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Quantum-Elastic Geometry: From Geometric Extension to Linear and Non Linear Regimes

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Part I: Mathematical and Operational Basements of Quantum-Elastic Geometry

A Background Independence and Lorentz-Structural Necessity of Dimensional Collapse

A.1 Geometric Status of the Fundamental Field

The fundamental degree of freedom in Quantum-Elastic Geometry (QEG) is a symmetric rank-2 tensor field

$$\mathcal{G}_{\mu\nu}(x), \quad (1)$$

defined on a smooth four-dimensional differentiable manifold \mathcal{M} .

Throughout this work, $\mathcal{G}_{\mu\nu}$ is not identified a priori with the spacetime metric. Rather, it represents the deformation tensor of an elastic substrate. In the weak-field regime one may introduce a local inertial chart and expand

$$\mathcal{G}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (2)$$

where $\eta_{\mu\nu}$ serves only as a perturbative reference configuration and does not constitute an independent background geometry.

All physical observables are constructed solely from $\mathcal{G}_{\mu\nu}$ and its covariant derivatives. No external metric structure or preferred tensor field is introduced.

A.2 Lorentz Covariance and Representation Structure

The tensor $\mathcal{G}_{\mu\nu}$ transforms under the $(\frac{1}{2}, \frac{1}{2}) \otimes (\frac{1}{2}, \frac{1}{2})$ representation of the Lorentz group $SO(1,3)$:

$$\mathcal{G}'_{\mu\nu}(x') = \Lambda_\mu^\alpha \Lambda_\nu^\beta \mathcal{G}_{\alpha\beta}(x), \quad (3)$$

where $\Lambda \in SO(1,3)$ satisfies

$$\Lambda_\alpha^\mu \Lambda_\beta^\nu \eta_{\mu\nu} = \eta_{\alpha\beta}. \quad (4)$$

The Lorentz matrices Λ_ν^μ are dimensionless elements of a Lie group generated by dimensionless generators $M_{\mu\nu}$ obeying the algebra

$$[M_{\mu\nu}, M_{\rho\sigma}] = \eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} + \eta_{\mu\sigma} M_{\nu\rho}. \quad (5)$$

The closure of this Lie algebra requires the generators and structure constants to be dimensionless.

A.3 Mixing of Longitudinal and Torsional Sectors

Under an infinitesimal boost in the x -direction (with rapidity parameter ϵ),

$$\Lambda_0^0 = 1, \quad \Lambda_0^1 = \epsilon, \quad \Lambda_1^0 = \epsilon, \quad \Lambda_1^1 = 1, \quad (6)$$

the components of $\mathcal{G}_{\mu\nu}$ transform as

$$\mathcal{G}'_{00} = \mathcal{G}_{00} + 2\epsilon\mathcal{G}_{01} + \mathcal{O}(\epsilon^2), \quad (7)$$

$$\mathcal{G}'_{01} = \mathcal{G}_{01} + \epsilon(\mathcal{G}_{11} + \mathcal{G}_{00}) + \mathcal{O}(\epsilon^2). \quad (8)$$

Thus the longitudinal (“compressive”) component \mathcal{G}_{00} and the torsional component \mathcal{G}_{0i} are linearly mixed by Lorentz boosts.

A.4 Impossibility of Sector-Dependent Dimensional Rescalings

Assume, for contradiction, that one introduces distinct dimensional rescaling constants α_L and α_T such that

$$\tilde{\mathcal{G}}_{00} = \alpha_L \mathcal{G}_{00}, \quad \tilde{\mathcal{G}}_{0i} = \alpha_T \mathcal{G}_{0i}. \quad (9)$$

Under a boost,

$$\tilde{\mathcal{G}}'_{00} = \alpha_L \mathcal{G}'_{00} = \alpha_L (\mathcal{G}_{00} + 2\epsilon\mathcal{G}_{01}) \quad (10)$$

$$= \tilde{\mathcal{G}}_{00} + 2\epsilon \left(\frac{\alpha_L}{\alpha_T} \right) \tilde{\mathcal{G}}_{01}. \quad (11)$$

For the transformation law to retain the canonical linear Lorentz form, the coefficient multiplying $\tilde{\mathcal{G}}_{01}$ must be unity. Hence

$$\frac{\alpha_L}{\alpha_T} = 1, \quad \Rightarrow \quad \alpha_L = \alpha_T. \quad (12)$$

The argument generalizes to arbitrary Lorentz transformations. While Lorentz covariance allows the introduction of scalar dimensional constants in the action, it does not permit independent dimensional rescalings between components that are linearly mixed by the symmetry group. Such rescalings would alter the linear representation structure of $\mathcal{G}_{\mu\nu}$ and effectively require the Lorentz matrices Λ_ν^μ to carry compensating dimensions, which is incompatible with the Lie algebra structure (5).

A.5 Consequence: Dimensional Collapse as Lorentz Necessity

Since all components of $\mathcal{G}_{\mu\nu}$ belong to a single irreducible tensor representation of $SO(1,3)$, they must share identical physical dimensions.

The elastic kinetic term has the schematic form

$$\mathcal{L}_{kin} = \kappa \nabla_\rho \mathcal{G}_{\mu\nu} \nabla^\rho \mathcal{G}^{\mu\nu}, \quad (13)$$

with a single scalar coupling constant κ .

The identification

$$[M] \equiv [L] \equiv [T] \quad (14)$$

therefore follows as a representation-theoretic consequence of:

1. Lorentz covariance,
2. The irreducible tensorial character of $\mathcal{G}_{\mu\nu}$,
3. The absence of independent sectorial conversion scales.

The dimensional collapse discussed in Section 2.1 is thus not a heuristic dimensional identification, but a structural requirement imposed by the symmetry of the theory.

A.6 Covariant Emergence of the Dissipative Rest Frame

A strictly background-independent theory forbids the introduction of absolute, non-dynamical vector fields to define dissipation. In QEG, the local rest 4-velocity u^μ entering the Rayleigh-type dissipation functional is not externally imposed, but dynamically emerges from the deformation state of the substrate.

For any localized finite-energy excitation, the effective Stress–Energy–Momentum tensor $T^{\mu\nu}$ admits (under standard energy conditions) a unique future-directed time-like eigenvector:

$$T^\mu{}_\nu u^\nu = -\rho u^\mu, \quad u^\mu u_\mu = -1, \quad (15)$$

where contractions are performed using the effective macroscopic metric induced by the coherent vacuum expectation value of the deformation tensor,

$$g_{\mu\nu}^{\text{eff}} \equiv \langle \mathcal{G}_{\mu\nu} \rangle. \quad (16)$$

The velocity field $u^\mu(x)$ is therefore dynamically determined and transforms covariantly under local Lorentz transformations.

The dissipative functional

$$\mathcal{R} \propto (u^\rho \nabla_\rho \mathcal{G}_{\mu\nu})^2 \quad (17)$$

introduces effective irreversibility at the coarse-grained level while preserving the fundamental covariance of the underlying action.

B Formal Derivation of the Effective Field Equations

This appendix provides the explicit weak-field derivation underlying the emergence of the gravitational and electromagnetic sectors from the unified deformation field $\mathcal{G}_{\mu\nu}$. It addresses the concern that the identification of distinct interactions from a single substrate field may constitute an assignment rather than a derivation.

B.1 Weak-field expansion

Consider small fluctuations around a homogeneous vacuum configuration:

$$\mathcal{G}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1, \quad (18)$$

where indices are raised and lowered with $\eta_{\mu\nu}$ at leading order¹.

The quadratic part of the QEG action takes the schematic form

$$S_{\text{QEG}}^{(2)} = \frac{\kappa}{2} \int d^4x (\partial_\alpha h_{\mu\nu})(\partial^\alpha h^{\mu\nu}) - \frac{1}{2} \int d^4x h_{\mu\nu} \mathcal{M}^{\mu\nu\rho\sigma} h_{\rho\sigma}, \quad (19)$$

where \mathcal{M} is the Hessian of the potential around the vacuum. In the infrared regime, we project onto the massless directions of \mathcal{M} , so that long-range propagation is governed entirely by the universal hyperbolic operator \square .

¹The Minkowski tensor $\eta_{\mu\nu}$ is not introduced as an independent background geometry, but represents a particular homogeneous vacuum solution of the dynamical field equations, around which perturbations are locally expanded

B.2 Irreducible decomposition and orthogonal projectors

On a homogeneous and isotropic background, $h_{\mu\nu}$ decomposes into irreducible representations of the little group (spatial rotations in the rest frame).

The spatial tensor sector:

$$h_{ij} = (\psi \delta_{ij} + (\partial_i \partial_j - \frac{1}{3} \delta_{ij} \nabla^2) E) + \partial_{(i} F_{j)} + h_{ij}^{TT}, \quad (20)$$

with

$$\partial_i F_i = 0, \quad \partial_i h_{ij}^{TT} = 0, \quad h_{ii}^{TT} = 0.$$

The mixed components:

$$h_{0i} = h_{0i}^T + \partial_i \sigma, \quad \partial_i h_{0i}^T = 0. \quad (21)$$

The projectors onto scalar (S), vector (V), and tensor (T) subspaces satisfy

$$P^S + P^V + P^T = I, \quad P^A P^B = \delta^{AB} P^A. \quad (22)$$

Since the quadratic kinetic operator is Poincaré invariant, it commutes with the symmetry projectors. Therefore,

$$\langle h^A, \mathbb{K} h^B \rangle \propto \delta^{AB}, \quad (23)$$

and the quadratic action is block-diagonal across irreducible sectors.

B.3 Gravitational sector: spin-2 dynamics

The transverse-traceless component h_{ij}^{TT} carries the propagating spin-2 degrees of freedom. Restricting (19) to this subspace yields

$$S_{\text{grav}}^{(2)} = \frac{\kappa}{2} \int d^4x (\partial_\alpha h_{ij}^{TT}) (\partial^\alpha h_{ij}^{TT}). \quad (24)$$

Rewriting the quadratic action in fully covariant form reproduces the Fierz–Pauli Lagrangian for a massless spin-2 field. By the Weinberg–Deser consistency argument, the unique nonlinear completion universally coupled to stress-energy reconstructs the Einstein–Hilbert action in the infrared limit:

$$S_{\text{grav,IR}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} 2\Lambda + \dots, \quad (25)$$

with

$$g_{\mu\nu} = \langle \mathcal{G}_{\mu\nu} \rangle.$$

Gauge fixing and emergence of the Green kernel

To extract the classical $1/r$ potentials, the quadratic operator must be inverted. Because of linearized diffeomorphism invariance $\delta h_{\mu\nu} = \partial_{(\mu} \xi_{\nu)}$, the operator possesses zero modes.

Imposing the harmonic (de Donder) gauge,

$$C^\mu(h) \equiv \partial_\nu h^{\mu\nu} - \frac{1}{2} \partial^\mu h = 0, \quad (26)$$

renders the operator invertible.

The gauge-fixed quadratic Lagrangian becomes

$$\mathcal{L}_{\text{eff}}^{(2)} = \frac{\kappa}{2} \left[\partial_\rho h_{\mu\nu} \partial^\rho h^{\mu\nu} - \frac{1}{2} \partial_\rho h \partial^\rho h \right]. \quad (27)$$

The field equations reduce to

$$\square h_{\mu\nu} = \frac{1}{\kappa} J_{\mu\nu}. \quad (28)$$

In the static limit,

$$G(r) = \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{\kappa |\vec{k}|^2} = \frac{1}{4\pi\kappa r}. \quad (29)$$

Thus the $1/(4\pi r)$ kernel follows directly from gauge fixing and operator inversion.

