

or more general:

$$F[g]=Tr\ \ f(O(g)) \tag{22c.}$$

and then

$$\delta F[g]=0 \tag{22d.}$$

means a condition of self-consistence of the spectrum of:

$$(O^{-1}\delta\ \ O)=0 \tag{22e.}$$

4.5. Connection to induced gravity:

In induced gravity, the term of R only generates by

$$\log(det(-\Box+m^2))\ \ . \tag{23a.}$$

This interpretation means that gravity is a collectice vacuum-effect.
Deep analogy to Casimir-effect, Casimir-effect in QED can be described in principle by

$$E_n=\frac{1}{2}\hbar\sum_n \omega_n \tag{23b.}$$

and the used functional by:

$$F=\frac{1}{2}\sum_n \log\lambda_n \tag{23c.}$$

Both are spectral-measurements, only with the difference, that Casimir-effect works on a fix background, where gravity works on a dynamic geometry.

4.6.Non-local terms:

If the heat-kernel-series isn‘t abandoned or not cut off, then there is:

$$\Gamma[g]=\int dx^4\sqrt{-g}(R\log\Box R+...) \tag{24.}$$

These non-local effects are pure loop-effects. In this picture here long-ranging coherence-corrections.

4.7. Short summary of the model-description:

$$F[g]=Tr(\log(O_{coh})) \tag{25a.}$$

with

$$O_{coh}=D^2=-\Box+V(g) \tag{25b.}$$

Then there follows automatically :
1. Cosmological constant,

2. Einstein-equation,
3. R^2 -corrections,
4. Non-local terms.

5. Construction of the coherence-operator:

5.1. There is:

$$F[g]=Tr(\log(O[g])) \quad (26.)$$

The question now is: what is $(O[g])$?
Used is the minimal structure of a most general covariant operator of second order and Laplace-type of:

$$O[g]=-\nabla^2+E(g) \quad , \quad (27a.)$$

where $E(g)$ -potential-term.

If $E(g):=\alpha R$ (27b.)

, then $O[g]=-\nabla^2+\alpha R$. (27c.)

Then heat-kernel-expansion automatically generates:
1. Cosmological constant,
2. Einstein-term,
3. R^2 term,
4. $R_{\mu\nu}R^{\mu\nu}$ -term.
That is structurally sufficient.

5.2. More natural in a geometric way is a Dirac-structure:

$$O=D^2 \wedge D=i\gamma^{\mu}\nabla_{\mu} \quad . \quad (27d.)$$

Then there is:

$$D^2=-\nabla^2+\frac{1}{4}R \quad . \quad (27e.)$$

Gravity then is directly codated in the spin-structure and this exactly is the access to spectral gravity.
Now the interpretation or meaning of the operator:

Iff coherence means “stability of a spectrum under geometrical variations“, then

$O=H_{coh}$ can be interpreted as an effective information-Hamiltonian. Then there is:
$F=Tr(\log(H))$ formal identical with free energy. This then exactly is the same structure-situation as by Casimir-effect.

5.3. Ghost-freedom as a spectral condition:

Ghosts generate in higher gravity by additional poles or wrong sign in propagator. This means in a spectral condition, that the eigenvalues $\lambda_{eig}(i)$ mustn‘t generate negative norm-states. Therefore the following conditions are needed.
Iff $F=Tr(\log(O_{coh}[g]))$, there is stability for $O_{coh}[g]>0$. This operator then is a positive, elliptic operator without existing of negative normstates and none unlimited energy-description. Why then generate ghosts of $S_\lambda=Tr\left(\frac{D}{\lambda}\right)$ type $R^2_{\mu\nu}$?

The answer is that the action there is supposed local not from ***log det*** of an elliptic operator.
Important insight:
A fundamental, spectral theory automatically is ghost-free, if the basic operator is positive.This is structural much more stable than local polynom-theory.

5.4. Spectral action and RG-flow:
After A. Connes there is a form of spectral action:

$$S_\lambda=Tr\left(\frac{D}{\lambda}\right) \tag{28a.}$$

with its asymptotical developement of:

$$S_\lambda=\sum_0^\infty f_n(\lambda^{4-n}a_n) \quad , \tag{28b.}$$

with a_n heat-kernel-coefficient. The RG-interpretation then is, that the scale S_λ is a cut-off and variation of λ leads to:

$$\lambda\frac{d}{d\lambda}S_\lambda \tag{28c.}$$

is a RG-flow and describes, how the action changes, if the energy-scale of λ is shifted. Therefore the fixpoint can be identified with a spectral fixpoint and this situation is near to asymptotic safety.
A non-trivial UV-fixpoint means:

$$\lambda\rightarrow\infty\rightarrow g_i(\lambda)\rightarrow g_i^{\,*} \quad , \tag{29.}$$

and coherence gets self-consistence at high scales.

5.5. Complete consistent formulation:

Mathematically precise:

Definition:There is an elliptic operator $O[g]$, defined on a manifold M .The associated structural functional is:

$$F[g]=Tr\ \log(O[g]) \quad . \tag{30a.}$$

For the dynamics there is the fixpoint-structure of:

$$\delta F[g]=0 \rightarrow O^{-1}\delta \ O=0 \quad , \tag{30b.}$$

which is the spectral selfconsistence.

