Deduction of Planck's Radiation Law using Boltzmann Statistics without Quantization Postulates

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Abstract

This work will demostrate that blackbody radiation is a thermodynamic phenomenon.

Keywords: thermodynamics, radiation, black body, Max Planck, Ludwig Boltzmann

1. Introduction

During the 1890s, physicists such as Lummer, Pringsheim, and Rubens conducted extremely detailed experiments at the Physikalisch-Technische Reichsanstalt in Berlin. These measurements showed that the intensity of radiation increased with frequency up to a maximum point and then decreased, forming a bell-shaped curve.

In June 1900, Lord Rayleigh published an article proposing the form of the law based on the equipartition theorem. However, the formula only worked for long wavelengths. In 1905, James Jeans corrected an error in the proportionality constant, resulting in the Rayleigh-Jeans formula as we know it today.

In October 1900 Max Planck published this famous law, which gave rise to quantum mechanics.

Planck's law, as he himself said, was born out of an act of desperation, assuming the quantization of energy; an argument that even he did not believe.

2. Planck constant (h)

The energy $\varepsilon = v_e^2 \mu_e$.Where $v_e$is the speed of the electron in the hydrogen atom and μ_e is the reduced mass of the electron.

The angular momentum of the electron is:

$$l_e = v_e r_b \mu_e = \hbar = 1.055 * 10^{-34}\text{Js} \ \ [1,2]$$

Where:

$$v_e = 2.2 * 10^6 \frac{m}{s} \text{ Electron speed}$$

$$r_b = 5.3 * 10^{-11}\, m \text{ Bohr radius}$$

$$\mu_e = 9.11 * 10^{-31}\text{Kg} \text{ Electron mass}$$

$$\varepsilon = v_e^2 \mu_e \quad v_e = \frac{\hbar}{r_b \mu_e} \quad v_e = \omega r_b$$

$$\varepsilon = \left(\frac{\hbar}{r_b \mu_e}\right)(\omega r_b)\mu_e$$

Therefore

$$\varepsilon = \hbar\omega \quad \omega \text{ Is the angular frequency of the electron}$$

$$\omega = 2\pi\nu \quad \text{So} \quad \varepsilon = 2\pi\hbar\nu$$

$$\text{But} \quad 2\pi\hbar = h$$

$$\text{Therefore} \quad \varepsilon = h\nu \tag{1}$$

3. Average Energy ($\bar{\varepsilon}$)

Rayleigh and Jeas used the Boltzmann probability density function [1] continuously, that is, using integration. They showed that the energy density is:

$$\rho_T(\nu) = \frac{8\pi\nu^2}{c^3} kT \ \ [1,2] \tag{2}$$

This result led to what they called the ultraviolet catastrophe, because the energy density could reach infinite values. Furthermore, it did not coincide with experimental results, which showed that at high energies the radiation density tended towards zero.

The calculations they used were the following:

$$\bar{\varepsilon} = \frac{\int_0^\infty \varepsilon \frac{e^{-\frac{\varepsilon}{kT}}}{kT} d\varepsilon}{\int_0^\infty \frac{e^{-\frac{\varepsilon}{kT}}}{kT} d\varepsilon} = kT$$

But Max Planck did not use integrals but summations, therefore assuming that energy is not continuous, and by performing the calculations he found that the formula is:

$$\rho_T(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kt}}-1}d\nu \quad [1,2]$$

Which coincided with the experimental results and avoided the ultraviolet catastrophe.

Rayleigh-Jeans made two crucial errors that, had they not made them, quantum mechanics would not exist.

The two errors were, firstly, assuming that the upper limit of integration is infinite, which implies the existence of infinite energies, which is not true. Secondly, they considered only one average energy, the average energy of ε, but in the system there is another energy, kT. The energy ε is the energy acquired by the electrons due to the temperature T, and the thermal energy is kT. Taking this into account, the formula for finding the average energy $\bar{\varepsilon}$ is:

$$\bar{\varepsilon} = \bar{\varepsilon}_T - \bar{\varepsilon}_\nu \tag{1}$$

$$\bar{\varepsilon}_T = \frac{\int_\varepsilon^0 kT\,\frac{e^{-\frac{\varepsilon}{kT}}}{kT}d\varepsilon}{\int_\varepsilon^0 \frac{e^{-\frac{\varepsilon}{kT}}}{kT}d\varepsilon} \tag{2}$$

$$\bar{\varepsilon}_\nu = \frac{\int_\varepsilon^0 \varepsilon\,\frac{e^{-\frac{\varepsilon}{kT}}}{kT}d\varepsilon}{\int_\varepsilon^0 \frac{e^{-\frac{\varepsilon}{kT}}}{kT}d\varepsilon} \tag{3}$$

Restando (2) – (3) You get the following:

$$\bar{\varepsilon} = \frac{\int_\varepsilon^0 \frac{(kT-\varepsilon)}{kTe^{\frac{\varepsilon}{kT}}}d\varepsilon}{\int_\varepsilon^0 \frac{e^{-\frac{\varepsilon}{kT}}}{kT}d\varepsilon} \tag{4}$$

$$\int_\varepsilon^0 \frac{(kT-\varepsilon)}{kTe^{\frac{\varepsilon}{kT}}}d\varepsilon = -\frac{\varepsilon}{e^{\frac{\varepsilon}{kT}}} \tag{5}$$

$$\int_\varepsilon^0 \frac{e^{-\frac{\varepsilon}{kT}}}{kT}d\varepsilon = \frac{1}{e^{\frac{\varepsilon}{kT}}} - 1 \tag{6}$$

Substituting (5) y (6) en (4)

$$\bar{\varepsilon} = \frac{-\frac{\varepsilon}{e^{\frac{\varepsilon}{kT}}}}{\frac{1}{e^{\left(\frac{\varepsilon}{kT}\right)}}-1} = \frac{\varepsilon}{e^{\left(\frac{\varepsilon}{kT}\right)}\left(1-\frac{1}{e^{\left(\frac{\varepsilon}{kT}\right)}}\right)} = \frac{\varepsilon}{e^{\left(\frac{\varepsilon}{kT}\right)}-1} \tag{7}$$

Therefore:

$$\rho_T(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}d\nu \tag{8}$$

Which is the same equation that Planck found in 1900, but without assuming quantization of energy.

4. Conclusions

Energy is continuous and non-quantized; it does not need to be discrete to justify the experimental results of black-body radiation. The key detail is knowing that the difference between the two average energies must be used.

Planck's constant h is simply the angular momentum of the electron in the hydrogen atom multiplied by 2π. It is not a magical or extraordinary quantity, as some claim. It is simply the minimum action of the electromagnetic system sof the hydrogen atom.

5. References

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