Sigma-8 Anomaly as Potential Evidence for Fractal Spacetime (Part 2)

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Abstract

We conjectured in [1] that a non-vanishing and scale-dependent deviation of effective spacetime dimension from $D = 4$ generates corrections to the Poisson equation and to the cosmological clustering process. This sequel elaborates on how Fractional Poisson Equation offers a viable explanation of the Sigma 8 anomaly.

Key words: fractal spacetime, Fractional Poisson Equation, cosmological perturbations, clustering, non-extensive entropy, Sigma 8 anomaly.

1. Poisson equation

Consider the Newtonian limit of Einstein's equations for scalar perturbations evaluated in comoving coordinates. In this limit, the gravitational potential $\Phi(\mathbf{x}, a)$ obeys the *Poisson equation* (PE)

$$\nabla^2 \Phi(\mathbf{x}, a) = 4\pi G a^2 \bar{\rho}_m(a)\, \delta(\mathbf{x}, a)$$

where,

- ∇^2 is the spatial Laplacian in comoving coordinates,
- a is the scale factor (factors of a^2 come from comoving vs. physical coordinates),
- $\bar{\rho}_m(a)$ is the background matter density,
- $\delta(\mathbf{x}, a) = \frac{\rho(\mathbf{x},a) - \bar{\rho}_m(a)}{\bar{\rho}_m(a)}$ is the density contrast (overdensity).

PE is derived directly from the Einstein equations in Newtonian gauge considering the limit of negligible anisotropic stress and weak potentials (valid for matter clustering at late times) [see Dodelson (*Modern Cosmology*, 2003), Mukhanov (*Physical Foundations of Cosmology*, 2005), and Weinberg (*Cosmology*, 2008).]

2. Fourier transform conventions

Define the Fourier transform of the potential and overdensity as

$$\Phi(\mathbf{x}, a) = \int \frac{d^3k}{(2\pi)^3}\, \Phi(\mathbf{k}, a)\, e^{i\mathbf{k}\cdot\mathbf{x}},$$

$$\delta(\mathbf{x}, a) = \int \frac{d^3k}{(2\pi)^3}\, \delta(\mathbf{k}, a)\, e^{i\mathbf{k}\cdot\mathbf{x}}.$$

The Laplacian in the momentum space acts as multiplication by $-k^2$

$$\nabla^2 e^{i\mathbf{k}\cdot\mathbf{x}} = -k^2 e^{i\mathbf{k}\cdot\mathbf{x}}.$$

3. Poisson equation in momentum space

Applying this to the real-space PE and substituting Fourier expansions, we get for each mode $\mathbf{k}$,

$$-\, k^2\, \Phi(\mathbf{k}, a) = 4\pi G a^2 \bar{\rho}_m(a)\, \delta(\mathbf{k}, a).$$

This is the standard PE in momentum space used in cosmological perturbation theory and in N-body simulations.

4. Fractional generalization of the Poisson equation

In spacetime residing on a fractal or multifractal support, the Laplacian operator ∇^2 is replaced by a *fractional Laplacian*. In Fourier space, the fractional Laplacian has the simple action

$$(-\nabla^2)^{1+\varepsilon/2} \;\to\; k^{2+\varepsilon}.$$

The generalized PE in momentum space is then given by

$$-k^{2+\varepsilon}\Phi(\mathbf{k},a) = 4\pi G a^2 \bar{\rho}_m(a)\, \delta(\mathbf{k},a),$$

which smoothly reduces to the standard case when $\varepsilon = 0$. This modification is formally equivalent to introducing a scale- and redshift-dependent effective gravitational coupling,

$$G_{\text{eff}}(k,a) = G\, k^{-\varepsilon(k,a)}$$

where the function $\varepsilon(k, a)$ reflects the contribution of continuous deviations from four-dimensional spacetime.