The low-energy-limit delivers over a heat-kernel:

$$F[g]=\int \sqrt{-g}\left(\Lambda+c_1 R+c_2 R^2+...\right) \quad . \tag{30c.}$$

The Einstein-equation then is generated as leading order.

5.6. Complete physical interpretation:

This model implicates the following statements:
1. Spacetime isn‘t fundamental,
2. Gravity is a spectral back-action,
3. Higher curvature-terms are corrections of fluctuations,
4- Ghost-freedom follows from positivity of the operator.

This situation is near to spectral geometry, induced gravity and asymptotic safety.
An analogy to Casimir effect is, when

$$E_{Cas}=\frac{1}{2} Tr\left(\log(O)\right) \tag{31a.}$$

but

$$F[g]=\frac{1}{2} Tr\left(\log(O[g])\right) \tag{31b.}$$

The only difference is, that at Casimir-effect in QED the background is fix but in this case here is dynamically. Therefore the interpretation comes to the conclusion, that:

Gravity is a selfconsistence-condition of a spectrum.

6. Cosmological constant in 4D:

6.1. Given as a starting point is:

$$F[g]=\frac{1}{2} Tr\left(logO[g]\right) \tag{32a.}$$

with an elliptic operator of second order:

$$O[g]=-\nabla^2+E(g) \quad . \tag{32b.}$$

Then the heat-kernel-expansion in 4D is:

$$Tr\, e^{-sO}=\frac{1}{(4\pi s)^2}\int d^4x\sqrt{-g}\left(a_0+a_1 s+a_2 s^2+...\right) \tag{32c.}$$

In four dimensions, the dimensions are crucial:

1. $a_0 \rightarrow$ --- Dimension of 4 --- Cosmological constant,
2. $a_1 \rightarrow$ --- Dimension of 2 --- Einstein-term,
3. $a_2 \rightarrow$ --- Dimension of 0 --- quadratic curvature.

Integration over s then delivers:

$$F[g]=\int d^4x\sqrt{-g}\left(\Lambda_{Eff}+\frac{1}{16\pi G_{Eff}}R+\alpha R^2+\beta R_{\mu\nu}R^{\mu\nu}+...\right) \quad . \tag{33.}$$

Anymore structure there isn‘t local existent in 4D until second order. The cosmological constant emerges inevitably as the leading term. This statement is no assumption but a necessity from dimension-analytics.

6.2. Matter as a spectral disturbation:

Now a matter-field ϕ is coupled in:

$$O[g,\phi]=-\nabla^2+m^2+V(\phi) \quad . \tag{34a.}$$

Then there is;

$$F[g,\phi]=\frac{1}{2}Tr(\log(O[g.\phi])) \tag{34b.}$$

Variation after $g_{\mu\nu}$ brings the result of:

$$\delta_g F=\frac{1}{2}Tr(O^{-1}\delta_g O) \quad . \tag{34c.}$$

This exactly is:

$$\langle T_{\mu\nu}\rangle \quad . \tag{35.}$$

Ergo matter appears as spectral back-reaction. This formulation reproduces structural the Einstein-equation:

$$G_{\mu\nu}=8\pi G\langle T_{\mu\nu}\rangle \tag{36.}$$

6.3. The complete 4D-structure up to R^2 -order:

In 4D there are exactly three independenent quadratic invariants:

$$(R^2)---(R_{\mu\nu}R^{\mu\nu})---(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) \quad . \tag{37.}$$

However, in 4D applies the pure topological term of Gauss-Bonnet, which doesn‘t connect to the metric and ergo doesn‘t influence it on a local dynamical way but acts as a pure topological base:

$$R^2_{\mu\nu\rho\sigma} = 4R^2_{\mu\nu} + R^2 \quad . \qquad (38a.)$$

Therefore the effective action ergo is restricted to:

$$S_{Eff} = \int d^4x \sqrt{-g}\left(\Lambda + \frac{1}{16\pi G}R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}\right) \quad . \qquad (38b.)$$

This is the complete local metric-structure in 4D up to second order.

6.4. Dynamics from $Tr(\log(O[g]))$:

After variation there is:

$$\delta F[g] = \frac{1}{2}Tr(O^{-1}\delta\ O) \quad . \qquad (39a.)$$

Because of:

$$\delta O[g] = -\nabla^2 + \delta E \quad , \qquad (39b.)$$

there generate terms proportional to $G_{\mu\nu}$ and additional derivations of fourth order at R^2 .

6.5. The resulting gravity field-equation in 4D then is:

$$G_{\mu\nu} + \alpha H^1_{\mu\nu} + \beta H^2_{\mu\nu} = 8\pi G\langle T\mu\nu\rangle \qquad (40a.)$$

Thereby are the definitions of:

$$H^1_{\mu\nu} = 2R\cdot R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2 - 2\nabla_\mu\nabla_\nu R + 2g_{\mu\nu}\Box R \qquad (40b.)$$

and variation of:

$$S_{RR} = \int d^4x\sqrt{-g}\,R_{\alpha\beta}R^{\alpha\beta} \qquad (40c.)$$

leads to

$$H^2{}_{\mu\nu} = 2R_{\mu\alpha\nu\beta}R^{\alpha\beta} - \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + \Box R_{\mu\nu} + g_{\mu\nu}\nabla_\alpha\nabla_\beta R^{\alpha\beta} - 2\nabla_\alpha\nabla(_\mu R^\alpha_\nu) \qquad (40d.)$$

where brackets mean symmetrization.
The important characteristics in 4D are: The description contains fourth derivations of metrics, contributes to massive spin-2 modus, generates ergo ghosts at description or interpretation of pure local polynom-action.

