"THE CLASSICAL ORIGINS OF QUANTUM SUPERPOSITION"

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ABSTRACT

According to modern concepts of physics, the phenomenon of "Quantum Superposition" is a characteristic of quantum processes of the microcosm, which has no classical analogue. In this part of the text, we will show that this phenomenon is not a characteristic of the microcosm only, and such a phenomenon can be introduced in the probabilistic method of describing the processes of the macrocosm. At the same time, we will indicate the fundamental principles of probabilities – common to both the micro and macrocosm, according to which the phenomenon of "quantum superposition" will be presented as a special case of these principles.

1: INTRODUCTION

In theoretical concepts of quantum mechanics, the assertion that the “quantum superposition” of probability amplitudes is a specific feature of quantum mechanics, caused by the peculiarities of the microcosm and having no classical analogue, is widespread and popular. As the main argument, it is pointed out that this phenomenon concerns the amplitudes of probabilities, which were introduced to describe the quantum processes of the microcosm, which do not have a classical analogue. Below we will show that this statement is not true.

Let's list the statements discussed in this part of the text:

1: When describing random events in macroscopic bodies, is it possible, or not, to introduce state vectors corresponding to probability amplitudes?

Answer - not only possible, but also necessary for a complete description of the outcome of an event;

2: Should the physical and mathematical principles of probability spaces be different for objects of the micro and macrocosm?

Answer - the principles of probability spaces should be universal and should not differ in any way when used in different mechanics;

3: Should the mathematical principles of the "Quantum Superposition" of state vectors of quantum objects differ from the mathematical principles of superposition of state vectors of macro objects, e.g., coins and dice?

Answer - The principle of superposition of state vectors is a universal characteristic of probability spaces, and it should be equally realized in classical and quantum mechanics;

2: - A BRIEF HISTORY OF THE ISSUE.

The introduction of the phenomenon of superposition into quantum-mechanical discussions was associated with de Broglie's consideration, according to which each corpuscular micro-object corresponds to a wave, the length of the period and amplitude of which are related to the quantitative values of the physical characteristics of this object (see [1]). De Broglie's consideration was related to empirical facts, according to which - electromagnetic radiation, along with wave properties, also has corpuscular properties. Somewhat later, more important empirical facts became known, such as diffractions and “interference” of fluxes of different quantum particles on holes, confirming the universality of de Broglie's opinion. In textbooks on quantum mechanics, you can read that when a stream of micro-objects passes through two micro-slits located at some distance from each other, then on the screen, behind these slits, the same interference images of quantum particle traces are obtained that we observe in the case of waves on the surface of water and when light passes through similar slits. On the one hand - wave ideas about light, and on the other hand - the repeatedly tested ancient Greek principle that the properties of the whole are determined by the properties of its component parts and that these properties of the whole should be attributed to its parts (see [2]), naturally led to the opinion that de Broglie's consideration was correct: the carriers of wave properties are not only the flows of micro-objects, but also these micro-objects themselves separately. This idea also turned out to be in line with the basic principle of probability theory, according to which observed patterns in a set of statistical data for identical objects should be attributed to individual objects in the form of probabilistic characteristics. On the basis of this, the following statement was introduced into quantum mechanical reasoning:

Under physical circumstances of one type, micro-objects behave like corpuscles and under these circumstances are corpuscles, and under circumstances of another type they behave like waves and under these circumstances are waves.

This statement has become the basis of many erroneous assumptions, including the quantum nature of the superposition of probability amplitudes. Therefore, it will be necessary to understand the essence of this phenomenon as well. We discuss this issue in [3], where it is shown that there are neither empirical nor theoretical grounds for attributing a wave nature to quantum objects of the microcosm, including photons. This means that there are no grounds for introducing the principle of wave-particle duality into the theoretical concepts of quantum mechanics. In turn, this means that the principle of superposition of probability amplitudes should be associated not with the wave nature of these amplitudes, but with their probabilistic nature.