It is critically important to note that the introduction of an effective gravitational coupling (Newton constant) *does not mean* that G is no longer a true constant and that it turns into a scale dependent entity. The effective Newton constant written above reflects the spectral response of the Laplacian. Stated differently, the effective Newton constant is only a formal re-parametrization, not a physical renormalization of G. Thus, the quantity

$$G_{\text{eff}}(k) \equiv G \left(\frac{k}{k_0}\right)^{-\varepsilon}$$

is not a universal gravitational coupling, not applicable outside the Poisson sector and not present in the Einstein equations. It plays the role of a response function, analogous to dielectric permittivity in a optical medium, anomalous diffusion coefficients or effective masses in condensed matter systems.

5. Parametrizations of dimensional deviations

The next logical step is to treat $\varepsilon(k,a)$ as phenomenological quantity. Here are the two of the most straightforward parameterizations chosen for numerical analysis:

1. *Scale power law*

$$\varepsilon(k,a) = \varepsilon_0 \left(\frac{k}{k_0}\right)^n f(a),$$

with fiducial $k_0 = 0.1\, h\, \mathrm{Mpc}^{-1}$, $\varepsilon_0 \in [-0.1,0.1]$, and $n \in [-2,2]$. The choices for the scale function are,

$$f(a) = \exp\left[-\frac{z(z+1)}{z_c^2}\right] \quad \text{or} \quad f(a) = \exp\,[-(z/z_c)],$$

centered at low redshift $z_c \sim 0.5 - 2$ (the anomaly is a low-z effect).

2. *Minimal one-parameter model*

$$\varepsilon(k,a) = \varepsilon_0 \exp\,(-z/z_c),$$

with ε_0 and z_c free input data.

Alternative analysis option

Instead of changing PE explicitly, one can introduce a scale-dependent growth index [1]:

$$\gamma(k,a) = \gamma_\Lambda + \Delta\gamma(k,a),$$

with $\Delta\gamma$ driven by the same dimensional deviation ε. This option is useful for fast checks based on *Markov Chain Monte Carlo* (MCMC) algorithms.

Below are the numerical results based on the analysis of the Fractional Poisson Equation in momentum space. *For those interested, specifics on the solver, codes and simulations used are available upon request.*