B.4 Electromagnetic sector from \mathcal{G}_{0i}

The mixed components h_{0i} transform as a spatial vector and define the unique vector channel within a symmetric rank-2 tensor.

Projecting onto the transverse part,

$$A_i = \lambda_{\text{EM}} h_{0i}^T, \quad \partial_i h_{0i}^T = 0. \quad (30)$$

The residual linearized diffeomorphism symmetry induces

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda, \quad (31)$$

establishing an emergent $U(1)$ redundancy.

Define

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (32)$$

The projected quadratic action becomes

$$S_{\text{EM}}^{(2)} = \int d^4x \left(\frac{c_g}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu \right). \quad (33)$$

At two derivatives, $F_{\mu\nu} F^{\mu\nu}$ is the unique local Lorentz scalar that is also invariant under the emergent gauge redundancy $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$. Varying yields

$$\partial_\mu F^{\mu\nu} = -\frac{1}{c_g} J^\nu. \quad (34)$$

Normalization and positivity

The coefficient c_g is not arbitrary. It is fixed by:

1. Positivity of the Hamiltonian density.
2. Canonical normalization of the radiative sector (equivalently, of the transverse massless pole in the current–current response).

Because the current J^μ is defined by functional variation of the same action, any rescaling of A_μ modifies both the kinetic term and the source coupling, altering the normalization of the radiative response. Requiring canonical normalization fixes the coefficient uniquely (up to conventional sign choices absorbed into field definitions), yielding the canonical Maxwell equations

$$\partial_\mu F^{\mu\nu} = J^\nu. \quad (35)$$

B.5 Infrared decoupling

The full theory contains nonlinear interactions,

$$S_{\text{QEG}} = S^{(2)} + S^{(3)} + S^{(4)} + \dots. \quad (36)$$

In the infrared limit, the quadratic operator dominates propagation. Because it is diagonal in irreducible sectors, each projected channel satisfies its own leading-order wave equation,

$$\mathcal{E}_{\mu\nu}^{(2)}[h] = J_{\mu\nu} + \mathcal{O}(h^2). \quad (37)$$

Nonlinear couplings are perturbatively suppressed at macroscopic scales. Gravity and electromagnetism therefore emerge as distinct irreducible projections of the unified deformation field $\mathcal{G}_{\mu\nu}$.

C Operator Consistency Check: Chernoff Semigroups and the QEG Generative Hierarchy

The constitutive hierarchy proposed in QEG admits a natural operator-theoretic interpretation. In the linear long-wavelength regime, the unified deformation field $\mathcal{G}_{\mu\nu}$ responds to the unified source tensor $J_{\mu\nu}$ through the stiffness operator K , as explicitly introduced in Sec. 2.1 of the Main paper:

$$K G = J, \quad K := \kappa \square, \quad (\text{static limit: } K \rightarrow -\kappa \nabla^2). \quad (38)$$

The associated compliance (Green) operator is defined as

$$C := K^{-1}, \quad G = C J, \quad (39)$$

which is the covariant field-theoretic analogue of Hooke’s law for a unified elastic medium.

Although QEG is physically motivated, it is important to verify that this hierarchy is compatible with rigorous propagation theory for (possibly variable-coefficient) generators. In this respect, the Chernoff semigroup framework developed by Remizov [230] provides a useful consistency anchor: it guarantees that a global propagator can emerge as the limit of repeated composition of elementary local response steps.

C.1 Resolvent Representation and Semigroup Propagation

Assume that K defines a closed densely defined operator on an appropriate Hilbert space of symmetric tensor fields². If K generates a strongly continuous semigroup $(U(t))_{t \geq 0}$, then

$$U(t) = e^{tK}, \quad (40)$$

is well-defined as the vacuum propagation operator in the linear regime.

The corresponding resolvent is defined by

$$R(\lambda, K) := (\lambda I - K)^{-1}, \quad \lambda > 0, \quad (41)$$

and the standard Laplace identity yields the integral representation

$$(\lambda I - K)^{-1} J = \int_0^\infty e^{-\lambda t} e^{tK} J dt, \quad (42)$$

which provides a rigorous meaning to the Green response even when an explicit kernel is not available in closed form.

In QEG language, Eq. (42) guarantees that the linear vacuum response is mathematically well-defined as a resolvent of the stiffness generator K , consistent with the compliance interpretation $C = K^{-1}$ used throughout the paper.

C.2 Constitutive Operators and Chernoff Approximation

In Sec. 3.1 of the Main paper, QEG introduces the constitutive “genotype” of the vacuum through two minimal Toeplitz ladder matrices M_c and M_α , encoding coherent and dissipative weighting rules across the modal carrier space. These matrices act locally on the physical deformation state S through Hadamard modulation, yielding the minimal constitutive closure

$$Q = \mathcal{G} \left[\mu_0 \Pi_{\text{sym}}(M_c \odot S) + \eta_0 \Pi_{\text{sym}}(M_\alpha \odot S) \right], \quad (43)$$

where \mathcal{G} denotes the appropriate Green operator, and Π_{sym} enforces the symmetric response sector.

From the viewpoint of Chernoff approximation theory, such a minimal constitutive rule naturally defines an *elementary propagation step* $S(\tau)$ acting on the state space, satisfying

$$S(0) = I, \quad \left. \frac{d}{d\tau} S(\tau) \right|_{\tau=0} = K. \quad (44)$$

Remizov’s formulation of the Chernoff product theorem implies that if K generates a strongly continuous semigroup, then the full propagator emerges as an infinite composition of such steps:

$$e^{tK} = \lim_{n \rightarrow \infty} \left(S(t/n) \right)^n. \quad (45)$$

Therefore, the QEG genotype–phenotype principle acquires a precise operator meaning:

Genotype	\rightarrow	Phenotype
(M_c, M_α) minimal modal masks	\rightarrow	K unified stiffness generator
Elementary update $S(\tau)$	\rightarrow	Full propagator e^{tK}

In particular, the repeated local action of the minimal constitutive rule converges to a global propagation law, even when the latter is not assumed explicitly in closed form.

²For instance, one may consider the space of square-integrable tensor fields with suitable gauge fixing and boundary conditions.

C.3 Dissipation as a Contractive Semigroup Sector

A key ingredient of QEG is the unavoidable emergence of dissipation as a consequence of coarse-graining and accumulation. In Sec. 2.5.1 of the Main Paper, this is encoded through a Rayleigh-type covariant damping term and through the identification of α as a universal damping invariant.

At the operator level, this corresponds to a decomposition of the generator into a coherent and a dissipative part,

$$K = K_{\text{coh}} + K_{\text{dis}}, \quad (46)$$

where K_{dis} is dissipative in the semigroup sense (contractive contribution). This provides a rigorous interpretation of irreversible modal leakage: whenever K_{dis} is dissipative, the associated semigroup satisfies

$$\|e^{tK_{\text{dis}}}\| \leq 1, \quad t \geq 0, \quad (47)$$

so that entropy production corresponds to contraction in the state norm. In the static (Euclidean) limit, this naturally yields exponentially decaying kernels, consistent with the thermo-entropic interpretation of the expansive sector.

Remark C.1 (Tensorial monism). *It is important to emphasize that the operator hierarchy of QEG does not introduce new physical fields. The objects $G_{\mu\nu}$, K , (M_c, M_α) and the dissipative component K_{dis} are not independent ontological entities: they are different functional descriptions of the same single substrate.*

- $G_{\mu\nu}$ is the **state**: the actual physical deformation of the vacuum.
- K is the **stiffness operator**: the linearized vacuum response generator.
- (M_c, M_α) are the **genotype**: the minimal modal weighting grammar.
- K_{dis} is the **dissipative sector**: the contractive component encoding irreversible accumulation loss.

Analogy. *If the substrate is a string, $G_{\mu\nu}$ is its shape, K is its tension law, (M_c, M_α) encode the local rule by which it responds to excitation, and K_{dis} is the friction that damps its vibration. All describe the same string.*

C.4 Conclusion

The Chernoff semigroup framework provides a rigorous mathematical interpretation of the QEG constitutive architecture. It supports the claim that the minimal response matrices (M_c, M_α) can be viewed as elementary propagation rules whose repeated action converges to a well-defined global propagator e^{tK} . Thus, the genotype–phenotype structure of QEG is not merely conceptual but is compatible with a standard operator-theoretic construction of vacuum dynamics.

Part II: Closure Schema and Perturbative Expansion in Quantum-Elastic Geometry

D Analytic expansion of higher-order corrections

In the QEG framework, higher-order corrections do not arise from ultraviolet divergences or loop integrals requiring regularization in the conventional QFT sense. Instead, they emerge from the intrinsic nonlinear structure of the unified substrate and from the coarse-graining procedure implicit in any effective description of a dissipative elastic medium. More precisely, once the unified deformation field is decomposed into slow coherent modes and unresolved fluctuations,

$$G_{\mu\nu} = G_{\mu\nu}^{(\text{slow})} + \chi_{\mu\nu}^{(\text{fast})}, \quad (48)$$

integrating out $\chi_{\mu\nu}^{(\text{fast})}$ generates an effective action $\Gamma[G^{(\text{slow})}]$. By construction, Γ must remain local and

covariant in the infrared static regime, and its induced corrections must respect the same isotropy and closure constraints derived in the previous sections.

The key structural point is that, in the minimal closure regime of QEG, *no new independent dimensionful scales are allowed*. Therefore, all corrections must be controlled by a single dimensionless control parameter. In the present formulation we take this parameter to be the *physical* fine-structure coupling α , operationally defined from a static IR response (see Appendix F). This implies that any renormalized dimensionless observable extracted from Γ can only be dressed through powers of α . Equivalently, higher-order corrections are not arbitrary: they are the inevitable algebraic residues of iterating the same geometric projection and coarse-graining steps which define the closure itself.

A further characteristic feature of the QEG closure scheme is that the resulting coefficients are *purely geometric*. In particular, isotropy reduces all tensorial corrections to traces of $O(3)$ projectors and angular averages on S^2 , which generically yield rational numbers. Hence, in the closure regime considered here, the expansion coefficients are expected to be rational rather than involving transcendental functions.

Finally, we emphasize that in the present static IR closure limit, no logarithmic corrections arise. This is not a phenomenological assumption but a direct consequence of the fact that the theory is evaluated at fixed coarse-graining resolution, with no additional running scale introduced. Thus, the effective quantities are organized as analytic power series in α without $\log(\mu)$ contributions.

Proposition D.1. *Assume:*

- (i) *locality and covariance of the effective action Γ in the static IR regime,*
- (ii) *isotropy and homogeneity of the vacuum substrate,*
- (iii) *minimal closure, in the sense that no new independent mass scales or dimensionful parameters are introduced beyond the reference normalization fixed at leading order, and*
- (iv) *the existence of a single dimensionless invariant α governing the strength of the coarse-grained response.*

Then, any renormalized dimensionless quantity X (coupling or effective constant extracted from Γ) admits, in a neighborhood of $\alpha = 0$, an analytic expansion of the form

$$X = X_{(0)} \left(1 + x_1 \alpha + x_2 \alpha^2 + x_3 \alpha^3 + \dots \right), \quad (49)$$

where the coefficients x_n are determined by geometric projector traces, spherical averages, and local operator contractions. In particular, under exact isotropy and projector closure, the coefficients x_n are expected to be rational numbers.

Proof. By assumptions (i)–(iii), the effective action Γ is a local covariant functional obtained from the microscopic substrate theory after integrating out unresolved degrees of freedom. Since the closure regime introduces no new dimensionful scales, all induced local operators in Γ must be weighted by powers of the unique dimensionless control parameter α . Therefore, any dimensionless observable X extracted from Γ must admit a formal analytic power series in α around $\alpha = 0$, yielding Eq. (49).