With that said, let's move on to discuss the first question from the above list. Let's start the discussion with a remark: the flows of micro-objects for which diffraction patterns are observed (to which the term "interference" has been erroneously ascribed; for more details, see [3]) are a set of non-interacting or weakly interacting objects. In the formation of a diffraction image, each object in the flow makes an independent movement, similar to dice when they are tossed at the same time. Empirically, this fact is confirmed:

If the micro-objects of the flow are directed to the slits individually, with a delay corresponding to long time intervals, then when traces of these particles are detected on the screen, the following picture is obtained: individual microparticles leave localized traces, which corresponds to their corpuscular nature. But at the same time, the total image of traces has the same spatial diffraction forms that are obtained when they are launched in a stream. That is, there is only a statistical unity between the traces of such a joint picture, and not a dynamic one, such as arises in the waves of any medium, for example - on the surface of the water.

The interpretation of empirical facts, on the basis of which the ideas of wave-corpuscular dualism were built, were considered indisputable, and at the same time, the facts of discreteness were observed in atomic processes, required an appropriate theoretical explanation. There was a feeling that since all these phenomena manifest themselves precisely in the processes of the microcosm, the explanation of the essence of these phenomena should be carried out within the framework of a new concept peculiar only to the microcosm. Many views were expressed. For example, Schrödinger believed that electrons in an atom form a cloud-like spatially distributed substance in which these particles acquire a wave nature. This opinion was not shared by many, since in all acts of observation the electron always manifested itself as a point localized particle. However, everyone agreed that without the introduction of wave functions, it would be impossible to explain the essence of the phenomenon of "Wave-Corpuscular Dualism". All of this, of course, required the introduction of adequate mathematical principles, and Max Born pointed out how this could be implemented:

Phenomena corresponding to the wave nature and discreteness should be attributed not to the micro-objects themselves in the form of their physical characteristics, but to the set of statistical data of the results of repeated events involving these objects. And since this set of results corresponds to the random outcomes of individual events, these phenomena should also be attributed to micro-objects in the form of probabilistic characteristics. To do this, the probability space - defined by ordinary numbers - must be extended, on the one hand - to the hyper numbers of matrix algebra, which will correspond to the phenomenon of discreteness, and on the other hand - to wave functions described by complex variables, which will correspond to the phenomenon of wave nature. With the help of quadratic forms of the elements of this extended space, it will be possible to return to ordinary numbers, i.e. - to probabilities. Wave functions written as columns of matrix algebra could be given an interpretation of probability amplitudes, which would successfully correspond to both the ideology of matrix algebra and the wave nature of the corresponding random outcomes (see [4]).

The formation of a logical chain corresponding to new ideas was completed when M. Born gave an interpretation of the Schrödinger equation (see [5]) as a dynamical equation for probability amplitudes. As a result, the probability space was expanded not only to abstract hyper numbers - corresponding to matrix algebra, but also to an even more abstract space of complex numbers. Since probability and probability amplitude are abstract mathematical constructions, then the attribution of wave properties to them did not require the existence of any really existing etheric medium. However, such an expansion of the probabilistic space can introduce such degrees of freedom into the corresponding mechanics that it will require great vigilance in the physical interpretation of the corresponding mathematical relations (and when such details are not given sufficient attention, the basis is set for the emergence of myths. see, e.g. [3] and [6]). But at the same time, it is necessary to say the following:

The introduction of probability amplitudes for the description of physical states was one of the most important facts from both a physical and a mathematical point of view.

The fact is that –

The introduction of state vectors as a mathematical principle of probability theory makes it possible to describe statistical reality more perfectly than would be possible without these vectors.

We will deal with this issue in the next subsection, which will be an effective "key" to clarifying the essence of the phenomenon of "quantum superposition".

3: – THE PRINCIPLE OF SUPERPOSITION IN THE CASE OF MACROSCOPIC OBJECTS.

Let us return to the problem with dice and coins, discussed in the first part of the text, and try to introduce state vectors for them - corresponding to the amplitudes of probabilities. By association with the spin, let's write the number (1/2) on one side of the coin and on the other side - the number (-1/2). Similarly, on the faces of the dice, we will add the numbers {+5/2; +3/2; +1/2; -1/2; -3/2; -5/2}. Let's start with a simple observation—the set of all possible numbers that appear in the results of tossing stationary objects to the top face is a complete set of mutually exclusive possibilities. This statement, which corresponds to empirical reality, can be represented as a mathematical principle by introducing state vectors. In the case of the dice, this can be done using six columns of matrix algebra:

$$V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \; V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \; V_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; V_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \; V_5 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \; V_6 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}; \tag{3.1}$$