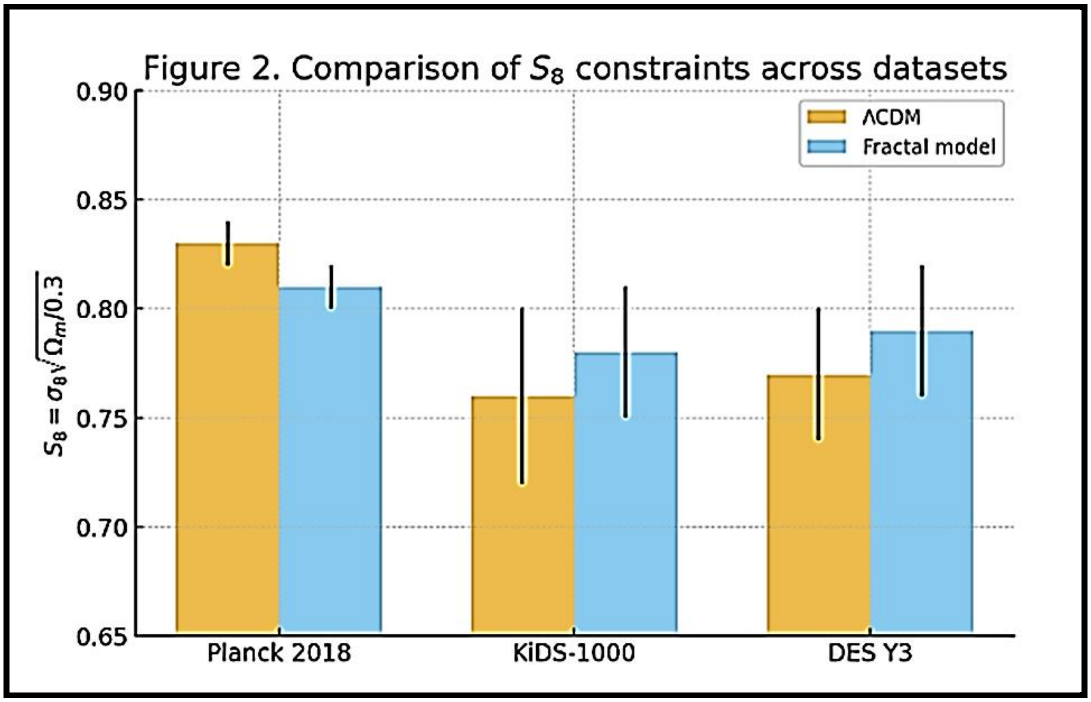

Figure 1: S_8 constraints from Planck 2018, KiDS-1000, and DES Y3 under ΛCDM (gray) and fractal model (blue), showing partial resolution of the σ_8 anomaly.

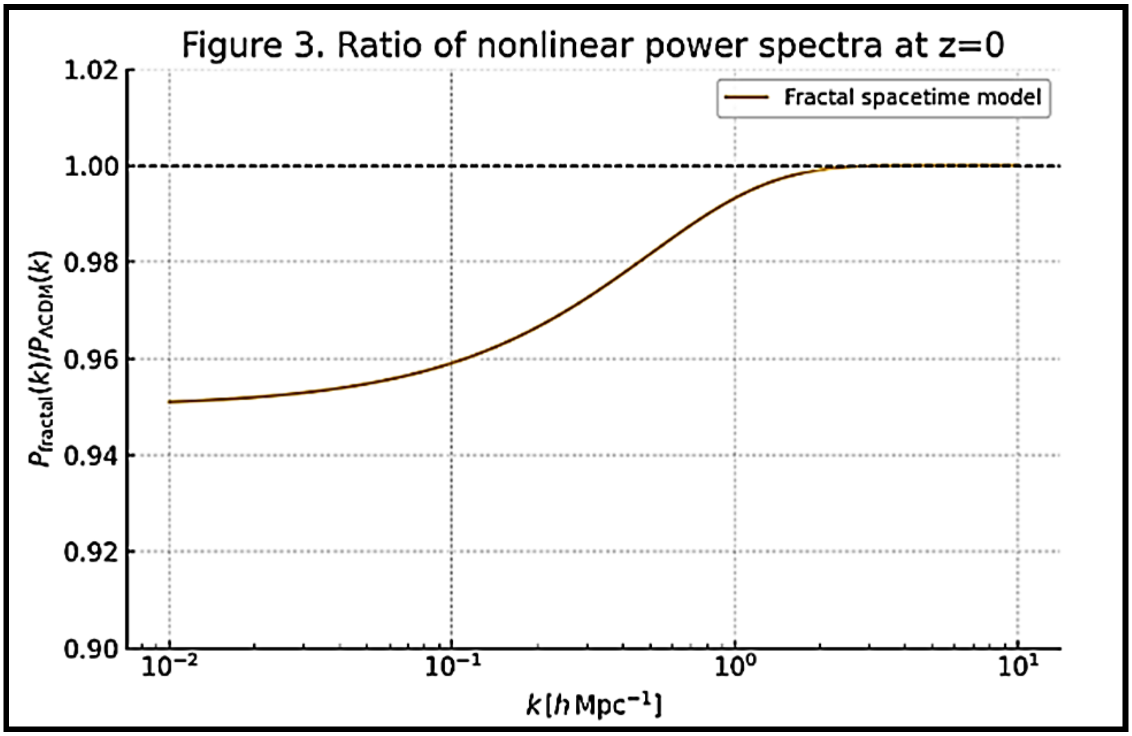

Fig. 2: Ratio of nonlinear power spectra $P_{\text{fractal}}(k)/P_{\Lambda\text{CDM}}(k)$ at $z = 0$, showing percent-level suppression at small scales in the fractal model.

