New Proof of Pythagorean Theorem

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1 Introduction

In this document, there will be a new proof of Pythagorean Theorem in $\overset{\triangle}{ABC} : a^2 = b^2 + c^2, where : \hat{A} = 90°$, using complex numbers and Euler's formula : $e^{ix} = \cos x + i \sin x$.

2 The Proof

We use the right triangle $\overset{\triangle}{ABC}$ in the figure 1 :

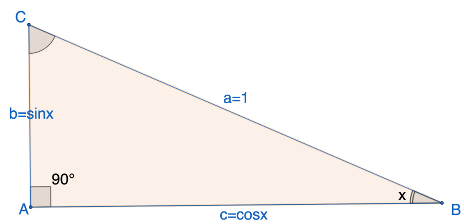

Figure 1: right triangle

As we see : $\hat{A} = 90°, (BC) = 1, \hat{B} = \hat{x}$

$\sin x = \frac{(AC)}{(BC} => (AC) = (BC)\sin x = 1 \sin x => \boxed{\sin x = (AC)}$ (1)

$\cos x = \frac{(AB)}{(BC} => (AB) = (BC)\sin x = 1 \cos x => \boxed{\cos x = (AB)}$ (2)

From Euler's Identity : $\boxed{e^{ix} = \cos x + i \sin x \ , \forall x}$ (3)

$(i^2 = -1)$ in complex numbers . We change x to -x in (3), so we have : $e^{-ix} = \cos(-x) + i\sin(-x) => \boxed{e^{-ix} = \cos x - i \sin x}$ (4)

if we mutiply (3) with (4), we take : $1 = e^{ix}e^{-ix} = (\cos x + i \sin x)(\cos x - i \sin x) = \cos x \cos x - i \cos x \sin x + i \sin x \cos x - i^2 \sin x \sin x = \cos^2 x + \sin^2 x =>$ $\boxed{\sin^2 x + \cos^2 x = 1}$ (5)

From (5), using (1),(2), we take : $(AC)^2 + (AB)^2 = 1 => \boxed{(AC)^2 + (AB)^2 = (BC)^2}$, which is the Pythagorean Theorem for the right $(\hat{A} = 90°)$ triangle : $\overset{\triangle}{ABC}$

3 Conclusion

In this document, i proved Pythagorean Theorem for a right triangle (as in figure 1), using Euler's formula in the way, you can see above.