Conditionally, column V_i let's compare the state of the dice, when after stopping and a number (7/2 - i) appears on the upper side; i – takes integer values from 1 to 6. Similarly, we can enter two status columns for a coin:

$$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \qquad v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \tag{3.2}$$

Also conditionally, let's attribute the state v_1 of the coin when the number (1/2) appears on the upper side, and v_2 - when it appears (-1/2). The abstract columns V_i and v_n correspond to the spatial "polarization states" of the objects that we observe on the upper side of the object after being tossed in the Earth's gravitational field and stopped on a horizontal surface. In matrix algebra, these columns are linearly independent objects that form the orthonormal basis of the corresponding vector probability space:

$$< \bar{V}_i \, | V_j > = \delta_{ij} \; ; \qquad < \bar{v}_n | \, v_m > = \delta_{nm} \; ; \tag{3.3}$$

When i ≠ j and m ≠ n, the matrix products of these columns give zero numeric values, what we call mutual orthogonality of columns. These conditions reflect the empirical facts of mutual exclusion of the corresponding physical "polarization states": if one of them is realized, then none of the others can be realized simultaneously with it. Here it should be remembered that the probabilistic space is built on the basis of our expectations, and these columns - in our expectations - should reflect the potential capabilities of objects - will be in a particular state. Therefore, these columns must be assigned to the corresponding objects as mathematical characteristics that reflect these "potential opportunities". To turn the above into a mathematical principle, let us introduce the "generalized state vectors" of these objects:

$$\Psi = \sum_{i=1}^{6} \Psi_i = \sum_{i=1}^{6} C_i \, V_i \; ; \qquad \psi = \sum_{n=1}^{2} \psi_n = \sum_{n=1}^{2} c_n \, v_n ; \tag{3.4}$$

We will select the coefficients C_i and c_n so that these "state vectors" satisfy the "condition of completeness" of potential possibilities:

$$< \overline{\Psi}\,|\Psi> = \sum_{i=1}^{6} C_i^2 = 1; \quad < \bar{\psi}\,|\psi> = \sum_{n=1}^{2} c_n^2 = 1; \tag{3.5}$$

These conditions are easily fulfilled using empirical data obtained as a result of phenomenological analysis of the empirical results of random events involving these objects. For example, in the "game mode" (see [2]):

$$C_i^2 = 1/6 \text{ и } c_n^2 = 1/2 \ ; \tag{3.6}$$

Squares of individual terms from (4):

$$< \overline{\Psi_i}\,|\,\Psi_i > = < \overline{C_i\,V_i}\,|C_i\,V_i > = C_i^2 \ ; \ < \overline{\psi_n}\,|\,\psi_n > = < \overline{c_n\,v_n}\,|c_n\,v_n > = c_n^2 \ ; \tag{3.7}$$

can be interpreted as the probabilities of the outcomes of events, and the and the terms themselves - $C_i\,V_i$ and $c_n\,v_n$ - as the amplitudes of these probabilities. Based on the above considerations, we can conclude:

The summation rule introduced in (3.4) is a complete analogue of the mathematical phenomenon of superposition of state vectors in quantum mechanics. This rule of summation, as a mathematical principle of probability theory, could be introduced into the theory independently of quantum mechanics. In this approach, the principle of superposition of quantum mechanics should be considered as a special case of the general mathematical principle.

In constructing the principles of probability theory, we must not forget that the probabilistic characteristics, which we ascribe to objects as their "potential opportunities", are in fact according to our expectations and are therefore only abstract mathematical characteristics. Indeed, how carefully we study the physical characteristics of the coins and dice themselves, we will not find anything similar to what the probabilities c_n^2 and C_i^2 might correspond to. What we will find are two sides in the case of a coin and six sides in the case of a dice. One might think that the numerical value c_n^2= 1/2 is due to the two faces of the coin, and the value C_i^2 = 1/6 is due to the six faces of the dice. However, this would be a misconception, since the "potential opportunities" of these objects, when tossed, include not only these numerical probabilities, but also any others that satisfy the conditions of completeness (2.5). And indeed, we can choose such "mechanical tricks" of flipping coins and dice, in the course of which these coefficients acquire other numerical values. As for the numbers of faces, they certainly take part in the formation of probability spaces, but only indirectly – they specify the number of mutually exclusive states: in the case of a coin – 2, and in the case of a dice – 6, but these numbers are not directly related to numerical values $c_n^2$and C_i^2. At the same time, probability amplitudes are assigned precisely to these states.