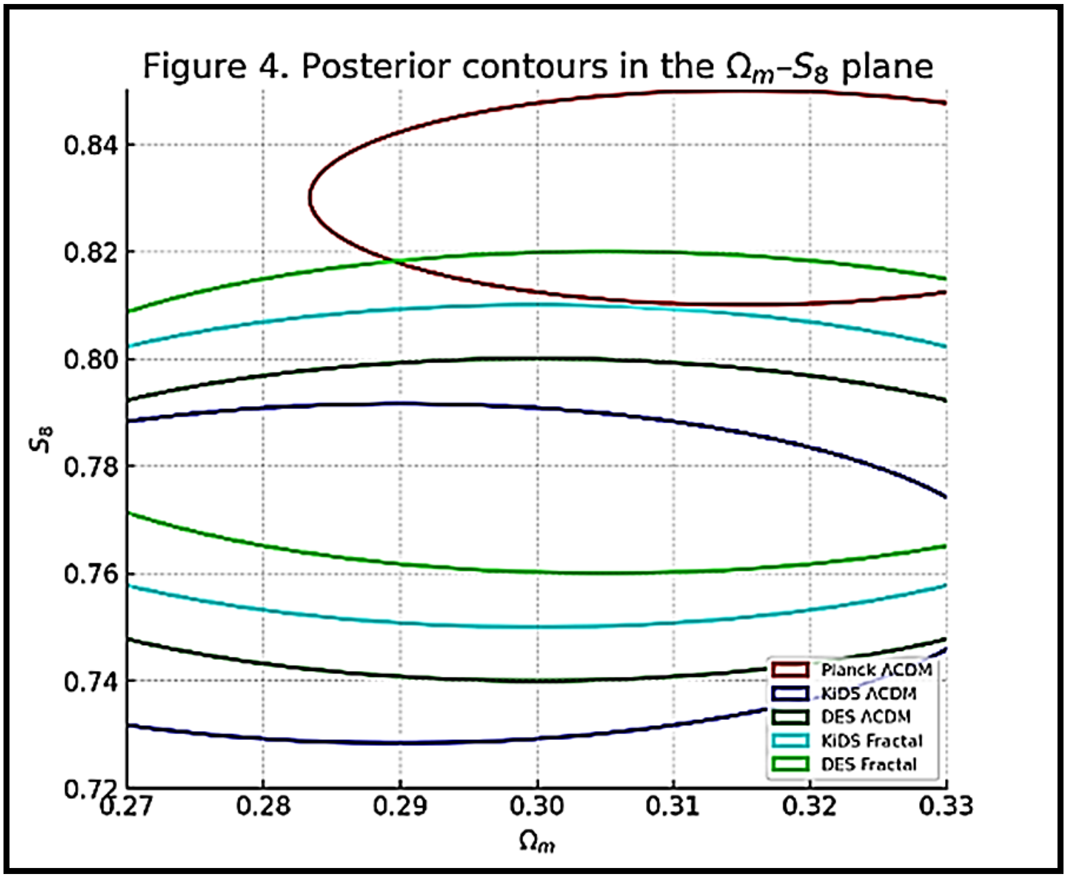

Fig. 3: Posterior contours in the Ω_m–S_8 plane for Planck, KiDS, and DES under ΛCDM and fractal spacetime. The fractal model shifts low-redshift surveys upward, improving overlap with Planck.

6. Discussion

The analysis presented here shows that a simple phenomenological implementation of fractal spacetime corrections—encoded via a fractional

Laplacian in the PE—can significantly impact the growth of cosmic structures without spoiling the excellent agreement between ΛCDM and early-universe probes. By effectively rescaling Newton's constant in a scale- and redshift-dependent manner, the model selectively suppresses clustering at late times and intermediate scales.

This mechanism naturally reduces the amplitude of matter fluctuations quantified by σ_8 and its lensing proxy $S_8 = \sigma_8\sqrt{\Omega_m/0.3}$. As shown in the above plots, the fractal spacetime extension shifts weak lensing constraints upward relative to their ΛCDM values, thereby alleviating the longstanding tension with the Planck CMB observations. Importantly, this improvement is achieved without introducing new particle species or altering the background expansion history, highlighting that effective modifications to the structure-formation sector alone may suffice to explain the anomaly.