By assumption (ii), isotropy reduces all tensor contractions entering the coefficients x_n to $O(3)$ -invariant combinations, i.e. traces of projector products and angular averages over S^2 . Such objects are rational by construction (they correspond to ratios of integer multiplicities and dimensions of invariant subspaces). Hence, in the exact closure regime, the coefficients are expected to be rational numbers. \square

A key structural feature of QEG is that the leading-order (minimal closure) value of any effective constant is itself not arbitrary. Rather, it admits the discrete form

$$X_{(0)} \equiv X^{(0)} = K_X \alpha^{k_X}, \quad k_X \in \mathbb{Z}, \quad (50)$$

where K_X is a purely geometric prefactor fixed by spherical closure factors, compatibility normalization, and projector conventions. Unlike the rational slope coefficients x_n , the prefactors K_X need not be rational: they may contain canonical geometric constants such as 4π arising from the Green function of the three-dimensional Laplacian and the associated isotropic closure identities.

As a consequence, once α is fixed operationally in the IR and no additional small parameters are introduced, *all* higher corrections must be expressible as a power series in α . We therefore adopt the self-consistent parameterization

$$Y = \left(K_Y \alpha^{k_Y} \right) \left(1 + D_1 \alpha + D_2 \alpha^2 + D_3 \alpha^3 + \dots \right), \\ X = \left(K_X \alpha^{k_X} \right) \left(1 + x_1 \alpha + x_2 \alpha^2 + x_3 \alpha^3 + \dots \right), \quad (51)$$

where D_i, x_i, \dots are dimensionless coefficients encoding higher-order geometric dressing of the kernel, higher-derivative operator contributions, and nonlinear backreaction of the substrate response.

In this sense, the QEG closure replaces the conventional renormalization machinery of divergent loop integrals with a finite, purely geometric dressing procedure. The appearance of rational coefficients is not accidental but reflects the fact that isotropic coarse-graining reduces the effective corrections to combinatorial weights fixed by the dimension of invariant projector subspaces.

D.1 Canonical geometric inputs and predictive summary

For the purposes of the main text, the closure scheme requires only a small set of *canonical geometric inputs*: the order-zero prefactors K_X (fixed by spherical closure, compatibility normalization, and $O(3)$ isotropy) and a minimal subset of first-order rational slopes (fixed by transverse kernel compatibility and S^2 projector averages). All remaining first-order coefficients follow algebraically from defining identities among constants (e.g. $Z_0 = \mu_0 c_0$, $c_0^{-2} = \mu_0 \varepsilon_0$, $Y_0 = 1/Z_0$).

To avoid interrupting the flow of the paper with technical derivations, we collect in Appendix E a telegraphic justification of the geometric prefactors K_X , and in Appendix F the corresponding canonical first-order slopes $C_1^{(X)}$. A compact symbolic and numerical summary of the resulting predictions is provided at the end of Appendix F.

E Geometric origin of the order-zero prefactors K_X

This Appendix summarizes (in a telegraphic but reproducible form) how the order-zero prefactors K_X entering $X_0 = K_X \alpha^{k_X}$ are fixed by geometric closure, projector algebra, and compatibility normalization. Unlike the rational slope coefficients C_i, D_i, \dots , the constants K_X are not expected to be rational in general: they may contain canonical geometric factors such as π or 4π arising from the Green function of the three-dimensional Laplacian and from isotropic closure measures.

E.1 Universal geometric ingredients

(i) Isotropic moments on S^2 .

For $\hat{\mathbf{k}} \in S^2$ one has, in $d = 3$,

$$\langle \hat{k}_i \hat{k}_j \rangle_{S^2} = \frac{1}{3} \delta_{ij}, \quad \langle \hat{k}_i \hat{k}_j \hat{k}_\ell \hat{k}_m \rangle_{S^2} = \frac{1}{15} \left(\delta_{ij} \delta_{\ell m} + \delta_{i\ell} \delta_{jm} + \delta_{im} \delta_{j\ell} \right). \quad (52)$$

For the transverse projector $P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i \hat{k}_j$ one also has

$$\text{tr } P = 2, \quad \langle P_{ij} \rangle_{S^2} = \frac{2}{3} \delta_{ij}. \quad (53)$$

(ii) 3D Green function normalization.

The static Coulomb/Laplace kernel is fixed by the universal identity

$$\int \frac{d^3k}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2} = \frac{1}{4\pi r}, \quad (54)$$

so any static $1/k^2$ propagator produces a long-range $1/r$ interaction with a canonical $1/(4\pi)$ factor.

(iii) Closure normalization and channel-dependent measures.

The closure scheme matches isotropic coarse-grained response kernels to canonical long-range static $1/r$ couplings. In the longitudinal sector this matching naturally averages over S^2 (directions), whereas in the torsional sector the physically relevant datum is the azimuthal polarization around $\hat{\mathbf{k}}$, so the minimal closure measure is instead the transverse circle S^1 .

E.2 Prefactor in the torsional sector: K_μ from azimuthal closure

The static transverse propagator in momentum space reads

$$D_T(0, \mathbf{k}) = \frac{1}{\mu_0 \mathbf{k}^2} P_T(\hat{\mathbf{k}}). \quad (55)$$

In coordinate space, the scalar Green's function contributes the standard Laplacian factor $(4\pi r)^{-1}$. However, unlike the longitudinal sector where closure involves directional averaging over S^2 , the torsional sector possesses an internal $U(1)$ degree of freedom corresponding to rotations around the propagation axis $\hat{\mathbf{k}}$.

Consequently, torsional closure is defined not by a directional average, but by the trace over the azimuthal polarization family S^1 . While the longitudinal measure normalizes the solid angle (averaging), the torsional measure sums over the available twist phases φ :

$$\mathcal{W}_{S^1} \equiv \oint_{S^1} d\varphi = 2\pi. \quad (56)$$

This reflects the fact that torsional rigidity encodes the response of the medium to the full range of transverse shear phases.

Applying this measure to the fundamental Green's function, the effective static kernel $\mathcal{K}_T(r)$ becomes:

$$\mathcal{K}_T(r) = \frac{1}{\mu_0} \underbrace{\left(\frac{1}{4\pi r}\right)}_{\text{3D Green}} \times \underbrace{(2\pi)}_{\text{Azimuthal trace}} = \frac{1}{2\mu_0} \frac{1}{r}. \quad (57)$$

The minimal closure principle dictates that, in natural geometric units, the canonically normalized torsional interaction must scale as the inverse 3D volume coupling $1/\alpha^3$, with a universal azimuthal normalization residue $1/(2\pi)$ inherited from the ratio between the transverse S^1 trace and the 4π Laplacian normalization.

Imposing this unitary closure normalization on Eq. (57) yields

$$\frac{1}{2\mu_0} \stackrel{!}{=} \frac{1}{2\pi} \frac{1}{\alpha^3}. \quad (58)$$

Substituting the minimal scaling ansatz $\mu_0 = K_\mu \alpha^3$, we obtain

$$\frac{1}{2K_\mu \alpha^3} = \frac{1}{2\pi \alpha^3} \implies \boxed{K_\mu = \pi}. \quad (59)$$

Remark E.1 (Geometric origin of π). *The factor π is not a fit parameter but a direct consequence of the azimuthal S^1 trace (2π) relative to the Laplacian normalization (4π), matched against the canonical cyclic weight $1/(2\pi)$ in the torsional closure. In the minimal limit, $K_\mu = \pi$ is thus the unique geometric residue of torsional isotropy.*

At order zero, the torsional rigidity therefore takes the form

$$\boxed{\mu_0 = K_\mu \alpha^3 = \pi \alpha^3}. \quad (60)$$

E.3 Induced prefactors from algebraic closure: K_Z, K_ϵ, K_Y

Once K_μ is fixed, the remaining order-zero prefactors follow without additional geometric input from the defining identities among constants:

$$Z_0 = \mu_0 c_0, \quad c_0^{-2} = \mu_0 \epsilon_0, \quad Y_0 = Z_0^{-1}.$$

In particular, if the impedance closure fixes

$$Z_0 \equiv K_Z \alpha^{-1} \quad \text{with} \quad K_Z = \sqrt{\frac{3}{5}} 4\pi, \quad (61)$$

then $\mu_0 c_0 = Z_0$ implies the induced causal prefactor

$$\boxed{K_\epsilon = \frac{K_Z}{K_\mu} = \frac{\sqrt{\frac{3}{5}} 4\pi}{\pi}} \quad (62)$$

and $c_0^{-2} = \mu_0 \epsilon_0$ yields

$$\boxed{K_\epsilon = \frac{1}{K_\mu K_c^2} = \frac{K_\mu}{K_Z^2}}. \quad (63)$$

Moreover, $Y_0 = Z_0^{-1}$ fixes

$$\boxed{K_Y = \frac{1}{K_Z}}. \quad (64)$$

With $K_\mu = \pi$ and $K_Z^2 = \frac{3}{5} 4\pi = \frac{12\pi}{5}$ this gives

$$\boxed{K_\epsilon = \frac{\pi}{12\pi/5} = \frac{5}{12}}. \quad (65)$$

Remark E.2 (Minimality). *The only independent geometric inputs at order zero are K_μ (torsional sector) and K_Z (impedance closure). All remaining prefactors are induced by identities among constants, so the order-zero layer of the theory contains no hidden freedom.*

F Derivation of the canonical correction coefficients

This Appendix records compact derivations of the *canonical* first-order slope coefficients $C_1^{(X)}$ used throughout the paper. The guiding principles are: (i) static IR extraction, (ii) isotropy ($O(3)$), (iii) projector algebra, and (iv) compatibility normalization. No logarithmic terms are retained in the minimal closure regime.

All first-order slopes are defined using the physical fine-structure coupling α as the unique expansion parameter:

$$X = X_0 \left(1 + \alpha C_1^{(X)} + \mathcal{O}(\alpha^2) \right).$$

F.1 Geometric slopes from isotropic projector averages

First-order canonical correction to the torsional rigidity: $C_1^{(\mu)}$

Proposition F.1 (Static torsional slope). *Let the transverse projector be $P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i \hat{k}_j$ and define the static transverse BB kernel as*

$$\Gamma_{BB}^T = \Gamma_{BB,0}^T \left(1 + \alpha b_B + \dots \right).$$

Extract the rigidity from the static IR limit,

$$\mu^{-1} = \lim_{\omega \rightarrow 0, k \rightarrow 0} \frac{\Gamma_{BB}^T(\omega, k)}{k^2}.$$

Then

$$\mu = \mu_0 \left(1 + \alpha C_1^{(\mu)} + \dots \right), \quad C_1^{(\mu)} = -b_B,$$

and, in the canonical trace-normalized scheme,

$$\boxed{C_1^{(\mu)} = 4 = \frac{20}{5}}.$$

Sketch. Work at $L = 1$ in the static limit. The transverse quadratic energy for a shear deformation ψ reads

$$E_{\text{shear}} = \frac{\mu}{2} \int \frac{d^3k}{(2\pi)^3} k^2 \psi_i(-\mathbf{k}) P_{ij}(\hat{\mathbf{k}}) \psi_j(\mathbf{k}).$$

At first order, the correction to μ^{-1} is governed by variations of the transverse projector structure. Compatibility requires that the effective quadratic kernel retains the full trace content of the transverse projector algebra, and isotropy forbids any additional independent tensor structures at this order.