Note that, in our expectations, all potential outcomes of events exist simultaneously. It should be noted, however, that the use of the term "simultaneously" does not imply the existence of any chronological order in probabilistic reasoning. A statistical set of all possible outcomes of events can be formed over a long period of time, but when describing the set of these outcomes, no chronological order is assumed for the elements of the set. The most important feature of the probabilistic method of description is a characteristic detail:

The probabilistic method of description deals only with the final results of events and does not concern the course of events, and does not imply either a dynamic description of the course of events, or any chronology of the consequences of events.

Using the example of events involving coins or dice, we will point out a detail due to which the probabilistic method of description does not imply the study of the issue in a chronological context. The fact is that the corresponding events occur in the gravitational field of the Earth. In the presence of this field, these events are formed, and the outputs of these events are defined as probabilistic. Without the gravitational field, of course, there would not have been such events, and the corresponding statistical set of results would not have been formed, on the basis of which it would be possible to construct the above-mentioned probability space. When these events occur, it can be assumed that the Earth's gravitational field is spatially homogeneous and invariant in time. Since external physical circumstances remain constant, both the statistics and the probability patterns corresponding to these repeating events are time-independent and completely stationary. Therefore, with the probabilistic method of describing the outcomes of events, there is no need to introduce chronological characteristics of these events, and for the same reason, the corresponding probabilities are assigned to objects as time-independent characteristics.

To summarize the above, we point out the following: all probabilistic events - involving objects such as coins and dice - occur in macroscopically repeating external conditions, and only those physical circumstances that we call the acts of "tossing" these objects can change. One of the types of such acts we call "game mode".

Note that our expectations about the potential outcomes of an event exist only until a specific outcome is realized:

At the moment of observing a specific outcome that has already been realized, all our expectations vanish just as they arose and existed in our imagination before the moment of observation. This phenomenon is characteristic of the probabilistic method of description and will be equally present, wherever probabilistic methods are used to describe the outcomes of events.

The same phenomenon in quantum mechanics is called **the "Collapse of the Wave Function".**

After the event's completion, the disappearance of our expectations, i.e. - the phenomenon of "collapse," corresponds to the disappearance of the probabilistic status of the superposition sum of the generalized state vector introduced in (3.4). This disappearance occurs upon the realization of a specific result, which represents **the phenomenon of "reduction" from the generalized state vector to a specific individual one**. In quantum mechanics, this specific detail of collapse is called "**Wave Function Reduction**," and it is clearly fully consistent with the classical understanding of the “collapse” phenomenon. Consequently, the quantum phenomenon of **"collapse"** corresponds only to a special case of a more general phenomenon, consistent with the probabilistic method of description, and not to the quantum-mechanical nature of the microcosm. We will return to the details of the issue of "quantum superposition" in Part 4 of the text.

Let's use the method of mathematical parameterization adopted in quantum mechanics and write the "generalized superposition state vector" of the coin as follows:

$$\Psi(\theta) = [a \cos\theta\, \Psi_1 + b \sin\theta\, \Psi_2] = [a \cos\theta \begin{pmatrix}1\\0\end{pmatrix} + b \sin\theta \begin{pmatrix}0\\1\end{pmatrix}];$$

$$a = \pm 1 ; \quad b = \pm 1 ; \tag{3.8}$$

The parameter θ is defined in the interval [0; π/2]. A specific numerical value θ_i corresponds to a certain set of statistical data on the outcome of events, which is generated by repeated tosses of a certain i-th type. By means of phenomenological analysis of this set, the numerical value θ_i is determined. In the state vector $\Psi\ (\theta_i)$, the statistical weight of the physical state Ψ_1 is $\cos^2\theta_i$, and the statistical weight of the physical state Ψ_2 is $sin^2\,\theta_i$. "Game Mode" corresponds to the value θ = π/4. Since the realization of $\Psi\ (\theta_i)$ and

$\Psi(\theta_j)$ implies different and mutually exclusive types of toss acts, the orthogonality relation must also be held for these state vectors:

$$< \overline{\Psi(\theta_i)}|\Psi(\theta_j) > = \delta_{ij} \; ; \qquad (3.8\text{-a})$$

It is clear that the state vectors written in the form (3.8) do not satisfy this condition. Therefore, in the format of a notation of type (3.8), the verbal principle of condition (3.8-a) should be used. Let's formulate the principle:

Regardless of whether the spectrum of basis vectors of probability amplitudes is discrete or continuous, the superposition state vectors corresponding to different sets of mixing coefficients will always correspond to macroscopically well-defined and mutually exclusive physical conditions. If a particular superposition vector $\Psi\ (\theta_i)$ is realized in physical circumstances of a certain i-th type, then in the same physical circumstances no other vector $\Psi(\theta_j)$ can be realized.