6.6. Ghost-freedom in 4D:

If the complete description is postulated as a local polynom-action, then there appears a ghost because of generating it as a spin-2-ghost at $R^2_{\mu\nu}$. But the description is complete ghostfree, iff it comes fundamental from: $F[g]=Tr(\log O[g])$ and iff:
1. The operator $O[g]$ is elliptic,
2. The spectrum is positive definit,
3. There are no negative norm-states.

These conditions then means in summary:
<u>Ghosts are artefacts and defects of local truncation.</u>
The complete spectral-theory is stable. The truncated Taylor-series may appear unstable.

<u>6.7. RG-interpretation in 4D:</u>

If a cutoff over λ is introduced, then

$$F_\lambda=Tr\left(\log\left(\frac{O}{\lambda^2}\right)\right) \quad . \qquad (41a.)$$

Then the couplings depend of λ .

$$G(\lambda)---\alpha(\lambda)---\beta(\lambda) \quad . \qquad (41b.)$$

A fixpoint means:

$$\lambda\frac{dG}{d\lambda}=0 \quad . \qquad (41c.)$$

This result exactly correspondens to UV-fixpoint of asymptotic safety.

<u>6.8. The complete physical interpretation in 4D:</u>

In 4D this ansatz consequently delivers:

1. Cosmological constant,
2. Einstein-term,
3. Quadratic curvature,
4. Non-local log-terms,
5. RG-flow of the couplings.

That automatically comes from the term:

$$F[g]=\frac{1}{2}Tr(\log O[g]) \quad . \qquad (42a.)$$

<u>6.9. The deepest structural point:</u>

In 4D gravity then is not a fundamental interaction but a spectral-selfconsistence of an elliptic operator. Spacetime-geometry is generated by:

$$\delta Tr(\log O[g])=0 \quad . \qquad (42b.)$$

This situation then is the 4D-description of the fixpoint principle.

7. Global processes:

7.1. Quantitative approximation of cosmological constant:

From $F=\frac{1}{2}Tr(\log(O))$ (43a.)

there is:

$$\Lambda_{Eff}\sim\frac{1}{4\pi^2}\int_0^{\Lambda_{UV}} s^{-3}ds \qquad (43b.)$$

with the quartic divergence of:

$$\Lambda_{Eff}\sim\Lambda_{UV}^4 \quad . \qquad (43c.)$$

Setting Planck-scales, then there is:

$$\Lambda_{UV}\sim M_{Pl} \quad , \qquad (44a.)$$

and

$$\rho_{\Lambda^{theory}}\sim M_{(Pl)}^4 \quad . \qquad (44b.)$$

But observed/measured is:

$$\rho_{\Lambda^{obs}}\sim 10^{-120}M_{(Pl)}^4 \quad . \qquad (44c.)$$

This is the problem of cosmological constant and in the image of this spectral theory it means, that the coherence-fixpoint must nearly totally cause a spectral erasing. The difference follows from using classical QFT-summing in flat Minkowski-background instead of using new ansätze.

7.2. Dark energy as a spectral effect:

If the heat-kernel series isn’t cut off, then there generate non-local terms:

$$\Gamma_{Nonloc}\sim\int\sqrt{-g}\,R\left(\log\left(\frac{\Box}{m^2}\right)\right)R \qquad (45.)$$

In cosmological FLRW-geometry this situation then leads to timedependent, effective contributions, to dynamical dark energy, possible self-accelerating expansion and dark energy could be interpreted as a form of spectral back-reaction.

7.3. Gravity-waves in this description:

Linearizing around Minkowski -metrics leads to the following terms:

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu} \quad (46a.)$$

With only Einstein-term:

$$\Box h_{\mu\nu}=0 \quad (46b.)$$

With R^2 -term appears an additional scalar wave of mass:

$$m_0^2 \sim \frac{1}{\alpha} \quad (46c.)$$

With $R_{\mu\nu}^2$ there would be a massive spin-mode and a ghost-sign in equation. In the full $Tr(\log)$ - ansatz, the propagator contains logarithmic corrections of form:

$$\frac{1}{k^2} \rightarrow \frac{1}{k^2}\left(1+c\cdot k^2 \log(k^2)\right) \quad . \quad (47.)$$

This description changes high-energy behaviour but not the lower structure of frequency. The LIGO-area concrete stays at Einstein-level.

7.4. Derivation of the nonlocal term $R\log(\Box R)$ in 4D:

From $\Gamma=\frac{1}{2}\log det(-\Box+m^2)$ (48a.)

for small curvatures there can be developed:

$$\Gamma=\int \sqrt{-g}\left(R\frac{1}{\Box}R+R\log\left(\frac{\Box}{m^2}R\right)...\right) \quad . \quad (48b.)$$

The logarithmic structure generates from:

$$\int_0^\infty \frac{ds}{s}e^{-s\cdot m^2} \quad (49.)$$

which means, that the UV-subtraction generates the log-term. This situation then leads physically to longrange correlations, scaledependent gravity and running of G.