Since the quadratic kernel involves the contraction of two transverse indices, the minimal trace-complete first-order dressing necessarily receives contributions from both the trace of the projector ($\text{tr } P$) and the trace of its idempotent square ($\text{tr } P^2$). In $d = 3$,

$$\text{tr } P = 2, \quad \text{tr } (P^2) = \text{tr } P = 2,$$

where the second identity follows from idempotency $P^2 = P$. Under canonical normalization, these two algebraic invariants contribute equally to the minimal dressing of the transverse BB kernel. Therefore,

$$b_B = -(\text{tr } P + \text{tr } (P^2)) = -(2 + 2) = -4.$$

Hence the rigidity slope is $C_1^{(\mu)} = -b_B = 4$. \square

Remark F.2 (Methodology). *This result relies only on idempotency of the projector algebra and on trace invariants of the transverse sector. It is basis-independent and fixed by compatibility plus isotropy in the static closure scheme.*

First-order canonical correction to the impedance: $C_1^{(Z)}$

Operational definition.

Define the IR vacuum impedance via the ratio of response kernels:

$$Z \equiv \sqrt{\frac{\mathcal{Z}_E(0,0)}{\mathcal{Z}_B(0,0)}}, \quad (66)$$

$$\Gamma^{(2)}[\mathbf{A}_T] = \frac{1}{2} \int \frac{d\omega d^3k}{(2\pi)^4} \left[\mathcal{Z}_E(\omega, k) |\mathbf{E}|^2 - \mathcal{Z}_B(\omega, k) |\mathbf{B}|^2 \right]. \quad (67)$$

Expanding to first order,

$$\frac{Z}{Z_0} = \sqrt{\frac{1 + \alpha a_E}{1 + \alpha a_B}} = 1 + \frac{\alpha}{2} (a_E - a_B) + \mathcal{O}(\alpha^2),$$

which yields

$$C_1^{(Z)} = \frac{1}{2} (a_E - a_B). \quad (68)$$

Canonical electric weight.

The electric susceptibility \mathcal{Z}_E receives two distinct vacuum polarization contributions: (i) a bulk trace contribution from the transverse gauge loop ($\text{tr } P = 2$), and (ii) the angle-dependent geometric screening. Decomposing

$$a_E = a_E^{(\text{tr})} + a_E^{(\text{geom})},$$

the trace part is fixed by

$$a_E^{(\text{tr})} = \text{tr } P = 2.$$

The geometric part corresponds to the standard isotropic quartic dressing on S^2 :

$$a_E^{(\text{geom})} = \langle \cos^2 \theta \rangle_{S^2} + \langle \hat{k}_x^2 \hat{k}_y^2 \rangle_{S^2} = \frac{1}{3} + \frac{1}{15} = \frac{2}{5}.$$

Combining both contributions yields

$$a_E = 2 + \frac{2}{5} = \frac{12}{5}. \quad (69)$$

Canonical magnetic weight.

The magnetic response is kinematic and fixed by the ratio of transverse to longitudinal projection averages. For any fixed probe \mathbf{u} , one has

$$\langle \|(\hat{\mathbf{k}}\hat{\mathbf{k}}^\top)\mathbf{u}\|^2 \rangle_{S^2} = \frac{1}{3} \|\mathbf{u}\|^2, \quad \langle \|(\mathbb{I} - \hat{\mathbf{k}}\hat{\mathbf{k}}^\top)\mathbf{u}\|^2 \rangle_{S^2} = \frac{2}{3} \|\mathbf{u}\|^2.$$

Thus

$$a_L = \frac{1}{3}, \quad a_T = \frac{2}{3} \implies \boxed{a_B = \frac{a_T}{a_L} = 2}.$$

Subtracting the weights gives

$$a_E - a_B = \frac{12}{5} - 2 = \frac{2}{5},$$

leading to

$$\boxed{C_1^{(Z)} = \frac{1}{2} \left(\frac{2}{5} \right) = \frac{1}{5}}.$$

Remark F.3 (Parsimony). *The construction integrates algebraic trace invariants ($\text{tr } P$) with the statistical S^2 moments. No independent input about c or μ is required.*

First-order linearization rules and propagation of slopes

Using the definitions

$$Z = \mu c, \quad Y = \frac{1}{Z}, \quad c = (\mu\varepsilon)^{-1/2},$$

the first-order variations satisfy

$$C_1^{(Z)} = C_1^{(\mu)} + C_1^{(c)}, \quad (70)$$

$$C_1^{(Y)} = -C_1^{(Z)}, \quad (71)$$

$$C_1^{(\varepsilon)} = -C_1^{(\mu)} - 2C_1^{(c)}. \quad (72)$$

Inserting the derived values $C_1^{(\mu)} = 4$ and $C_1^{(Z)} = 1/5$, we obtain the full set of canonical slopes:

$$\boxed{C_1^{(c)} = \frac{1}{5} - 4 = -\frac{19}{5}} \quad \boxed{C_1^{(\varepsilon)} = -4 - 2\left(-\frac{19}{5}\right) = \frac{18}{5}} \quad \boxed{C_1^{(Y)} = -\frac{1}{5}}$$

Remark F.4 (Arithmetic closure). *Despite the large integer contribution from the trace sector, the canonical slopes retain a rigid rational structure:*

$$\boxed{C_1^{(\mu)} = \frac{20}{5}, \quad C_1^{(Z)} = \frac{1}{5}, \quad C_1^{(c)} = -\frac{19}{5}, \quad C_1^{(\varepsilon)} = \frac{18}{5}, \quad C_1^{(Y)} = -\frac{1}{5}}$$

All coefficients remain integer multiples of $1/5$, demonstrating that the algebraic trace corrections (+2, +4) are commensurate with the geometric S^2 averages ($1/15, 1/3$).

Global summary tables (canonical closure slopes)

Table 1. Symbolic summary: leading prefactors, scaling in α , and canonical first-order slopes.

Quantity X	Leading form X_0	Scaling $\sim \alpha^p$	$C_1^{(X)}$
μ	$K_\mu \alpha^3$	$p = 3$	$\frac{20}{5}$
c	$K_c \alpha^{-4}$	$p = -4$	$-\frac{19}{5}$
ε	$K_\varepsilon \alpha^5$	$p = 5$	$\frac{18}{5}$
Z	$K_Z \alpha^{-1}$	$p = -1$	$\frac{1}{5}$
Y	$K_Y \alpha^{+1}$	$p = +1$	$-\frac{1}{5}$

Part III: Thermodynamics and Emergent Gravity in Quantum-Elastic Geometry

Table 2. Numerical benchmark. CODATA in SI and model with first-order correction

X	CODATA (SI)	$X_{\text{model}} = X_0(1 + C_1^{(X)}\alpha)$	Rel. Error % we evaluate
μ	1.256637×10^{-6}	1.256438×10^{-6}	-0.0158
c	299792458	2.996790×10^8	-0.0379
ϵ	8.854188×10^{-12}	8.848655×10^{-12}	-0.0625
Z	3.767303×10^2	3.768327×10^2	+0.0272
Y	2.654419×10^{-3}	2.653692×10^{-3}	-0.0274

Note. Relative error (%) is
 $(X_{\text{model}} - X_{\text{CODATA}})/X_{\text{CODATA}} \times 100$.

G The Origin of Zero-Point Energy from Geometric Admittance

The foundations of quantum mechanics predict a non-zero ground-state energy for any stable quantum system: the Zero-Point Energy (ZPE),

$$E_0 = \frac{1}{2} \hbar \omega. \quad (73)$$

In the QEG framework, where the vacuum is a quantized elastic substrate, this energy represents the irreducible quantum “tension” of spacetime. In this section we derive the ZPE from the theory’s constitutive identities and then give a first-principles explanation of the exponent appearing in the resulting admittance law. Throughout, we use the dimensional-collapse identification of energy with length, $[E] \equiv [L]$.

G.1 Algebraic Derivation from Constitutive Identities

Our goal is to establish the exact geometric identity

$$E_0(L_{\text{ref}}) = \frac{\hbar c}{2L_{\text{ref}}} = Y_0^{10} L_{\text{ref}}, \quad (74)$$

where Y_0 is the vacuum geometric admittance,

$$Y_0 := \frac{1}{Z_0} = \sqrt{\frac{\epsilon_0}{\mu_0}}. \quad (75)$$

From the expressions derived in the main text, the Vacuum Constitutive Equation states that

$$\frac{\hbar c}{L_{\text{ref}}} \equiv e \cdot \mu_0 \equiv 2\alpha k_B L_{\text{ref}} \quad (\text{with } [L] \equiv [T]). \quad (76)$$

Substituting the geometric expression for the elementary charge,

$$e \equiv \frac{\mu_0^3}{4\pi} L_{\text{ref}}, \quad (77)$$

we obtain

$$\begin{aligned} \hbar c &= \frac{\mu_0^4}{4\pi} L_{\text{ref}}^2, \\ \frac{\hbar c}{2} &= \frac{\mu_0^4}{(4\pi)^2} L_{\text{ref}}^2, \\ \sqrt{\frac{\hbar c}{2}} &= \frac{\mu_0^2}{4\pi} L_{\text{ref}}. \end{aligned} \quad (78)$$

Noting that $e = \frac{\mu_0^3}{4\pi} L_{\text{ref}}$ can be factored as

$$e = \mu_0 \left(\frac{\mu_0^2}{4\pi} L_{\text{ref}} \right), \quad (79)$$

we substitute Eq. (78) to find the dynamic identity

$$e = \sqrt{\frac{\hbar c}{2}} \mu_0. \quad (80)$$

Multiplying both sides by the causal speed c and using the impedance identity $Z_0 = \mu_0 c$ yields

$$e c = \sqrt{\frac{\hbar c}{2}} Z_0. \quad (81)$$

On the other hand, using the fundamental capacitive and thermal identities established in the main text,

$$\hbar \equiv \epsilon_0^3 L_{\text{ref}}^2, \quad k_B \equiv \epsilon_0 \mu_0^2, \quad (82)$$

$$\frac{\hbar}{k_B} \equiv \frac{\epsilon_0^3 L_{\text{ref}}^2}{\epsilon_0 \mu_0^2} = \left(\frac{\epsilon_0}{\mu_0} \right)^2 L_{\text{ref}}^2 = Y_0^4 L_{\text{ref}}^2. \quad (83)$$

From Eq. (76) we also have $\hbar/k_B \equiv e c L_{\text{ref}}$, hence

$$e c = Y_0^4 L_{\text{ref}}. \quad (84)$$

Equating Eq. (84) with Eq. (81) gives

$$\sqrt{\frac{\hbar c}{2}} Z_0 = Y_0^4 L_{\text{ref}}. \quad (85)$$

Since $Z_0 = 1/Y_0$, multiplying both sides by Y_0 yields

$$\sqrt{\frac{\hbar c}{2}} = Y_0^5 L_{\text{ref}}. \quad (86)$$

Squaring and dividing by L_{ref} produces the definitive ZPE formulation:

$$E_0(L_{\text{ref}}) = \frac{\hbar c}{2L_{\text{ref}}} = Y_0^{10} L_{\text{ref}}. \quad (87)$$

G.2 The Exponent 10 from the Internal Tensor Configuration Space

The exponent $N = 10$ is not introduced ad hoc. It is the algebraic signature of the internal configuration space of the unified deformation tensor $\mathcal{G}_{\mu\nu}$ and the way orthogonal elastic response channels compose in QEG.

(i) Internal response channels vs. propagating degrees of freedom.

A symmetric 4×4 tensor has $4 \cdot 5/2 = 10$ independent components. In QEG these components define the *internal response channels* of a causal cell, i.e. the dimensionality of the constitutive operator acting on $\mathcal{G}_{\mu\nu}$. This counting is a statement about the dimension of the internal tensor configuration space per cell, not a claim that there are ten freely propagating gauge-independent particles in the infrared.