Let's consider another important detail related to the question of how to construct superposition sums in the case of multiple objects. As we mentioned in the first part of the text (see [2]), the state vectors of individual objects are introduced through a statistical ensemble. We can obtain this ensemble in two ways:

1) as a result of repeating events in macroscopically repetitive physical circumstances, performed repeatedly by a single object;
2) or as a result of collective actions performed simultaneously by many identical objects in the same physical circumstance.

Nevertheless, from the standpoint of the principles of statistical description, these two realities are not completely identical, which we can easily demonstrate using the example of two coins. To do this, we write the state vectors of each coin as superposition sums:

$$\Psi_1 = \sum_{n=1}^{2} \alpha_n\, v_n \; ; \qquad \Psi_2 = \sum_{n=1}^{2} \beta_n\, v_n \; ; \qquad (3.9)$$

For a visual demonstration, let's consider two coins of different colors in the "game mode". In state vectors, mark the color difference with the appropriate indices and write them as follows:

$$\Psi^{(1)} = (1/\sqrt{2}) \sum_{n=1}^{2} \Psi^{(1)}_{[n]} = (1/\sqrt{2})[\begin{pmatrix}1\\0\end{pmatrix}^{(1)} + \begin{pmatrix}0\\1\end{pmatrix}^{(1)}] \; ;$$

$$\Psi^{(2)} = (1/\sqrt{2}) \sum_{n=1}^{2} \Psi^{(2)}_{[n]} = (1/\sqrt{2})\,[\begin{pmatrix}1\\0\end{pmatrix}^{(2)} + \begin{pmatrix}0\\1\end{pmatrix}^{(2)}] \; ; \qquad (3.10)$$

If these two coins are tossed at the same time, the superposition sum of the state vectors of the system will look like this:

$$\Psi^{(1\,;\,2)} = \Psi^{(1)(2)}_{[1][1]} + \Psi^{(1)(2)}_{[1][2]} + \Psi^{(1)(2)}_{[2][1]} + \Psi^{(1)(2)}_{[2][2]} \; ; \qquad (3.11)$$

$\Psi^{(i)(j)}_{[m][n]}$ corresponds to the physical state of a two-coin system, in which the pair of indices [n] and [m] indicates a specific number observed on the upper face of the coin; indices (i) and (j) indicate the color of the coin. In "game mode," we easily discover that all possible physical states from (3.11) are generated by all possible physical states of individual coins, which are formed independently of one another. Typically, the mathematical realization of this empirical fact is realized by the rule of "direct multiplication" of the state vectors of individual coins, which is determined by the rules of the tensor product:

$$\Psi^{(i)(j)}_{[n][m]} = \Psi^{(i)}_{[n]} \otimes \Psi^{(j)}_{[m]} = \Psi^{(j)}_{[m]} \otimes \Psi^{(i)}_{[n]}; \tag{3.12}$$

Tensor products defined in this way satisfy the permutation condition:

$$\Psi^{(i)(j)}_{[n][m]} = \Psi^{(j)(i)}_{[m][n]}; \tag{3.13}$$

This permutability condition does not mean that invariance is relative to the relocation of indices within the same line, since the corresponding state vectors - $\Psi^{(i)(j)}_{[n][m]}$ and - $\Psi^{(i)(j)}_{[m][n]}$, or - $\Psi^{(i)(j)}_{[n][m]}$ and - $\Psi^{(j)(i)}_{[n][m]}$, correspond to different physical states of this system. Due to the different colors of the coins, we can easily distinguish these realities from each other, so (3.11) should be written as follows:

$$\Psi^{(1\,;\,2)} = [\binom{1}{0}^{(1)} \otimes \binom{1}{0}^{(2)} + \binom{1}{0}^{(1)} \otimes \binom{0}{1}^{(2)} + \binom{0}{1}^{(1)} \otimes \binom{1}{0}^{(2)} + \binom{0}{1}^{(1)} \otimes \binom{0}{1}^{(2)}]/2\,; \tag{3.14}$$