7.5. The whole interpretation in 4D:

The model delivers in 4D mandatory over heat-kernel expansion with Seeley-de Witt coefficients [10.] c_n :

$$F[g]=\frac{1}{2}Tr\log(O[g]) \rightarrow \int d^4x\sqrt{-g}(\Lambda+c_1 R+c_2 R^2+c_3 R^{\mu\nu}R_{\mu\nu}+...) \quad (50.)$$

and therefore a complete spectral structure of:

1. $Rlog(\Box R)$,

2. RG-flow leads to scale-dependent couplings,
3. Fixpoint leads to selfconsistent spectrum.

In Interpretation Gravity in 4D then is:
1. Spectral backcoupling of an elliptic operator,
2. Casimirlike effects appear of the complete vacuum,
3. Fixpoint-condition of an information-Hamiltonian.

8. FLRW- Cosmology with R^2 and non-local terms:

1. Discussed is the running in $G(k)$,
2. Avoidance of singularities by using R^2 ,
3. Black Holes with spectral corrections.

8.1. Working with effective action in 4D:

$$S=\int d^{4x}\sqrt{-g}\left(\Lambda+\frac{1}{16\pi G}R+\alpha R^2+\beta(R^{\mu\nu}R_{\mu\nu})+R\log\left(\frac{\Box}{m^2}\right)R\right) \qquad (51a.)$$

Then take the lineelement of:

$$ds^2=-dt^2+a^2(t)(d\vec{x})^2 \qquad (51b.)$$

and Ricci-scalar with its Hubble-function of;

$$H(t)=\frac{\dot{a}(t)}{a(t)} \quad . \qquad (51c.)$$

A.) Only the R^2 -term:

The modified Friedman-equation comes to:

$$3H^2+36\alpha\left(2H\ddot{H}+6H^2\dot{H}+\dot{H}^2\right)=8\pi G\rho \qquad (52.)$$

Important is he fact, that at high curvature the R^2 term dominates, Then there is a solution with $H=const.$ and exponential expansion, which exactly is the case of **Starobinsky-inflation.** The connecting inflaton is the scalar modus from R^2 .

B.) Non-local term $R\log(\Box R)$:

In FLRW there is:

$$\Box R=-\ddot{R}-3H\dot{R} \quad . \qquad (53a.)$$

Then the logarithm generates retarded integral-equations, effective memory-terms and the consequences are a form of dynamically dark energy and self-accelerating solution without an explicite Λ .

8.2. Running of $G(k)$:

From the spectral structure follows:

$$G(k)=\frac{G_0}{1+c\,G_o k^2} \qquad (53b.)$$

for high moments of k .

At UV there is:

$$G(k)\sim\frac{1}{ck^2} \quad . \qquad (53c.)$$

Gravity then gets weaker asymptotically. This correspondens to the fixpoint of asymptotic safety. Now in cosmolocic applications there is set:

$k\sim H$. Then $G(H)$ gets time-dependent.

<u>8.3. Avoiding of singularity:</u>

Look a high curvature with $R\rightarrow\infty$:

With R^2 -domination, the field-equation reduces to:

$$\Box R-m^2 R=0 \qquad (53d.)$$

with

$$m^2\sim\frac{1}{\alpha} \quad .$$

(53e.)Solutions stay finite. As a result there is, that Big-Bang singularity is substituted by Big Bounce and Ricci-scalar doesn't diverge. This situation in principle is possible in 4D-description.

<u>8.4. Black Holes:</u>

For static, spheric solution there is:

$$ds^2=-f(r)\,dt^2+\frac{dr^2}{f(r)}+r^2 d\Omega^2 \qquad (54a.)$$

In Einstein there is:

$$f(r)=1-\frac{2GM}{r} \quad , \qquad (54b.)$$

with R^2 -correction, then there is:

$$f(r)=1-\frac{2GM}{r}+\epsilon\frac{e^{-Mr}}{r} \qquad (54c.)$$

with $m^2 \sim \frac{1}{\alpha}$ → Yukawa-correction.

Nonlocal terms generate:

$$f(r) = 1 - \frac{2G(r)M}{r} \tag{54d.}$$

with running $G(r)$.

In the middle of center for $\lim r \to 0$ the singularity could be weaken and possibly generated a regularly center.

<u>8.5. Gravity waves:</u>

The propagator changes from:

$$\frac{1}{k^2} \tag{55a.}$$

to

$$\frac{1}{k^2(1 + c\,k^2 \log(k^2))} \ . \tag{55b.}$$

This situation then generates Einstein-behaviour at low frequencies and weakening/damping of the ampliude at high frequencies. There is no additionally observed polarization, iff α is small.

<u>8.6. The complete structure in 4D:</u>

This ansatz consequently leads to:
1. Inflation over R^2 ,
2. Dynamical dark energy-terms over nonlocal log-terms,
3. UV-weakening of gravity,
4. Possible erasing of singularities,
5. Modified Black Holes.

This all description comes from:

$$F[g] = \frac{1}{2} Tr(\log(O[g])) \tag{56.}$$

<u>8.7. Physical interpretation:</u>

In 4D the structural core of gravity then is spectral selfconsistence, Casimirlike vacuum-backcoupling, RG-fixpoint of an elliptic operator and the Einstein-equation only is the leading order of spectral expansion.