Under the modal decomposition used throughout the paper, the 10 channels decompose into orthogonal sectors:

- **1 Contractive channel:** \mathcal{G}_{00} (1 component),
- **3 Torsional/transport channels:** \mathcal{G}_{0i} (3 components),
- **1 Expansive volumetric channel:** $\chi = \text{Tr}(\mathcal{G}_{ij})$ (1 component),
- **5 Shear channels:** the transverse-traceless sector S_{ij} (5 components),

so that $1 + 3 + 1 + 5 = 10$.

(ii) Multiplicative composition as a determinant law.

In QEG, independent orthogonal response channels compose multiplicatively at the level of the *internal constitutive operator*. Let \mathbb{K} denote the internal stiffness operator acting on the 10-dimensional tensor space within a causal cell, and let $\mathbb{Y} := \mathbb{K}^{-1}$ be the corresponding admittance/compliance operator. The conservative “capacity” of the vacuum to support ground-state fluctuations is controlled by the internal phase-volume measure set by $\det(\mathbb{Y})$ (equivalently, by the product of the eigen-admittances of the orthogonal channels).

At leading order in the isotropic vacuum, the internal operator is diagonal in the modal basis and its eigen-admittances are degenerate up to the universal baseline scale:

$$\lambda_n(\mathbb{Y}) = Y_0 (1 + \delta_n), \quad |\delta_n| \ll 1, \quad (88)$$

where the δ_n encode small channel-dependent dressings from mixing, projection constraints, and polarization

traces (of the same structural type discussed elsewhere as $\mathcal{O}(\alpha)$ corrections).

Therefore,

$$\det(\mathbb{Y}) = \prod_{n=1}^{10} \lambda_n(\mathbb{Y}) = Y_0^{10} \prod_{n=1}^{10} (1 + \delta_n) = Y_0^{10} \left(1 + \mathcal{O}\left(\sum_n \delta_n\right) \right) \quad (89)$$

Equation (89) provides the precise meaning of the Y_0^{10} scaling: it is the leading-order isotropic result of the determinant composition law over the full internal tensor configuration space. In other words, the exponent 10 is fixed by the dimension of the internal response space (the symmetric tensor channels), while deviations from perfect degeneracy produce higher-order dressings consistent with the perturbative structure of the framework.

Interpretive Remark: A Topological Signature of Four-Dimensional Spacetime.

The integer 10 admits a profound geometric duality. The total number of independent planes of rotation (bivectors) across all nested geometric layers from 1D to d -dimensions follows the series $\sum_{k=1}^d k(k-1)/2 = d(d^2-1)/6$. On the other hand, the independent degrees of freedom of a symmetric tensor scale as $d(d+1)/2$. Setting these two quantities equal,

$$\frac{d(d^2-1)}{6} = \frac{d(d+1)}{2},$$

yields a unique non-trivial integer solution: $\mathbf{d} = 4$. This means that exactly and only in four dimensions does the tensorial capacity of the vacuum (10 degrees of freedom) perfectly match the accumulated rotational complexity of its nested geometric hierarchy ($0 + 1 + 3 + 6 = 10$). In this light, the identity $E_0 \propto Y_0^{10}$ is not merely a statement about zero-point energy, but a topological signature of why the unified elastic substrate must manifest as a four-dimensional spacetime.

G.3 Numerical Check and Sensitivity of the Exponent

Because the exponent is large, the identity (87) is extremely sensitive to the correct channel-counting. Taking $L_{\text{ref}} = 1$ m, the quantum-mechanical ZPE scale is

$$E_0 = \frac{\hbar c}{2L_{\text{ref}}} \approx \frac{(1.05457 \times 10^{-34} \text{ J s})(2.9979 \times 10^8 \text{ m/s})}{2 \cdot 1 \text{ m}} \approx 1.58 \times 10^{-26} \text{ J} \quad (90)$$

Using $Z_0 \simeq 376.73 \Omega$ gives $Y_0 = 1/Z_0 \simeq 2.654 \times 10^{-3} \Omega^{-1}$, hence

$$Y_0^{10} L_{\text{ref}} \approx (2.654 \times 10^{-3})^{10} \cdot 1 \text{ m} \approx 1.76 \times 10^{-26} \text{ J} \quad (\text{in collapsed } [E] \equiv [L] \text{ units}) \quad (91)$$

The agreement at the 10^{-26} scale is highly non-trivial given the exponent. Shifting the exponent by ± 1 would move the prediction by roughly three orders of magnitude. The residual discrepancy at the $\sim 10\%$ level is consistent with the expected channel-dependent dressing factors δ_n in Eq. (89), which encode $\mathcal{O}(\alpha)$ corrections from internal-mode mixing, isotropic averaging, and polarization traces (as systematically discussed elsewhere in the perturbative dressing analysis).

Conclusion

As a result, the vacuum ZPE scale becomes a structural consequence of the determinant composition law of the internal admittance operator over the ten-dimensional symmetric-tensor response space per causal cell, where the exponent 10 is fixed by the intrinsic configuration space of $\mathcal{G}_{\mu\nu}$ and the multiplicative composition of orthogonal geometric response channels.

H The Macroscopic Casimir Pressure from Substrate Elasticity

The Casimir effect [53, 161] is traditionally interpreted as a direct macroscopic manifestation of the mechanical stress exerted by the quantum vacuum. Within the QEG framework, we derive the fundamental quantum of this vacuum pressure strictly from first principles, replacing standard perturbative regularizations with the exact geometric and thermodynamic continuum mechanics of the elastic substrate.

H.1 Modal Stress-Energy and the Longitudinal Force Density

In the QEG representation of the vacuum, each normal mode of the elastic substrate behaves as an independent quantum harmonic oscillator. As established throughout this work, the irreducible zero-point energy (ZPE) of a coherent mode confined to a characteristic causal length L_{ref} is:

$$E_0(L_{\text{ref}}) = \frac{\hbar c}{2L_{\text{ref}}} \quad (92)$$

In continuum mechanics, the force exerted by an elastic mode along its axis of propagation is governed by the longitudinal component of the effective stress-energy tensor ($\langle T_{zz} \rangle$). By the principle of virtual displacement for a confined geometric strain, the macroscopic force conjugate to this characteristic scale is strictly:

$$F_0(L_{\text{ref}}) = \left| -\frac{\partial E_0}{\partial L_{\text{ref}}} \right| = \frac{\hbar c}{2L_{\text{ref}}^2} \quad (93)$$

This represents the absolute baseline 1D geometric force (or momentum flux) exerted by a single fundamental zero-point excitation of the vacuum.

H.2 Isotropic Bulk Pressure: Laplacian Flux Normalization

To transition from a 1D modal force to a 3D macroscopic physical pressure, we must project this stress through the spatial geometry of the substrate. In the static, unpolarized limit, the vacuum's elastic response is isotropic and conservative, rigorously governed by the 3D Laplacian Green's function.

Gauss's flux theorem dictates that a point-like longitudinal stress natively distributes its restoring force isotropically over the boundary of a sphere of radius L_{ref} , carrying a surface area $A = 4\pi L_{\text{ref}}^2$. The intrinsic, bare isotropic vacuum pressure quantum is therefore geometrically forced to be:

$$P_0(L_{\text{ref}}) = \frac{F_0(L_{\text{ref}})}{4\pi L_{\text{ref}}^2} = \frac{\hbar c}{8\pi L_{\text{ref}}^4} \quad (94)$$

This formula establishes the universal unit of isotropic vacuum stress. Its scaling ($\hbar c/L_{\text{ref}}^4$) is dimensionally exact, while the prefactor $1/(8\pi)$ is not a perturbative artifact, but the unique geometric residue of 3D flux conservation.

H.3 Thermodynamic Consistency Check

The robustness of this geometric derivation is confirmed by recovering it identically through bulk macroscopic thermodynamics. The equation of state defines bulk pressure as the negative volumetric gradient of internal energy:

$$P = -\frac{\partial E}{\partial V} \quad (95)$$

Assigning the natural spherical volume $V = \frac{4}{3}\pi L_{\text{ref}}^3$ to the isotropic causal cell, the differential volume element is $dV = 4\pi L_{\text{ref}}^2 dL_{\text{ref}}$. Applying the chain rule to the zero-point energy:

$$P_0(L_{\text{ref}}) = -\frac{\partial E_0}{\partial L_{\text{ref}}} \cdot \frac{dL_{\text{ref}}}{dV} = \left(\frac{\hbar c}{2L_{\text{ref}}^2} \right) \left(\frac{1}{4\pi L_{\text{ref}}^2} \right) = \frac{\hbar c}{8\pi L_{\text{ref}}^4} \quad (96)$$

This strict convergence between elastostatics (stress-tensor flux) and thermodynamics confirms that $1/(8\pi)$ is the exact, parameter-free geometric constant for the macroscopic zero-point pressure of free space.

H.4 From the Geometric Baseline to the Empirical Casimir Force

We can now contrast this pure substrate baseline against the standard quantum electrodynamic (QED) calculation for the Casimir force between two perfectly conducting parallel plates separated by a distance d :

$$\frac{F_C}{A} = \frac{\pi^2 \hbar c}{240 d^4} \approx \frac{1.300 \times 10^{-27} \text{ N} \cdot \text{m}^2}{d^4} \quad (97)$$

By comparison, evaluating the QEG intrinsic geometric pressure (Eq. 94) at the same scale yields:

$$P_0(d) = \frac{\hbar c}{8\pi d^4} \approx \frac{1.258 \times 10^{-27} \text{ N} \cdot \text{m}^2}{d^4} \quad (98)$$

The theoretical geometric baseline strictly matches the magnitude and exact dimensional scaling of the standard macroscopic Casimir force. The minimal fractional difference between the prefactors ($\pi^2/240 \approx 0.0411$ versus $1/8\pi \approx 0.0398$, a deviation of merely $\sim 3.2\%$) carries a transparent physical interpretation: the QEG factor ($1/8\pi$) describes the pure, unconstrained *isotropic* 3D expansion of the vacuum cell, whereas the QED factor ($\pi^2/240$) inherently relies on the Riemann Zeta function regularization ($\zeta(-3)$) specifically required to account for the highly anisotropic, 1D Dirichlet boundary conditions imposed by idealized parallel metallic plates. The near-perfect numerical convergence demonstrates that the classical Casimir effect is simply the local anisotropic geometric polarization of the vacuum's foundational $1/8\pi$ elastic bulk pressure.

I The Geometro-Elastic Unity of Thermal Radiation and Quantum Dispersion

In standard theoretical physics, macroscopic thermodynamics and microscopic quantum electrodynamics are treated as formally distinct domains. Blackbody radiation is a statistical consequence of thermalized photon gases, while London dispersion forces arise from the zero-point energy (ZPE) correlations of the quantum vacuum.

In Quantum-Elastic Geometry (QEG), this distinction vanishes. Both phenomena are macroscopic manifestations of the exact same underlying structure: the network of quantum harmonic oscillators comprising the elastic substrate. To prove this, we demonstrate that the fundamental constants governing these two phenomena—the Stefan-Boltzmann constant (σ) and the London dispersion coefficient (C_6)—form a perfect dimensional and geometric complementary pair, collapsing into the fundamental equation of state for the vacuum's power density.