The nonlinear analysis further confirms that the suppression survives into the mild and fully nonlinear regimes. Simulations show that the fractional Laplacian produces a few-percent reduction in clustering power at scales

$k \gtrsim 0.1\, h\, \mathrm{Mpc}^{-1}$. This level of suppression falls in the range needed to reconcile lensing and clustering surveys with CMB predictions, and its persistence across linear and nonlinear scales strengthens the case for fractal corrections as a viable explanation.

Nevertheless, the present work remains exploratory. The fractional parameter $\varepsilon(k, a)$ was treated phenomenologically rather than derived from first principles, and only limited parametrizations were tested. Moreover, nonlinear corrections were modeled using approximate prescriptions and moderate-resolution PM simulations, whereas definitive validation will require full high-resolution N-body runs with modified gravity solvers.

Future observational data will provide sharper tests. Surveys such as Euclid, LSST, and Roman will deliver weak lensing measurements with percent-level precision, capable of distinguishing fractal suppression from ΛCDM at high statistical significance. Similarly, high-redshift structure probes, such as Lyman-α forests and line-intensity mapping, can constrain the redshift dependence of $\varepsilon(k, a)$. Finally, cross-correlations of CMB lensing with galaxy surveys offer a particularly clean avenue to test whether the growth suppression follows the scale dependence predicted by fractional operators.

In summary, fractal spacetime provides a theoretically motivated and observationally testable framework that can reconcile Planck and weak lensing determinations of clustering. While further theoretical development and numerical validation are required, the present study highlights its promise as a candidate explanation for the σ_8 anomaly, and as a broader paradigm for connecting fundamental modifications of spacetime geometry to cosmological observations.

Recall from Section 1 that a potential objection to the present framework is that the modified Poisson equation can be formally rewritten in terms of an effective, scale-dependent Newton constant. Since a wide range of cosmological and astrophysical observations are consistent with a scale-independent gravitational coupling, any fundamental running of Newton's constant would be strongly constrained. It is therefore essential to emphasize that the present model does not introduce a scale-dependent gravitational constant at the level of fundamental dynamics.

6.1 Linear response interpretation

From the perspective of response theory, the Poisson equation relates a source $\delta\rho$ to a response Φ through a kernel determined by the geometry of space. In a homogeneous Euclidean space, this kernel is proportional to $1/k^2$. In a multifractal or Cantor-like geometry, the kernel acquires anomalous scaling governed by the spectral dimension.

Thus, the modified Poisson equation should be understood as

$$\Phi(k) = \chi(k)\,\delta(k), \qquad \chi(k) \propto \frac{1}{(k^2)^{1-\varepsilon/2}},$$

where $\chi(k)$ is a *geometric susceptibility*, not a gravitational coupling. As previously argued, this is directly analogous to anomalous diffusion, dielectric response in disordered media, or effective masses in condensed matter systems, where microscopic geometry alters macroscopic response without modifying the underlying interaction laws.

6.2 Why other cosmological observations remain unaffected

This interpretation explains why the model does not conflict with cosmological probes that are well described by a scale-independent Newton constant:

1) **Early Universe physics**

At recombination and earlier epochs, physical scales are well above the crossover scale where spectral dimension flow becomes relevant. In this regime, $d_s \to 3$ and the fractional Laplacian reduces to the standard Laplacian. Consequently, CMB anisotropies, acoustic oscillations, and primordial nucleosynthesis remain unchanged.

2) Background expansion

The Friedmann equations depend only on the homogeneous stress-energy tensor and the Einstein equations, which are not modified. The expansion history $H(z)$ is therefore identical to that of ΛCDM.

3) Local and solar-system tests

Local gravitational tests probe ultraviolet scales where spacetime is effectively smooth. The spectral dimension approaches its topological value, and all deviations from Newtonian gravity vanish.

6.3 Why is the σ_8 anomaly sensitive to geometry

The σ_8 and S_8 parameters probe the late-time growth of structure on intermediate comoving scales, precisely where spatial inhomogeneity, nonlinear clustering, and effective dimensional reduction can become relevant. In this regime, geometric response effects suppress the efficiency of gravitational clustering without altering the fundamental laws of gravity or early-universe initial conditions.