For each term of this sum, with the help of permutations of the factors, the permutability ratios specified in (3.12) are realized. The state vectors of (3.14) correspond to mutually exclusive physical states that must satisfy orthogonality relations:

$$4 < \overline{\Psi}^{(i)(j)}_{[n][m]} \mid \Psi^{(i)(j)}_{[k][l]} > = < \overline{\Psi}^{(i)}_{[n]} \otimes \overline{\Psi}^{(j)}_{[m]} \mid \Psi^{(i)}_{[k]} \otimes \Psi^{(j)}_{[l]} > =$$

$$= < \overline{\Psi}^{(i)}_{[n]} \mid \Psi^{(i)}_{[k]} > \otimes < \overline{\Psi}^{(j)}_{[m]} \mid \Psi^{(j)}_{[l]} > = \delta_{nk} \otimes \delta_{ml} = \delta_{nk}\, \delta_{ml}\,; \tag{3.15}$$

In the given dot products of the state vectors of the system, the multiplication operation occurs only between the elements of the probability space of a given object. At the same time, multiplication between the state vectors of objects of different colors does not occur. This principle corresponds to a very specific empirical fact: observing the physical state of a coin of the i-th color - $\Psi^{(i)}_{[1]}$, we detect only this specific state of this coin and in this same state it is impossible to detect - (a) the same coin in the state $\Psi^{(i)}_{[2]}$; (b) the physical states of an object of the second color - $\Psi^{(j)}_{[l]}$. The mathematical realization of the empirical facts corresponding to cases (a) is easily achieved by the relations:

$$< \overline{\Psi}^{(i)}_{[1]} \mid \Psi^{(i)}_{[1]} > = < \overline{\binom{1}{0}^{(i)}} \mid \binom{1}{0}^{(i)} > = < (1;0)^{(i)} \mid \binom{1}{0}^{(i)} > = 1; \tag{3.16.1}$$

$$< \overline{\Psi}^{(i)}_{[2]} \mid \Psi^{(i)}_{[1]} > = < \overline{\binom{0}{1}^{(i)}} \mid \binom{1}{0}^{(i)} > = < (0;1)^{(i)} \mid \binom{1}{0}^{(i)} > = 0; \tag{3.16.2}$$

We cannot achieve a mathematical realization of the two facts corresponding to cases (b) – the orthogonality of $\Psi^{(i)}_{[1]}$ with respect to the vectors $\Psi^{(j)}_{[1]}$ and $\Psi^{(j)}_{[2]}$, in terms of a mathematical realization of the type (3.16.1-2). Therefore, we only verbally express the impossibility of such a multiplication. To demonstrate the above statement about the incomplete identity of two different realities of realization of the statistical ensemble – from the point of view of the possibilities of mathematical realization, we consider the completeness condition for the state vectors, in the mathematical parameterization described in this paper:

$$< \overline{\Psi}^{(1)(2)} \mid \Psi^{(1)(2)} > = |\Psi^{(1)(2)}_{[1][1]}|^2 + |\Psi^{(1)(2)}_{[1][2]}|^2 + |\Psi^{(1)(2)}_{[2][1]}|^2 + |\Psi^{(1)(2)}_{[2][2]}|^2 =$$

$$= 1/4 + 1/4 + 1/4 + 1/4 = 1 \; ; \qquad (3.17)$$

The corresponding results of these mathematical relations are also confirmed empirically. Let's see what happens if these coins are of the same color and we cannot distinguish them from each other. We can no longer distinguish physical states corresponding to the state vectors $\Psi^{(1)}_{[n]}$ and $\Psi^{(2)}_{[n]}$, and in (3.11) - all terms must be replaced by index-free representations:

$$\Psi^{(1\,;\,2)} = \Psi_{[1][1]} + \Psi_{[1][2]} + \Psi_{[2][1]} + \Psi_{[2][2]} =$$

$$= (1/2)\,[\, \binom{1}{0} \otimes \binom{1}{0} + \binom{1}{0} \otimes \binom{0}{1} + \binom{0}{1} \otimes \binom{1}{0} + \binom{0}{1} \otimes \binom{0}{1} \,]; \qquad (3.18)$$