I.1 Dimensional Convergence in Phase Space

London dispersion forces arise from the instantaneous dipole-dipole correlations of the vacuum's zero-point fluctuations. Consistent with the Coherent Stiffness Ladder (Sec. 3.5 of the Main paper), the macroscopic phase-averaged zero-point energy of the causal cell is $\mathcal{E}_{\text{ZPE}} = \hbar c/2\pi L_{\text{ref}}$. We can therefore define the *fiducial vacuum dispersion coefficient* ($C_{6,\text{vac}}$) associated with

the correlated fluctuations of two adjacent macroscopic cells (each of volume $V = L_{\text{ref}}^3$) strictly as:

$$C_{6,\text{vac}} = \mathcal{E}_{\text{ZPE}} \cdot V^2 = \left(\frac{\hbar c}{2\pi L_{\text{ref}}} \right) L_{\text{ref}}^6 = \frac{\hbar c}{2\pi} L_{\text{ref}}^5 \quad (99)$$

In the standard SI system, the coefficient C_6 carries the convoluted dimensions of $[\text{J} \cdot \text{m}^6]$, while the Stefan-Boltzmann constant ($\sigma = \pi^2 k_B^4/60\hbar^3 c^2$) carries $[\text{J} \cdot \text{s}^{-1} \cdot \text{m}^{-2} \cdot \text{K}^{-4}]$. Multiplying them in SI produces a dimensionally chaotic quantity lacking direct physical interpretation.

However, applying the universal QEG dimensional collapse (Table 1 of the Main paper), where energy, time, and temperature are geometrically equivalent to length ($[E] \equiv [T] \equiv [K] \equiv [L]$), the true geometric signature emerges:

$$[\sigma] \equiv [L^{-6}], \quad [C_{6,\text{vac}}] \equiv [L^7] \quad (100)$$

Physically, the $[L^7]$ dimensionality of London dispersion does not imply a literal 7-dimensional physical space. Rather, it is the strict geometric signature of a *two-body quantum correlation* in phase space: integrating the spatial volume of the first cell ($[L^3]$) against the second cell ($[L^3]$) over the continuous energetic spectrum of the vacuum ($[E] \equiv [L^1]$).

By multiplying these two constants, we extract a profound structural invariant:

$$[\sigma \cdot C_{6,\text{vac}}] = [L^{-6}] \cdot [L^7] = [L] \equiv [E] \quad (101)$$

This mathematically isolates a pure, intrinsic *vacuum energy scale*, confirming that macroscopic thermal radiation and microscopic quantum dispersion are the orthogonal dissipative and elastic projections of the exact same continuous medium.

I.2 Analytical Evaluation: The Ultimate ZPE Collapse

We can definitively prove this dimensional convergence by analytically evaluating the product $\sigma \cdot C_{6,\text{vac}}$ using strictly the exact QEG geometric identities derived in the main text.

The product of the classical Stefan-Boltzmann constant and the geometric vacuum London coefficient is:

$$E_{\text{vac}} = \left(\frac{\pi^2 k_B^4}{60\hbar^3 c^2} \right) \cdot \left(\frac{\hbar c}{2\pi} L_{\text{ref}}^5 \right) = \frac{\pi^2 k_B^4 L_{\text{ref}}^5}{60\hbar^2 2\pi c} \quad (102)$$

We now apply the fundamental QEG structural identities for the thermodynamic and quantum scales: $k_B \equiv \mu_0/c^2$ (Eq. 174 of the Main paper) and $\hbar \equiv L_{\text{ref}}^2/c^4$ (Eq. 216 of the Main paper). Substituting these yields a spectacular algebraic collapse where the c^8 kinematic scaling factors exactly cancel out:

$$E_{\text{vac}} = \frac{\pi^2 (\mu_0/c^2)^4 L_{\text{ref}}^5}{60(L_{\text{ref}}^2/c^4)^2 2\pi c} = \frac{\pi^2 (\mu_0^4/c^8) L_{\text{ref}}^5}{60(L_{\text{ref}}^4/c^8) 2\pi c} = \frac{\pi^2 \mu_0^4 L_{\text{ref}}}{60 \cdot 2\pi c} \quad (103)$$

To physicalize this pure inductive invariant, we recall the geometric identity for the elementary charge, $e \equiv \mu_0^3 L_{\text{ref}}/4\pi$ (Eq. 215 of the Main paper), allowing the substitution $\mu_0^4 L_{\text{ref}} = 4\pi \mu_0 e$:

$$E_{\text{vac}} = \frac{\pi^2}{60 \cdot 2\pi c} (4\pi \mu_0 e) \quad (104)$$

Applying the foundational Vacuum Constitutive Equation (Eq. 187 of the Main paper), the potential of the bare charge is structurally identical to the fundamental zero-point energy: $\mu_0 e \equiv \hbar c/L_{\text{ref}}$. Substituting this, we obtain the definitive analytic solution:

$$\sigma \cdot C_{6,\text{vac}} = 2 \cdot \left(\frac{1}{4} \right) \cdot \left(\frac{\pi^2}{15} \right) \cdot \frac{\hbar}{L_{\text{ref}}} \quad (105)$$

I.3 The Topological Signature of Macroscopic Power

This final formulation is a masterpiece of geometric deconstruction. The product of macroscopic thermal dissipation (σ) and spatial elastic stiffness ($C_{6,\text{vac}}$) returns the unreduced quantum of action frequency (\hbar/L_{ref}) dressed by three exact topological signatures:

- 2: The multiplicity of the two independent transverse polarizations of the substrate's coherent modes.
- 1/4: The holographic *bulk-to-boundary* projection, mapping the 3D volumetric gas of oscillators onto a 2D emitting surface.
- $\pi^2/15$: The absolute mathematical saturation limit of the 3D quantum lattice, derived from integrating the Bose-Einstein distribution over a spherical momentum space ($\zeta(4)$).

This structural deconstruction allows us to completely re-envision the Stefan-Boltzmann constant itself. By substituting the QEG identities directly into σ , we obtain:

$$\sigma = \frac{\pi^2 \mu_0^4 c^2}{60 L_{\text{ref}}^6} = \frac{\pi^2 c^2}{60} \left(\frac{\mu_0^2}{L_{\text{ref}}^3} \right)^2 \quad (106)$$

Recognizing μ_0^2 as the Unitary Vacuum Power (P_{unit}) established in Eq. 207 of the Main paper, and L_{ref}^3 as the macroscopic volume (V_{macro}), the Stefan-Boltzmann constant is revealed as:

$$\sigma = \left(\frac{1}{4} \right) \left(\frac{\pi^2}{15} \right) \left(\frac{P_{\text{unit}}}{V_{\text{macro}}} \right)^2 c^2 \quad (107)$$

Conclusion: The Macroscopic Hooke's Law of Thermodynamics.

This parameter-free identity reveals that the constant of macroscopic thermal emission (σ) is not an independent thermodynamic postulate. Structurally, it is the exact continuum-mechanics realization of a quadratic Hookean response for a dissipative medium (Response = Geometry \times Stiffness \times Stress²).

Here, the causal speed squared (c^2) acts as the dynamic Young's modulus of the substrate, while the Inductive Power Density ($P_{\text{unit}}/V_{\text{macro}}$) acts as the fundamental geometric stress. Scaled by the topological constraints of a 3D lattice emitting through a 2D boundary, this proves that macroscopic thermal radiation is fully derived from the geometric inertia and elasticity of the spacetime substrate.

J Geometric Interpretation of Classical Force Denominators: Tension vs. Shear

Having derived the nature of the fundamental coupling constants μ_0 , K_e , k_B , and G as modal projections of the substrate's response matrices in the main text, we can now reinterpret the classical force laws they govern. Rather than being disparate empirical axioms, Newton's and Coulomb's laws emerge as complementary manifestations of a single underlying mechanism: *momentum exchange through an elastic and dissipative vacuum*.

J.1 Newton's and Coulomb's Laws as Damped Momentum Transfer

From the previously derived first-order expressions, the gravitational coupling and the Boltzmann constant are related to the vacuum permeability by $G \equiv \mu_0 \alpha^2$ and $k_B \equiv \mu_0 / c^2$. It algebraically follows that:

$$G \equiv k_B \cdot \alpha^2 \cdot c^2 \quad (108)$$

Using this substitution, we can rewrite Newton's law of gravitation to reveal its thermo-entropic nature. The gravitational force is mediated by the exchange of *damped, longitudinal momentum*, and can be expressed in terms of

damped relativistic momenta. Taking $\zeta = \alpha$ as the scalar norm of the structural damping tensor $\zeta_{\mu\nu}$, we obtain:

$$F_g = G \frac{Mm}{r^2} = k_B \frac{(Mc \cdot \zeta)(mc \cdot \zeta)}{r^2} \quad (109)$$

Thus, the terms in the numerator correspond to effective, damped relativistic momenta, whose propagation is modulated by the incoherent damping structure encoded in the vacuum. Gravity emerges as the effective resistance to the coherent alignment of these projected momenta through the substrate's dissipative tensorial geometry.

By symmetry, the Coulomb force can be reformulated as the exchange of *undamped, transverse momentum*. Using $K_e = \frac{1}{4\pi\epsilon_0}$ and the causal identity $c^2 = 1/(\mu_0\epsilon_0)$, we write:

$$F_e = K_e \frac{Q_1 Q_2}{r^2} = \left(\frac{\mu_0 c^2}{4\pi} \right) \frac{Q_1 Q_2}{r^2} = \mu_0 \frac{(Q_1 c)(Q_2 c)}{4\pi r^2} \quad (110)$$

This structure perfectly mirrors the gravitational expression, with (Qc) playing the role of the transverse modal momentum and the vacuum magnetic permeability μ_0 acting as the transverse field stiffness. Thus, Newton's and Coulomb's laws appear as complementary modal projections of the same unified tensorial response.

J.2 Physical Interpretation: Structural Differences and Spatial Geometry

The distinct mathematical forms of Eqs. 109 and 110 are not accidental; they reflect profound differences in the physical nature of the momentum exchange. Comparing the rewritten laws reveals two fundamental structural differences that provide deep insight into their distinct mechanics.

The Selective Role of Damping.

A crucial difference is the explicit presence of the damping factor, ζ , in the gravitational force, while it is absent in the static Coulomb force. This does not imply that electromagnetism is an undamped phenomenon—indeed, the propagation of radiation is damped by the vacuum. Rather, it reveals the fundamental character of each interaction:

- The **Coulomb force** is a *conservative interaction* between static charges. Its formulation corresponds to the coherent elastic channel of the substrate, meaning it does not involve dissipation at leading order. The dissipative effects of electromagnetism only arise in dynamic phenomena (e.g., radiation resistance).
- The **gravitational force**, when expressed in this thermo-entropic form, is revealed to be an inherently *dissipative and entropic interaction*. It is not a static potential force in the classical conservative sense, but the result of momentum exchange through the substrate's incoherent (dissipative) medium. Therefore, it *must* explicitly include the damping factor ζ as part of its fundamental definition.

This indicates that damping is the key feature distinguishing the entropic (longitudinal) modes of the vacuum from the conservative (transverse) static modes.

The Geometry of Propagation (r^2 vs. $4\pi r^2$).

The second key difference lies in the geometry of the spatial denominator.

- The **Coulomb force** (Eq. 110) explicitly isolates the factor $4\pi r^2$, the surface area of a sphere. This reflects the isotropic, wave-like nature of the interaction, where the influence of a point charge propagates outwards uniformly in all directions, spreading over a spherical surface. This is the hallmark of a **transverse shear wave**.

- The re-expressed **gravitational force** (Eq. 109), by contrast, lacks the 4π solid angle factor. Its pure r^2 dependence represents a direct interaction strictly between two points. This supports the interpretation of a **longitudinal interaction**, where the force acts as a direct “pressure” or “tension” along the line connecting the two masses, rather than as a field radiating spherically over a wavefront.

Remark: Geometric Interpretation.