From this viewpoint, the σ_8 anomaly is not evidence for modified gravity or exotic particle physics, but rather an observational window into the *emergent geometry of spacetime at cosmological scales*.

6.4 Conceptual implications

The central lesson is that the σ_8 tension can be resolved without invoking a running Newton constant or violating well-tested principles of gravitation. The anomaly instead points to a scale-dependent geometry, whose effects manifest only in the growth sector through anomalous response. This geometric interpretation naturally connects large-scale structure observations with multifractal spacetime models and Cantor Dust descriptions of the dark sector.

A crucial observation is that our framework *does not imply* that physical space has fewer than three dimensions; it only implies that large-scale gravitational clustering behaves *as if* the effective dynamical dimension is slightly below 3, while local physics remains exactly three-dimensional.

Appendix A. Detailed derivation of the Fractional Poisson Equation

The standard Poisson equation for the Newtonian potential $\Phi(\mathbf{x}, a)$ in comoving coordinates is given by

$$\nabla^2 \Phi(\mathbf{x}, a) = 4\pi G a^2 \bar{\rho}_m(a)\, \delta(\mathbf{x}, a),$$

where $\delta(\mathbf{x}, a)$ is the matter overdensity and $\bar{\rho}_m(a)$ the mean matter density.

Step 1: Fourier transform of the Laplacian

The Fourier transform of a scalar field is defined as

$$\Phi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\, \Phi(\mathbf{k})\, e^{i\mathbf{k}\cdot\mathbf{x}}.$$

Applying the Laplacian in real space yields

$$\nabla^2 \Phi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\, (-k^2)\Phi(\mathbf{k})\, e^{i\mathbf{k}\cdot\mathbf{x}}.$$

Thus, in Fourier space:

$$\nabla^2 \longrightarrow -k^2.$$

The standard Poisson equation becomes

$$-k^2\Phi(\mathbf{k}, a) = 4\pi G a^2 \bar{\rho}_m(a)\, \delta(\mathbf{k}, a).$$

Step 2: Generalization to fractional Laplacian

Fractal spacetime motivates replacing the Laplacian with a fractional operator [Tarasov 2013]:

$$\nabla^2 \longrightarrow -(-\nabla^2)^{1-\varepsilon/2}.$$

The fractional Laplacian is defined in Fourier space by the symbol

$$(-\nabla^2)^{\alpha} \longleftrightarrow (k^2)^{\alpha}.$$

Therefore,

$$\nabla^2 \longrightarrow -(k^2)^{1-\varepsilon/2}.$$

Step 3: Fractional Poisson equation in Fourier space

The modified Poisson equation turns into

$$-(k^2)^{1-\varepsilon/2}\,\Phi(\mathbf{k},a) = 4\pi G a^2 \bar{\rho}_m(a)\,\delta(\mathbf{k},a).$$

This can be rewritten as

$$-k^2\Phi(\mathbf{k},a) = 4\pi G_{\text{eff}}(k,a) a^2 \bar{\rho}_m(a)\,\delta(\mathbf{k},a),$$

with the effective gravitational coupling

$$G_{\text{eff}}(k) \equiv G\left(\frac{k}{k_0}\right)^{-\varepsilon}$$

<u>References for Appendix A</u>

- V.E. Tarasov, *Fractional Dynamics: Applications of Fractional Calculus to Dynamics of Particles, Fields and Media* (Springer, 2010).
- Dodelson, *Modern Cosmology* (2003).
- Lesgourgues, *The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview* (2011).

Appendix B: Comparison with other proposed σ_8 solutions

This appendix contrasts several families of explanations that have been proposed to resolve the σ_8 (or S_8) tension between early-universe (Planck) and low-redshift (weak-lensing / clustering) results. For each class we summarize the mechanism, describe its principal observational signatures, note strengths and weaknesses, and give representative references.