<u>9. Construction of coherence-operator as an information-Hamiltonian:</u>

<u>9.1. Basic idea is a Hamilton-function H from information-theory, where the conditions hold of:</u>

1. H is a generator of dynamics,
2. It is the spectral-carrier of the states,
3. It determines the entropy over

$$S=Tr(\rho\log\rho) \qquad (57a.)$$

.

For a Gaussian-field there is:

$$Z=\int D\phi e^{-\frac{1}{2}\phi H\phi}=(det(H))^{-\frac{1}{2}} \qquad (57b.)$$

with its free energy of:

$$F=\frac{1}{2}Tr(\log(H)) \quad . \qquad (57c.)$$

This formulation exactly is the used structure.

9.2. Now defined is the Hamiltonian H in 4D:

Definition of the information-Hamiltonian as:

$$H[g]=-\nabla^2+\xi R+m^2 \qquad (58.)$$

Characteristics are:
1. Elliptic operator,
2. Positive for $m^2>0$,
3. Completely geometric covariant,
4. Well-defined on a 4D-manifold.

This operator measures spectral-coherence. How strong does geometry modulate the eigenvalues? The eigenwertproblem then is:

$$H\,\psi_n=\lambda_n\,\psi_n \qquad (59.)$$

and the spectral density contains all relevant geometric informations.

9.3. Spectral functional as an information measurement:
Define:

$$F[g]=\frac{1}{2}Tr(\log(H[g])) \qquad (60a.)$$

then spectral is:

$$F=\frac{1}{2}\sum_n(\log(\lambda_n)) \qquad (60b.)$$

with interpretation of: large eigenvalues show high incoherence but near the fixpoint there is a stationary spectrum. Now the heat-kernel in 4D:

$$F=\int d^4 x\sqrt{-g}\left(\Lambda+\frac{1}{16\pi G}R+\alpha R^2+\beta R^{\mu\nu}R_{\mu\nu}+R\log\left(\frac{\Box}{\mu^2}\right)R\ldots\right) \tag{60c.}$$

This situation leads to the interpretation, that gravity generates as a spectral answer of the information-Hamiltonian.

<u>9.4. Dynamics from information principle:</u>

Variation brings:

$$\delta F=\frac{1}{2}Tr\left(H^{-1}\delta H\right) \quad , \tag{61a.}$$

because of

$$\delta H=\xi\delta R+\delta\left(-\nabla^2\right) \tag{61b.}$$

there is then:

$$G_{\mu\nu}+\alpha H^1_{\mu n\nu}+\beta H^2\mu\nu=8\pi G\,\mathrm{T}_{\mu\nu} \tag{61c.}$$

Then the Einstein-equation is stationary point of spectral information.

<u>9.5. Entropic interpretation:</u>

The spectral entropy is:

$$S_{spec}=\frac{\partial}{\partial m^2}F \tag{62.}$$

is proportional to density of states. Therefore gravity evolves to extremal condition of the spectral entropy and this fact connects the model with entropic gravity theories.

<u>9.6. Applications in 4D:</u>

I. Cosmology:
In FLRW there is:

$$R=6\left(\dot{H}_{Hub}+2H^2_{Hub}\right) \tag{63.}$$

with its information-Hamiltonian of differential-operator and potential term:

$$H_{Ham}\,\phi(t)=-\left(\frac{\partial^2\phi}{\partial t^2}+3H_{Hub}\frac{\partial\phi}{\partial t}\right)+\left(\xi R+m^2\right)\phi(t) \quad . \tag{64.}$$

Eigenvalues are modulated by

$$H_{Ham}\,\phi(t) \quad . \tag{65a.}$$

and fixpoint is:

$$\delta Tr\left(\log\left(H_{Ham}\right)\right)=0 \tag{65b.}$$

.

This condition leads to modified Friedman-equation and ergo at large R to R^2 domination. This situation then is *Starobinsky* [11.] .

II. Running of $\left(G(k)\right)$:

The spectral density is $\rho(\lambda)\sim\lambda^2$, so there is regularization at the lambda-scale.

$$G(\lambda)=\frac{G_0}{1+c\,G_0\,\lambda^2} \quad . \tag{65c.}$$

In UV there is condition of: $G(\lambda)\rightarrow 0$. This exactly is behaviour of asymptotic safety.

III. Black Holes:

For static geometry there is:

$$H_{Ham}=-\Delta_{Lap}r+\xi R(r) \tag{65d.}$$

The spectrum is strongly modified near $(r=0)$. If (R) gets large, then:
eigenvalues increase, spectral backcoupling dominates. This situation could weaken the central singularity.

9.7. Ghost-freedom:

Important is the condition: As long as $H_{ham}>0$, there are no negative norm-states, no unlimited energy. Ghosts only appear at using local Taylor-truncation. The complete spectral-theory is stable.

9.8. Short summary:

In 4D the model consistently is described by:

$$F[g]=\frac{1}{2}Tr\left(\log\left(-\nabla^2+\xi R+m^2\right)\right) \quad . \tag{66.}$$

Gravity then is a form of spectral self-consistence, extremum of an information measure and Casimirlike effect of complete geometry. The truly central point is, that the coherence operator is mathematically a geometrically curved information Hamiltonian. The Einstein equation then is the condition that its spectrum is stationary. This could be an extremely strong structural statement.