The distinction between the denominators r^2 and $4\pi r^2$ should not be read as an empirical asymmetry in the spatial geometry of the two forces, but rather as a difference in modal propagation. Both interactions obey Gauss-like flux conservation and are experimentally consistent with isotropic $1/r^2$ decay. Standard physics absorbs the 4π into the definition of the corresponding coupling constant (G or K_e) so it carries no independent physical weight in the final numerical value.

However, in the QEG unified framework, pulling the factors apart demonstrates that the absence of the explicit 4π in the gravitational term expresses a longitudinal projection of the vacuum’s stress tensor—an exchange of compressive momentum directly along the line of centers. Conversely, the electromagnetic interaction represents a transverse, rotational projection whose influence is necessarily distributed isotropically over the spherical wavefront. Hence, the two geometries differ not in their spatial metric, but in the internal symmetry of their modal excitation: tension versus shear.

J.3 Synthesis: Momentum, Symmetry, and Strength

In summary, both interactions reflect momentum exchange across a structured medium, but what differs is the spatial symmetry of the exchange. The immense difference in their strengths arises not from arbitrarily chosen coupling constants, but from the physical properties of the vacuum itself:

- **Gravity** emerges as a highly suppressed, entropy-weighted longitudinal momentum flow. It is weak not because its coupling is arbitrarily small, but because the vacuum is extremely rigid against this type of compressional deformation. The Boltzmann constant k_B quantifies the high entropic cost required to induce such a strain.
- **Electromagnetism**, in contrast, reflects a much more efficient, transversely mediated momentum exchange. It is amplified by the vacuum’s comparatively soft resistance to shear-like deformations, a reactive process natively governed by μ_0 .

K Dual Emergence of the Gravitational Coupling: Topological Closure and Spectral Dressing

In Quantum-Elastic Geometry (QEG), the macroscopic gravitational coupling G represents the global, isotropic response of the vacuum substrate. In this section, we demonstrate that G can be derived through two fundamentally distinct, yet mathematically convergent, procedures: a static topological compactification and a dynamic spectral dressing by vacuum fluctuations. This dual emergence highlights the deep structural consistency of the QEG framework.

K.1 Raw Kinematic Action and Coherent Phase Normalization

The vacuum substrate is modeled as a four-dimensional elastic medium whose accumulated deformation defines a geometric action functional (see Section 1.1.2 of the main text). Dimensional collapse implies

$$[S] \equiv [L^2], \tag{111}$$

so that action measures accumulated surface-like geometric deformation rather than energy multiplied by time.

For a reference causal cell of scale L_{ref} , the purely kinematic (undressed) accumulated action in 1D is fixed by dimensional consistency and the causal modulus c :

$$S_0 \equiv \frac{L_{\text{ref}}^2}{c} \tag{112}$$

This S_0 represents the raw spacetime area swept by a causal length. However, coherent propagation in the torsional sector carries an internal $U(1)$ phase degree of freedom (Section 5.5.1 of the main text). A physically admissible, stable excitation must therefore close on a minimal phase orbit:

$$\oint_{S^1} d\phi = 2\pi. \tag{113}$$

The coherent, phase-normalized action per cycle is thus:

$$S \equiv \frac{S_0}{2\pi} = \frac{L_{\text{ref}}^2}{2\pi c} \tag{114}$$

This normalization is structurally enforced by the angular trace closure of the torsional projector (Appendix C), where universal $1/(2\pi)$ residues arise from phase averaging over the $U(1)$ orbit.

K.2 Microscopic Elastic Response

The static vacuum stiffness is governed by the Green function of the Laplacian in three spatial dimensions,

$$\nabla^2 G(\mathbf{x}) = -\delta^{(3)}(\mathbf{x}), \tag{115}$$

whose unique isotropic solution is

$$G(r) = \frac{1}{4\pi r}. \tag{116}$$

Evaluated at the boundary of the reference cell, $r = L_{\text{ref}}$, the geometric stiffness is

$$k_0 = \frac{1}{4\pi L_{\text{ref}}}. \tag{117}$$

Using the constitutive relation for the elastic medium ($F = k^2 S$), the phase-normalized microscopic restoring force becomes:

$$\begin{aligned} F_{\text{micro}} &= k_0^2 S \\ &= \left(\frac{1}{4\pi L_{\text{ref}}} \right)^2 \left(\frac{L_{\text{ref}}^2}{2\pi c} \right) \\ &= \frac{1}{16\pi^2 c} \cdot \frac{1}{2\pi} = \frac{1}{32\pi^3 c}. \end{aligned} \tag{118}$$

We must now relate this directional, microscopic force amplitude to the macroscopic gravitational coupling, which we previously established in the Unified Response Identity (Section 4.6 of the main text) as:

$$G \equiv \frac{1}{16\pi c}. \tag{119}$$

The ratio between the macroscopic coupling and the microscopic response is exactly $2\pi^2$. We now demonstrate that this universal geometric factor emerges independently from both the static topology of the substrate and its quantum dynamics.

K.3 Topological Derivation: Compact S^3 Closure

Finite-action boundary conditions compactify spatial infinity,

$$\mathbb{R}^3 \cup \{\infty\} \simeq S^3, \quad (120)$$

so static, globally coherent configurations are effectively defined on a compact three-sphere.

The microscopic force F_{micro} represents the response amplitude per radial direction. The macroscopic monopole coupling G corresponds to the fully isotropic, symmetric projection (the $l=0$ mode) over this compact domain. Integrating this directional response over the full solid angle of S^3 yields:

$$\oint_{S^3} d\Omega_3 = \text{Vol}(S^3) = 2\pi^2. \quad (121)$$

Therefore, the macroscopic coupling is:

$$G = F_{\text{micro}} \times 2\pi^2 = \frac{1}{32\pi^3 c} \times 2\pi^2 = \frac{1}{16\pi c}. \quad (122)$$

Gravity thus emerges as the globally isotropized completion of a local, phase-normalized elastic response.

K.4 Spectral Derivation: Vacuum Fluctuation Determinant

Alternatively, we may derive the coupling dynamically. Begin with the raw, un-normalized kinematic action S_0 from Eq. (112). The undressed, bare microscopic force is:

$$F_{\text{bare}} = k_0^2 S_0 = \left(\frac{1}{4\pi L_{\text{ref}}} \right)^2 \left(\frac{L_{\text{ref}}^2}{c} \right) = \frac{1}{16\pi^2 c}. \quad (123)$$

To obtain the macroscopic coupling, this bare force must be dressed by the quantum fluctuations of the deformation field. In the semiclassical path integral, these fluctuations generate a functional determinant contribution. For a one-dimensional harmonic oscillator, the Gel'fand–Yaglom theorem yields the fluctuation determinant:

$$\det(-\partial_t^2 + \omega^2) \propto \frac{\sin(\omega T)}{\omega T}. \quad (124)$$

Using Euler's infinite product representation,

$$\frac{\sin x}{x} = \prod_{n=1}^{\infty} \left(1 - \frac{x^2}{\pi^2 n^2} \right), \quad (125)$$

and evaluating at the "causal quadrature" $x = \pi/2$ (the point of maximal orthogonal action transfer between conjugate variables), we find:

$$\frac{\sin(\pi/2)}{\pi/2} = \frac{2}{\pi}. \quad (126)$$

The collective geometric dressing factor Γ_{1D} imposed by the vacuum on a single degree of freedom is the inverse of this determinant:

$$\Gamma_{1D} = \left[\frac{2}{\pi} \right]^{-1} = \frac{\pi}{2} = \prod_{n=1}^{\infty} \frac{4n^2}{4n^2 - 1}, \quad (127)$$

which is exactly the classical Wallis Product.

Because the propagating torsional sector (which defines dynamic inertia) possesses exactly **two independent transverse polarizations**, the total spectral dressing factor becomes:

$$\Gamma_{\text{vac}} = 2 \times \Gamma_{1D} = 2 \times \frac{\pi}{2} = \pi. \quad (128)$$

The macroscopic gravitational constant G is therefore the bare elastic force (F_{bare}) dressed by the total vacuum fluctuation determinant (Γ_{vac}):

$$G = F_{\text{bare}} \times \Gamma_{\text{vac}} = \left(\frac{1}{16\pi^2 c} \right) \times \pi = \frac{1}{16\pi c}, \quad (129)$$

identical to Eq. (122).

K.5 4D Closure Identity and Conclusion

The convergence of these two derivations is structurally enforced by the QEG 4D accumulation architecture. In Section 3.4.1 of the main text, the minimal coherent 4D cell satisfies:

$$\frac{\hbar}{2} \equiv \frac{L_{\text{ref}}^2}{8\pi^2 \alpha c^4}. \quad (130)$$

The numerical factor $8\pi^2$ within the denominator naturally factorizes as:

$$8\pi^2 = (4\pi) \times (2\pi), \quad (131)$$

where 4π represents the spherical flux closure of S^2 (the Gauss normalization) and 2π represents the minimal torsional $U(1)$ phase orbit.

Thus, the gravitational coupling

$$G = \frac{1}{16\pi c} \quad (132)$$

emerges as the unique value consistent with:

1. Coherent $U(1)$ phase normalization of the accumulated 4D action (2π);
2. Global isotropic closure on the compactified space S^3 ($2\pi^2$);
3. Spectral dressing by transverse vacuum fluctuations (the Wallis product, π).

The exact agreement between the topological compactification and the spectral determinant highlights the profound structural consistency between global geometry and quantum dynamics within Quantum-Elastic Geometry.

L First-Principles Derivation of the On-Shell Einstein-Hilbert Action

In the QEG framework, the macroscopic dynamics of spacetime are intimately linked to the microscopic quantum fluctuations of the vacuum. We demonstrate this structural unity by explicitly deriving the quantum of the classical Einstein-Hilbert action evaluated on a single macroscopic causal cell of the QEG vacuum state. This proves that the coherent geometric seed of macroscopic gravity matches exactly the *Level 2 Metric Tension* invariant (\mathcal{T}_2) derived independently in the Coherent Stiffness Ladder (Sec. 3.1 of the main text).

L.1 Step 1: The Effective Geometric Action on a de Sitter Background

We begin with the standard Einstein-Hilbert action governing macroscopic General Relativity. Using the canonical temporal convention ($x^0 = t$), where the spacetime integration measure $d^4x \equiv d^3x dt$ carries mixed geometric-kinematic dimensions of $[L^3 T]$, the action including the cosmological constant Λ is:

$$S_{EH} = \frac{c^4}{16\pi G} \int (R - 2\Lambda) \sqrt{-g} d^3x dt \quad (133)$$

In a maximally symmetric vacuum governed entirely by the cosmological tension (a de Sitter geometry), the trace of the Einstein field equations strictly dictates the scalar curvature as $R = 4\Lambda$. Substituting this on-shell geometric condition yields the effective constant Lagrangian density $\mathcal{L}_{\text{eff}} = R - 2\Lambda = 2\Lambda$.

L.2 Step 2: Coarse-Graining over the Macroscopic Causal Cell

General Relativity is an effective macroscopic theory; its continuous integrals emerge from the coarse-graining of the discrete microscopic substrate. We evaluate this geometric action over the fundamental coarse-grained building block of the QEG vacuum: a macroscopic causal cell of spatial volume $V_3 = L_{\text{ref}}^3$ and duration τ_{ref} .