In the physical states of the two-coin system, we can no longer distinguish between the physical states corresponding to the state vectors $\Psi_{[1][2]}$ and $\Psi_{[2][1]}$, and the corresponding mathematical representations of these vectors:

$$\Psi_{[1][2]} = \binom{1}{0} \otimes \binom{0}{1} = \binom{0}{1} \otimes \binom{1}{0} = \Psi_{[2][1]}; \qquad (3.19)$$

As a result, we can rewrite (3.18) as follows:

$$\Psi^{(1\,;\,2)} = \Psi_{[1][1]} + 2\Psi_{[1][2]} + \Psi_{[2][2]} = (1/2)\,[\, \binom{1}{0} \otimes \binom{1}{0} + 2\binom{1}{0} \otimes \binom{0}{1} + \binom{0}{1} \otimes \binom{0}{1} \,]; \qquad (3.20)$$

In the case of parameterization by such mathematical principles, the condition for the completeness of the probability space of the system will take the form:

$$< \overline{\Psi} \mid \; \Psi > = (1/4)\, |\binom{1}{0} \otimes \binom{1}{0} + 2\binom{1}{0} \otimes \binom{0}{1} + \binom{0}{1} \otimes \binom{0}{1}\,|^2 =$$

$$= (1/4)\,[1 + 4 + 1] = 3/2; \qquad (3.21)$$

From the resulting expression, it can be seen that the interpretation of the basis of the probability space is destroyed. And therefore, the state vectors introduced by such principles no longer represent probability amplitudes. You can try to save the mentioned base by introducing new superposition coefficients in (3.20):

$$\Psi^{(1\,;\,2)} = K_1\, \Psi_{[1][1]} + K_2\, \Psi_{[1][2]} + K_3\, \Psi_{[2][2]} \; ; \qquad (3.22)$$

and the condition of completeness will take the form:

$${K_1}^2 + {K_2}^2 + {K_3}^2 = 1 \; ; \qquad (3.23)$$

However, in this case, we no longer have any preliminary theoretical considerations about what ratios the mixing coefficients K_i can satisfy, using which we could predict their specific numerical values. Therefore, we have the opportunity to specify the coefficients K_i only empirically. It should be noted here that the physical mechanism, which in "game mode" is outside our deterministic control and which governs individual coin movements and event outcomes, will not be "sensitive" to changes in coin color. Therefore, the same ratios will be fulfilled for both slugs – colored and unmarked identical coins – the same ratios will be met:

$${K_1}^2 = {K_3}^2 = {K_2}^2/\,2 \; ; \qquad (3.24)$$

which corresponds to the ratio (3.17) obtained with the help of the corresponding mathematical principles. Therefore, it can be said unequivocally that the mathematical principles by which the state vectors of the probability space corresponding to (3.21) and the corresponding probability weights are obtained will contradict both the empirical data and the fundamental principles of probability theory.

As we can see from the above discussion, it is not possible to orthogonalize the probability spaces of different coins by introducing indices to denote individual coins. However, the introduction of such indices makes it possible to factor the probability spaces of different coins, which allows you to control the independence of these spaces from each other. In turn, this creates an opportunity to verbally prohibit the scalar products of state vectors of different coins:

$$< \overline{\Psi}^{(1)}_{[1]} |\Psi^{(2)}_{[1]} > ; \qquad (3.25)$$

by equating them to zero, we would have to realize the condition of orthogonality. Since this cannot be achieved by the rules of standard matrix algebra, we are forced to use the rule of verbal prohibition.

On the basis of the above considerations, it can be concluded that the following principle should be introduced into the theory of probability:

In order to introduce state vectors into the probabilistic description of multi-object systems, it is necessary that these objects be labeled in the probability space, regardless of whether we can label them in reality or not. This would allow us to indicate that the real physical states of these objects are mutually exclusive. In the language of probabilities, this means that under no physical circumstances can another object be found in one particular object, and it does not matter whether these objects are not distinguishable for our subjective considerations or objectively for reasons independent of us, as it happens, for example, with particles of the microcosm.

Taking into account the results obtained, we move on to the discussion of the phenomenon of "**Quantum Superposition**", which we will discuss in Part IV of the text.

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