10. The Standard-model of matter description as a spectral distortion:

The SM of matter can be assumed as a disturbation of the pure spacetime-spectrum. Starting point is the information Hamiltonian of:

$$H[g]=-\nabla^2+\xi R+m^2 \tag{67a.}$$

10.1. Including of matter-fields:

Every field (ϕ_i) (scalar, fermion, vector) delvers an additional operator of form:

$$H_i[g,\phi_i]=-\nabla^2+V_i(g,\phi_i) \quad . \tag{67b.}$$

Examples are:

1. Scalars: $(V_\phi=m_\phi^2+\xi_\phi R)$,
2. Fermions: $(D^2+m_\psi^2)$, (68a.-c.)
3. Vectors: $(-\nabla^2 g_{\mu\nu}+R_{\mu\nu}+m_A^2 g_{\mu\nu})$.

Then there is for the complete functional:

$$F[g,\phi_i]=\frac{1}{2}\sum_i(-1)^{F_i}Tr(\log(H_i[g,\phi_i])) \tag{69.}$$

with $F_i=0$ for bosons and $F_i=1$ for fermions.

10.2. Spectral backcoupling/backaction:

Variation after $g_{\mu\nu}$ delivers:

$$\delta_g F=\frac{1}{2}\sum_i(-1)^{F_i}Tr(H_i^{-1}\delta g H_i)\langle T_{\mu\nu}^{SM}\rangle \quad . \tag{70.}$$

This term reproduces the Einstein-equations with standard-model energy-momentum. Every sort of field contributes additional to spectral gravity. No further additive assumptutions necessary, all information comes from spectral description.

10.3. Physical consequences:

If a scalarfield is included, then there can be generated an inflaton from term H_ϕ . Dark energy, caused from nonlocal terms like $(R\log(\Box)R)$ stays unchanged. UV-behaviour through all spectra modulated ➔ asymptotic safety in 4D possible.

10.4. Complete axiomatics of the information-Hamiltonian:

I. Basic axioms:

1. Geometric covariance:
   $\forall g_{\mu\nu}$ there is a covariant transformation of $O[g]$ under $Diff(M^4)$,
2. Ellipticity:
   $O[g]>0$. The operator is positive definite, which condition leads to Ghost-freedom,
3. Spectral dynamics:

Gravity generates as a stationary point of the spectral functionality.
$\delta Tr(\log(O[g]))=0$,
4. Matter-coupling:
Matter-fields appear as additional spectral distortion of: $(O[g])\rightarrow(O[g,\phi_i])$,
5. Hierarchy of scales:
Cutoff (Λ) regulates UV-flow which situation leads to RG-flow of coupling:
$G(\Lambda);\alpha(\Lambda);\beta(\Lambda)$

II. Consequences:
Cosmological constant and Einstein-term appear automatically as leading heat-kernel-coefficients in 4D. Quadratic terms and non-local descriptions deliver inflation and dynamical dark energy. Matter is complete integrated by spectral distortions. Fixpoint-principle generates selfconsistent spacetime and in this way a spectral gravity-description.

III. The complete information-Hamiltonian:

$$O[g\phi_i]=-\nabla^2+\xi R+m^2+\sum_i H_i[g,\phi_i] \quad . \tag{71a.}$$

The assiocated functional is:

$$F[g,\phi_i]=\frac{1}{2}Tr(\log(O[g,\phi_i])) \tag{71b.}$$

This formula is the fundamental 4D- coherence-operator, from which the Einstein-equation directly follows, the standard-model backcoupling, the dynamics of inflation, the RG-flow, the ghost-freedom and non-local corrections can be derived.

<u>10.5. Physical interpretation:</u>

Physical definition/notation	**Comment**
Coherence: vacuum-information	He operator quantified, how coherent the 4D-geometry and the fields are.
Gravity: extremum of information	$\delta F=0$ → Stationary point of spectral back-coupling
Matter: integrated over spectrum	Every standard-model component modulates gravity over its eigenwert spectrum
UV-fixpoint: asymptotic safety	Operator positive definite, stable coupling, no ghosts
Non-local effects:	Log terms $(R\log(\Box)R)$, dynamical dark energy, long-distance coherence.

<u>Table 4: Short summary of gravity and matter-fields, generated by coherence-operator.</u>

<u>11. 4D-Casimir-analogy including standard-model fields:</u>

The coherence-operator connects to vacuum-energy, spectrum and dynamical gravity.

$$O[g,\phi_i]=-\nabla^2+\xi R+m^2+\sum_i H_i[g,\phi_i] \quad . \tag{72a.}$$

The index i goes over all fields: scalar, fermion,vector. H_i contains mass m_i and if applicable coupling to (R) . Positivity and ghost-freedom is guaranteed by the spectral functional of :

$$F[g,\phi_i]=\frac{1}{2}\sum_i (-1)^{F_i} Tr(\log H_i[g,\phi_i]) \quad . \tag{72b.}$$

11.1. GRT-physical concepts of casimir-analogy:

In this description-case geometry is dynamic, all fields contribute active parts to vaccum-energy, eigenfrequencies ω_n are modulated by local metric. This means, that vacuumenergy generates a spectral backcoupling to spacetime: $\delta F[g,\phi_i]=0$ ➔ modified Einstein-equations.
In contrast at classical Casimir-effect from QED there is a fix background and there only appears $E_{Cas}=\frac{1}{2}\hbar\sum_n \omega_n$ with constant bounding conditions and ergo fix vacuum-energy.