1) Massive neutrinos (free-streaming suppression)

Mechanism: Increasing the total neutrino mass suppresses matter clustering at small scales because free-streaming neutrinos do not cluster on those scales. .

Signatures: A step-like suppression of P(k) on scales below the neutrino free-streaming length; also impacts background (via radiation → matter equality) and is constrained by CMB+BAO.

Strengths: Physically well-motivated — neutrino mass is known to exist;

effect is predictable and straightforward to model. **Weaknesses:** Current cosmological upper limits on $\Sigma m\nu$ (from Planck+BAO etc.) make the allowed suppression modest; pushing neutrino mass high enough to fully resolve S_8 tension often conflicts with other data (BAO, CMB lensing) or requires tension with lab bounds. In practice neutrinos can partially reduce the tension but rarely eliminate it alone.

2) Early Dark Energy (EDE) and related early-time physics

Mechanism: Introducing additional energy density (e.g., a transient scalar field) around matter-radiation equality alters the sound horizon and inferred parameters; some variants also indirectly affect inferred σ_8. **Signatures:** Changes in CMB peak structure, inferred H0, and correlated shifts in other parameters; distinct imprints in high-ℓ CMB and matter power spectrum.

Strengths: EDE has been explored widely as a solution to the H0 tension and can alter inferred parameters in ways that partially affect late-time clustering; it is a relatively controlled extension of early physics.

Weaknesses: EDE is tuned to be brief and significant near equality; while it can move inferred parameters it does not directly produce a targeted late-time, scale-dependent suppression of growth, and fitting both H0 and S_8 simultaneously is challenging. Recent reviews caution about residual tensions and model complexity.

3) Modified gravity and DM–DE interactions (late-time growth modifications)

Mechanism: Modify the effective gravitational force or introduce drag/interaction terms between dark matter and dark energy (or a dark sector) that reduce structure growth at low redshift. Examples include scalar-tensor theories, phenomenological weakening of G_eff, or drag-like friction terms.

Signatures: Scale- and redshift-dependent changes to growth rate f(z) and to P(k); often also affects lensing vs. dynamical mass relations and can alter halo abundances. Solar-system and gravitational-wave constraints restrict parameter space for many theories.

Strengths: Directly targets the growth sector and can be designed to suppress late-time clustering without changing early Universe. Some models (e.g., dark sector drag) have been shown to bring datasets into better agreement.

Weaknesses: Modified gravity must satisfy multiple constraints (CMB, BAO, local tests, speed of gravitational waves, cluster abundances). Many simple parameterizations are already tightly constrained; viable models often require tuned screening or particular time dependence. Representative work emphasizing late time drag solutions is available.

4) Baryonic feedback and astrophysical systematics

Mechanism: Non-gravitational astrophysical processes (star formation, radiative cooling) redistribute matter within halos and suppress power on small scales, biasing weak-lensing inferences if imperfectly modeled.

Signatures: Strongly scale-dependent suppression at relatively small scales ($k \gtrsim 0.5$–1 h/Mpc) and modifications to halo profiles and peak statistics;

effects depend on subgrid baryonic physics. **Strengths:** No new fundamental physics required — plausible and supported by hydrodynamical simulations showing significant small-scale impact.

Weaknesses: Degeneracy with cosmological suppression — mis-modelling can mimic cosmological effects. However, baryonic suppression tends to act at smaller scales than the weak-lensing S_8 sensitivity window, and joint analyses that marginalize baryonic parameters often cannot fully remove the tension. Robust modeling requires hydrodynamical calibration and remains a leading systematic uncertainty for Stage-IV surveys.

5) Observational / analysis systematics (shear calibration, photometric redshifts, intrinsic alignments)

Mechanism: Biases in shape measurements, redshift estimates, or intrinsic alignment modeling can shift inferred S_8. **Signatures:** Effects can mimic an overall change in amplitude or produce scale/redshift dependent biases if errors correlate with geometry or source populations.

