A Note on a Degenerate Circle and a Line via Division by Zero Calculus

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Abstract

We consider the degenerate case of the general circle equation. By applying Division by Zero Calculus, we observe that two different finite quantities arise depending on the algebraic form of the radius. One corresponds to zero area, while the other gives the signed distance from the origin to the line.

1 Introduction

Consider the general equation

$$S(x,y) = a(x^2+y^2) + 2gx + 2fy + c.$$

For $a \neq 0$, this represents a circle, while for $a = 0$ it reduces to the line

$$2gx + 2fy + c = 0.$$

We examine this degenerate case using Division by Zero Calculus (DBZC), in which

$$\frac{z}{0} = 0.$$

2 Main Observation

For $a \neq 0$, the radius R of the circle is given by

$$R = \frac{\sqrt{g^2+f^2-ac}}{|a|},$$

and hence

$$R^2 = \frac{g^2+f^2-ac}{a^2}.$$

(i) Interpretation of R^2

Applying DBZC to R^2, we obtain

$$R^2 = 0.$$

Thus,

$$\pi R^2 = 0,$$

which reflects the fact that a line does not enclose any area.

(ii) Interpretation of R

Applying DBZC directly to the expression of R, we obtain

$$R = -\frac{c}{2\sqrt{f^2 + g^2}}.$$

This value coincides with the signed distance from the origin to the line

$$2gx + 2fy + c = 0.$$

3 Conclusion

For the degenerate case of a circle into a line:

- the squared radius yields $R^2 = 0$ (area structure),
- the radius yields a finite value corresponding to distance.

Thus, different algebraic forms extract different geometric quantities from the same degenerate configuration.

Division by Zero Calculus distinguishes geometric structures through algebraic form.

This demonstrates that algebraic form encodes geometric structure.

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- the radius yields a finite value corresponding to distance.

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References

[1] H. Okumura and S. Saitoh, Division by Zero Calculus in Figures – Our New Space Since Euclid, viXra:2106.0108, 2021.

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