Concept	**Classical QED-Casimir-effect**	**The GRT- coherence-operator**
Geomatry	Fix	Dynamically
Energy	By Vacuum-fluctuations	Spectral backcoupling of action to gravity
Stability	Through boundary-conditions	Special elliptic operator, positive
Action	$E=\frac{1}{2}\hbar\sum_n \omega_n$	$F=\frac{1}{2}Tr(\log(H[g,\phi_i]))$
Matter	None	Standardmodel-fields ➔ additional spectral-distortion
Cosmology	None	Dynamics of inflation, dark energy, RG-flow.

Table 5: Gravity in 4D as a selfconsistent spectral Casimir-analogy of energy of complete vacuum structure including standard-model.

11.2. Heat-kernel-expansion in 4D:

For every operator H_i there is:

$$Tr\, e^{-sH_i}=\frac{1}{(4\pi s)^2}\int d^4x\sqrt{-g}\sum_{n=0}^{\infty} a_n^i s^n \tag{73.}$$

Thereby is set:
a_o^i --- cosmological constant,
a_1^i --- Einstein-Hilbert-term,
a_2^i --- quadratic curvature (R^2, Ric^2) ,
higher a_n^i --- nonlocal corrections of type $(R\log(\Box)R)$ resp.
$\sum a_n^{(i)}\, O_n^{(i)} \sim (Rlog(\Box)R)$.

Summing over all fields then delivers:

$$F = \frac{1}{2} \sum_i (-1)^{F_i} \int \sqrt{-g} \left(\Lambda_i + c_i R + \alpha_i R^2 + \beta_i R_{\mu\nu} R^{\mu\nu} + \ldots \right) \tag{74.}$$

Interpretation: Bosons and fermions contribute with different signs, which leads to a partly cancellation analog to Casimir-effect. The complete effect describes dynamical vacuum-energy and modified Friedman-and Schwarzschild-solutions for global and local application.

11.3. Dynamic gravity from 4D-Casimir:

Variation after $g_{\mu\nu}$ then delivers:

$$\delta F = \frac{1}{2} \sum_i (-1)^{F_i} Tr\left(H_i^{-1} \delta_g H_i\right) = 0 \tag{75.}$$

This variation leads to a modified Einstein-equation of:

$$G_{\mu\nu} + \alpha H_{\mu\nu}^{(1)} + \beta H_{\mu\nu}^{(2)} + \ldots = 8\pi G \langle T_{\mu\nu}^{SM} \rangle . \tag{76.}$$

T

Hereby $\left(\langle T_{\mu\nu}^{SM} \rangle\right)$ the expected value of matter-tensor-operator is spectral generated, in analogy to Casimir-effect. Important and interesting is the fact, that no extern energy is needed but gravity generates self-consistent from vacuum-structures.

11.4. Cosmology in 4D:

The FLRW-metric $\left(ds^2 = -dt^2 + a(t)^2 d(\vec{x})^2\right)$ (77.)

leads to spectral Friedman-equation of:

$$3H^2 + \sum_i (-1)^{F_i} \left[\alpha_i \left(R^2 - modifications\right) + \ldots \right] = 8\pi G \sum_i \langle \rho_i \rangle_{vac} \quad . \tag{78.}$$

There generate three classical effects:
1. Low-energy Einstein-term ➔ classical Friedman-behaviour,
2. High-energy R^2 dominance: Starobinsky-inflation,
3. Nonlocal $\left(R \log(\Box) R\right)$ -terms ➔ dynamical dark energy.
Casimirlike vacuum-fluctuations determine the expansion of 4D-spacetime.

11.5. Black Holes in 4D:

Static sphere:

$$\left(ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega^2\right) \tag{79.}$$

with its spectral vacuum-energy of:

$$\langle T_{\mu\nu} \rangle_{vac} = \frac{1}{2} \sum_i (-1)^{F_i} Tr\left(H_i^{-1} \delta_g H_i\right) \quad . \tag{80.}$$

then modified Schwarzschild appears with $f(r)$:

$$f(r)=1-\frac{2G(r)M}{r}+\epsilon\frac{e^{-mr}}{r}+\dots \quad . \tag{81.}$$

Conditions: situation ghostfree, because operator is positive definit. Central region could be described regularly, possible neglection of singularity because of:

$$\lim_{r\to 0}\epsilon\left(\frac{e^{-mr}}{r}\right)=-\epsilon m \quad . \tag{82.}$$

12. Spectral sum over standard-fields:

12.1.With the functional of:

$$F[g,\phi_i]=\frac{1}{2}\sum_i(-1)^{F_i}Tr(\log H_i[g,\phi_i]) \tag{83a.}$$

and the conditions where i goes over all standard-model fields (scalars, vectors, fermions), $F_i=0$ for all bosons, $F_i=1$ for all fermions and $(H_i=-\nabla^2+m_i^2+\xi_i R)$. The leading contribution for cosmological constant then is:

$$F\supset\frac{1}{32\pi^2}\sum_i(-1)^{F_i}m_i^4\int d^{4x}\sqrt{-g}\log\left(\frac{m_i^2}{\mu^2}\right) \tag{83b.}$$

This situation is an analogy to 4D-Casimir-energy of:

$E_n=\frac{1}{2}\hbar\sum_n\omega_n$. Because it is UV-divergent, a UV cut-off Λ_{UV} is needed.