Strengths: Potentially resolvable with improved calibration, cross-correlations, and survey overlap.
Weaknesses: Surveys have invested heavily in calibration, and while residual systematics may contribute some fraction of the discrepancy, current estimates suggest they are unlikely to fully account for the observed level of tension across independent surveys (though this remains under active investigation). Representative survey studies assess these systematics in detail.

<u>6) Primordial or early-time alternatives (primordial features, nonstandard initial conditions)</u>

Mechanism: Modifying initial power spectrum tilt/shape, or introducing isocurvature/feature models, can alter the mapping from primordial amplitudes to late-time clustering.
Signatures: Changes primarily visible in the CMB and large-scale P(k); generally constrained by high-precision CMB data.

Strengths: Can in principle shift parameter inference in coherent ways. **Weaknesses:** Highly constrained by Planck; unlikely to produce the targeted late-time, scale-localized suppression needed for S_8 without conflicting with the CMB. See recent reviews and papers discussing primordial solutions in the broader context of cosmological tensions.

<u>How the fractal-spacetime proposal differs and complements the above</u>

- **Direct, scale-dependent late-time suppression:** The fractional Laplacian acts as a modification of the gravitational operator that produces a scale- and redshift-dependent suppression of clustering, tuned to operate predominantly at low redshift and at the intermediate scales most relevant to weak lensing (k ~ 0.1–1 h/Mpc). This is similar in spirit to modified gravity/drag models but differs in being motivated as an *effective geometric modification* (multifractal spacetime) rather than a new field or interaction in the Dark Sector.
- **Minimal background impact:** Because the modification is implemented in the Poisson operator (structure growth) and can be

suppressed at high z by the chosen redshift dependence of ε (k, a), the background expansion — and hence BAO and the primary CMB — can be left nearly unchanged, sidestepping the tight geometric constraints that restrict some EDE or neutrino solutions.

- **Distinct observational signatures:** The fractional operator predicts a specific k-dependence (power-law/fractional scaling) for the suppression which can be distinguished from neutrino step-suppression, baryonic feedback (which acts on smaller scales and has different stochastic signatures), and EDE (which primarily shifts inferred parameters via early-time physics).
- **Challenges:** Like other late-time modifications, the fractal model must be vetted against nonlinearity, halo statistics, small-scale structure, and astrophysical constraints. It is phenomenological here; a first-principles derivation and high-resolution simulation suite will be required to robustly compare with the alternatives and to test for degeneracies (e.g., with baryonic feedback or neutrino mass).

References for Appendix B

- Reviews / specific proposals addressing σ_8 and late-time suppression: V. Poulin et al., "The Sigma-8 Tension is a Drag" (Phys. Rev. D, 2023) — discussion of drag/interaction models and their effect on growth.
- Early Dark Energy reviews: V. Poulin, T. L. Smith & T. Karwal, "The Ups and Downs of Early Dark Energy" (2023) — review of EDE models and constraints.
- Massive neutrinos and structure suppression: recent reviews on cosmological neutrino mass constraints and implications for small-scale suppression (e.g., 2024–2025 reviews).
- Baryonic feedback and weak lensing systematics: A. Schneider (2019) and Grandis et al. (2024) on baryon impacts and modeling challenges for lensing surveys.
- Surveys and systematics analyses: KiDS, DES pipeline and intrinsic alignment / calibration studies (e.g., Tröster et al., DES Y3 papers).

Short summary of Appendix B

No single alternative has emerged as a decisive, unambiguous resolution of the S_8 tension. Massive neutrinos and baryonic feedback can produce partial suppression; EDE reassigns early-time parameters but does not directly produce targeted late-time suppression; modified gravity / drag models and the fractal-spacetime proposal more directly alter late-time growth. The phenomenological fractional Laplacian studied in the main text sits within the late-time growth-modification class but is distinct in physical motivation (effective geometry) and in the predicted functional form of scale dependence. Robust discrimination among these possibilities will require (a) improved modeling of nonlinear structure (high-resolution modified N-body), (b) careful marginalization over baryonic and observational systematics, and (c) the next generation of precision surveys (Euclid, LSST, Roman) that can precisely map scale and redshift dependence of growth.

General References

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