12.2. Effectice cosmological constant:

$$\Lambda_{eff}=\frac{1}{32\pi^2}\sum_i(-1)^{F_i}m_i^4\left(\log\frac{m_i^2}{\mu^2}\right) \quad . \tag{84.}$$

Bosons increase Λ_{Eff} , fermions decrease Λ_{Eff} . Partly cancellation, a supersymmetric behaviour would result near zero.

Numeric orders of sizes:

Type of field	Mass m_i	Contributio $\frac{(m_i^4)}{(32\pi^2)}$
Higgs	125 GeV	$E^4\sim 10^8\,Gev^4$
$W^{\pm}$; Z^0	80-91 GeV	$E^4\sim 10^7\,GeV^4$
Top-quark	173 GeV (fermion)	$E^4\sim -10^9\,GeV^4$

Table 6: Contribution-sum from all matter-fields to effective cosmological constant. Sum of all SM-fields: $\Lambda_{Eff}^{SM}\sim(10^8-10^9)\,GeV^4$ **.**

But observed is: $(\rho_{\Lambda}^{obs}\sim 10^{-47}GeV^4)$ .Large difference, which means, that the spectral fixpoint conditions must nearly deliver complete cancellation.

12.3. Non-local dark energy:

Higher heat-kernels generate non-local structure:

$$F_{nonlocal} \sum_i (-1)^{F_i} \int d^{4x} \sqrt{-g}\, R \log\left(\frac{-(\Box)+m_i^2}{\mu^2}\right) R \qquad (85.)$$

This situation leads to dynamical scale-dependent dark energy. Effective like:

$$\rho_{DE}(t) \sim R(t) \log\left(\frac{\Box}{\mu^2}\right) R(t) \qquad (86.)$$

and in FLRW-metrics like: $\Box R = -\ddot{R} - 3H\dot{R}$. Memoryeffect causes dynamical acceleration which leads to description of a selfaccelerating universe without explicite Λ .

12.4 Modified Friedman-equations in 4D:

$$3H^2 + \{\alpha\left(2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2\right)_{R^2-Correction}\} + \rho_{DE}(t)\, Nonlocal \log - Term = 8\pi G \rho SM \qquad (87.)$$

Inflationphase: R^2 domination leads to Starobinsky-inflation and latephase is nonlocal generated and determined by dynamical dark energy.

12.5. Black-Holes in 4D-description:

Spectral vacuum-energy

$$e(\langle T_{\mu\nu}\rangle_{vac}) \qquad (88.)$$

modifies Schwarzschild

$$f(r) = 1 - \frac{2GM}{r} + \epsilon \frac{e^{¿} - mr}{r} + \dots \qquad (89.)$$

This situation generates Yukawa-corrections by R^2 -operator and non-local log-corrections which causes small longrange deviations.Singularities are weaken and are ghost-free.

12.6. Physical interpretation:

1. Casimir-analogy: selfconsostent gravity SM-fields = dynamical bounding conditions of coherence-operator,
2. Vacuum-energy: selfconsisting gravity ➔ Einstein-equation + corrections,
3.Dark energy: nonlocal spectral-correction of form $R(\log(\Box)R)$,
4. UV-fixpoint = asymptotic safe gravity ➔ stable couplings.

This all description comes from the coherence-operator as an information-Hamiltonian in 4D.

Appendix A:

Nonlinear Cosmic Dynamics

Area	Dynamic/Action	Spectral source	Typical effects/comment

Cosmology-early phase	R^2 -Inflation	Vukawa-like corrections small, R^2 dominates	$H \approx const.$; $a(t) \sim \exp(Ht)$; 60 e-folds possible
Cosmology-middle phase	Standard-model fields	Vacuumenergy SM-fields	Modulating SM-spectra, Expansion, Radiation and Matter-domination
Cosmology-late phase	Dynamical dark energy	Nonlocal term $R \log(\Box) R$	Logarithmic memory-effect ➔ accelerated expansion
Black-Hole: center/inner area/Kruskal-area	Local Casimirlike Backcoupling	Yukawalike terms from SM-fields	$f(r) \approx 1 - \frac{2GM}{r} + \sum \epsilon_i \frac{e^{-m_i r}}{r}$; regular center, ghost-free
Black Hole: Outer area/Schwarzschild-area	Non-local long range correction	$R \log(\Box) R$	Small $f(r)$ -deviations at large r , dynamical dark energy-effect localisated
General:	Fixpoint of spectral dynamics	$\delta Tr(\log H[g, \varphi_i]) = 0$	Einstein-equation + corrections, spectral gravity, 4D-Casimir-analogy

Table 7: Core-idea: All description of universe, from inflation over dynamics of dark energy until modifications of Black-Hole-metrics generates selfconsistent from the spectrum of coherence-operator without extern sources of energy or any finetuning.

Appendix B:
“Science errs forward and progresses!“, Sir Karl Popper.

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14. Verification:

This paper definitely is written without support from an AI, LLM or chatbot like

Grok or Chat GPT 4 or other artificial tools. It is fully, purely human work in every universe.