In the local neighborhood of this cell ($L_{\text{ref}} \ll R_{\text{Hubble}}$), the metric determinant is locally Minkowskian ($\sqrt{-g} \simeq 1$). Integrating the constant density over this

cell yields the discrete quantum of macroscopic geometric action:

$$\Delta S_{EH} = \frac{c^4}{16\pi G} (2\Lambda) L_{\text{ref}}^3 \tau_{\text{ref}} = \left(\frac{c^4 \Lambda}{8\pi G} \right) L_{\text{ref}}^3 \tau_{\text{ref}} \quad (134)$$

In standard General Relativity, the cosmological constant is strictly coupled to the vacuum energy density (ρ_{vac}) through the classical identity $\Lambda = \frac{8\pi G}{c^4} \rho_{\text{vac}}$, meaning that the prefactor precisely isolates the density: $\frac{c^4 \Lambda}{8\pi G} = \rho_{\text{vac}}$. Substituting this reveals that the macroscopic geometric action per cell is identically the product of energy density, volume, and time:

$$\Delta S_{EH} = \rho_{\text{vac}} \cdot L_{\text{ref}}^3 \cdot \tau_{\text{ref}} \quad (135)$$

L.3 Step 3: Applying the QEG Dimensional Collapse and Vacuum Density

To evaluate this action, we input the exact QEG vacuum energy density. As established in Section 5.4.4 of the Main paper, the density is regularized by the fundamental UV cutoff of the substrate (the electromagnetic aperture, l_{em}) and phase-averaged over the macroscopic volume:

$$\rho_{\text{vac}} = \frac{\hbar c}{2\pi \cdot l_{\text{em}} \cdot L_{\text{ref}}^3} \quad (136)$$

Substituting our foundational geometric identity for the electromagnetic resolution ($l_{\text{em}} \equiv L_{\text{ref}}/c^2$), the density simplifies to $\rho_{\text{vac}} = \frac{\hbar c^3}{2\pi L_{\text{ref}}^4}$. Inserting this into our geometric action equation (Eq. 135) yields:

$$\Delta S_{EH} = \left(\frac{\hbar c^3}{2\pi L_{\text{ref}}^4} \right) L_{\text{ref}}^3 \cdot \tau_{\text{ref}} = \frac{\hbar c^3}{2\pi} \left(\frac{\tau_{\text{ref}}}{L_{\text{ref}}} \right) \quad (137)$$

In the QEG dimensional collapse framework, the static spatial reference length and the unscaled reference time are structurally identical ($\tau_{\text{ref}} \equiv L_{\text{ref}}$), meaning their ratio is strictly 1. We thus obtain the definitive parameter-free value for the invariant geometric action of a causal cell:

$$\Delta S_{EH} = \frac{\hbar c^3}{2\pi} \quad (138)$$

L.4 Physical Interpretation: Gravity as the Macroscopic Equation of State

This exact derivation reveals a breathtaking structural unity. First, the 2π term in the denominator represents the universal normalization associated with a one-dimensional angular phase cycle, verifying that General Relativity acts as the phase-averaged macroscopic envelope of the vacuum's zero-point modes. More profoundly, this result directly bridges macroscopic spacetime curvature to the *Level 2 Metric Tension* invariant (\mathcal{T}_2) derived in the Coherent Stiffness Ladder. In Section 3.5.1 of the Main paper, we established that the absolute rigidity of the metric structure is $\mathcal{T}_2 = \hbar c^3 / L_{\text{ref}}$, which collapses geometrically into the scaled causal time interval ($\tau_{\text{causal}} = L_{\text{ref}}/c$). By factoring our result, we can rewrite the macroscopic action quantum of the universe as:

$$\Delta S_{EH} = \frac{1}{2\pi} \left(\frac{\hbar c^3}{L_{\text{ref}}} \right) L_{\text{ref}} \implies \boxed{\Delta S_{EH} = \frac{\tau_{\text{causal}} \cdot L_{\text{ref}}}{2\pi}} \quad (139)$$

This confirms that the classical Einstein-Hilbert functional evaluates strictly to the fundamental dynamic tension supported by the macroscopic metric, scaled by the reference length and normalized by the macroscopic phase cycle.

The Ontological Status of General Relativity.

This thermodynamic and elastic reinterpretation provides a definitive physical mandate. In classical theoretical physics, the Einstein-Hilbert action is traditionally treated as an abstract mathematical functional. In the QEG framework, it acquires a rigorous mechanical identity: it is the accumulated free energy (or total elastic strain) of the macroscopic vacuum.

Consequently, QEG formally aligns with the emergent gravity paradigm proposed by Jacobson and Verlinde. General Relativity is revealed not as a microscopic quantum interaction requiring a fundamental gauge boson (the graviton), but as the macroscopic thermodynamic equation of state of the quantum-elastic vacuum. The expansion of the cosmos and the deformation of geometry around massive bodies are macroscopic statistical expressions of a single underlying imperative: the quantum-elastic substrate minimizing its internal action by dissipating mechanical stress across the causal limit.

Einstein's Field Equations as the Macroscopic Hooke's Law.

This thermodynamic and elastic reinterpretation breathes new mechanical life into the Einstein Field Equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \left(\frac{8\pi G}{c^4} \right) T_{\mu\nu} \quad (140)$$

Rather than an abstract statement about differential geometry, this equation is revealed as the ultimate macroscopic generalization of Hooke's Law for the spacetime continuum (Strain = Compliance \times Stress):

- **The Mechanical Stress ($T_{\mu\nu}$):** Massive particles and energy fields are not external entities occupying an empty stage; they are non-linear topological defects (knots) within the continuous elastic network. The energy-momentum tensor represents the localized mechanical stress that these defects exert on the surrounding vacuum lattice.
- **The Geometric Strain ($G_{\mu\nu}$):** The Einstein tensor represents the geometric strain—the pure elastic response of the vacuum. Gravity is not an attractive force that "pulls" masses together; it is the physical gradient of tension within the deformed spacetime lattice as it accommodates local stress to prevent topological rupture.
- **The Vacuum Compliance ($8\pi G/c^4$):** This universal coupling constant is strictly the macroscopic mechanical compliance (the inverse of rigidity) of the vacuum. It quantifies exactly how much the elastic substrate will geometrically deform (strain) per unit of applied energetic stress.
- **The Geometric Surface Tension ($\Lambda g_{\mu\nu}$):** As derived previously, the cosmological constant acts as the intrinsic, static pre-stress of the macroscopic canvas. It is the isotropic outward elastic surface tension generated by the zero-point quantum fluctuations, preventing the geometric lattice from collapsing under its own topological weight.

Conclusion.

By bridging the quantum of action (\hbar) and the macroscopic geometry of the universe through the QEG scaling ladders, we confirm that General Relativity and Quantum Mechanics are not incompatible theories awaiting a complex unification. Rather, gravity is simply the thermodynamic and elastic macroscopic manifestation of the quantum vacuum. The expansion of the cosmos, the dilation of time near a massive body, and the orbital mechanics of galaxies are all macroscopic expressions of a single underlying imperative: the quantum-elastic substrate continuously minimizing its internal action by dissipating mechanical stress at the causal velocity limit.

M Advanced Mechanical and Field-Theoretic Interpretations of the Cosmological Constant

As derived in Section 5.4.4 of the main text, the cosmological constant emerges from the zero-point fluctuations of the quantum-elastic substrate, yielding the fundamental identity:

$$\Lambda = \frac{4(l_p \cdot c)^2}{L_{\text{ref}}^4} \quad (141)$$

While the main text explores its primary interpretation as an elastic surface tension and its connection to Bekenstein-Hawking entropy, this expression admits additional profound interpretations in both classical mechanics and gauge field theory.

M.1 Complementary Mechanical Interpretations

Kinetic Momentum Interpretation.

Within the QEG dimensional collapse, velocity is a dimensionless kinematic scalar. Consequently, the quantity $p_{pl} = l_p \cdot c$ carries the exact dimensions of a fundamental geometric *Planck momentum* ($[L]$). It represents the momentum of the most primordial quantum fluctuation: a Planck-scale excitation propagating at the maximum causal speed. With this identification, Eq. 141 becomes:

$$\Lambda = \frac{4 p_{pl}^2}{L_{\text{ref}}^4} \quad (142)$$

Given that the square of momentum (p^2) is directly proportional to kinetic energy, this expression reveals that the cosmological constant can be formally understood as the *density of kinetic pressure* exerted by the zero-point fluctuations of the vacuum. The accelerated expansion of the universe is thus the macroscopic manifestation of the incessant kinetic impulse of the spacetime substrate's own fundamental quantum vibrations.

Torsional Inertia Interpretation.

Alternatively, the term $(l_p \cdot c)^2$ has dimensions of $[L^2]$, which in rotational mechanics is dimensionally equivalent to the *moment of inertia per unit mass* ($I/m \sim r^2$). We can therefore identify this term with the *fundamental rotational inertia* of a primordial cell of the vacuum substrate. The formula for Λ then acquires a complementary meaning:

$$\Lambda = 4 \frac{I_{\text{fund}}/m_{\text{fund}}}{L_{\text{ref}}^4} \quad (143)$$

From this perspective, the cosmological constant measures the *density of the vacuum's torsional inertia*. It reflects spacetime's intrinsic resistance to rotational or shear-like microscopic deformations. This view powerfully connects the cosmic expansion to the torsional modes of the spatial substrate, suggesting that the universe's expansion on the largest scales is the result of the substrate globally "unwinding" a primordial torsional stress, governed by its intrinsic geometric inertia.

M.2 The Cosmic Spin Anomaly as a Torsional Fossil

Beyond its ability to unify field couplings and resolve standard cosmological tensions, the QEG framework offers a covariant physical mechanism for outstanding observational puzzles that challenge the isotropic background of the standard model. One such puzzle is the recently reported evidence for a large-scale spin asymmetry in the universe, suggesting a preferred axis or "cosmic spin" dipole in the angular momentum of galaxies [246, 247]. Such an observation, if confirmed, represents a profound violation of the cosmological principle of isotropy and is structurally difficult to reconcile with Λ CDM.

The QEG framework accommodates this possibility as a natural topological residue. This connection arises directly from three of the theory's core constitutive tenets:

- 1. Primordial Torsional Modes:** The symmetric deformation tensor $G_{\mu\nu}$ natively admits an intrinsic transverse-traceless shear sector (the torsional modes S_{ij}). It is therefore entirely mathematically consistent that the initial state of the universe contained a net topological winding or coherent torsional strain—a primordial macroscopic angular momentum. The observed spin asymmetry in galaxies would be a "fossil" of this initial condition, a macroscopic remnant of the spacetime fabric's own intrinsic rotational state.
- 2. Inertia of the Vacuum:** As expanded in the preceding paragraphs, the cosmological constant Λ can be mechanically interpreted as the density of the vacuum's torsional inertia. A non-zero primordial spin is the natural dynamical counterpart of a substrate possessing such an intrinsic rotational impedance. The observed cosmic axis would simply correspond to the principal axis of this primordial inertia.
- 3. Intrinsic Dissipation and the Fading Anisotropy:** The most elegant aspect of this geometric explanation lies in its compatibility with the extreme isotropy of the Cosmic Microwave Background (CMB). The QEG substrate is not ideally elastic but inherently *dissipative*, governed by the covariant Rayleigh functional whose scale is set by the universal viscosity parameter α . This irreversible accumulation property ensures that any primordial anisotropy, such as a large-scale coherent rotation, is naturally damped over cosmic time.

This leads to a compelling phenomenological narrative: the universe may have been born with a significant torsional winding (spin), but the inherent viscosity of the quantum-elastic spacetime has smoothed out this macroscopic anisotropy over 13.8 billion years. What we observe today is not a violently rotating cosmos, but a nearly completely relaxed, isotropic one, retaining only a ghostly, statistically subtle residue of its primordial spin. Thus, the potential confirmation of a cosmic axis would stand as powerful observational evidence for the torsional, dissipative, and elastic nature of the spacetime substrate itself.

M.3 The Inevitable Gauge-like Structure of the Thermo-Entropic Field

The principles of QEG demand that every mode of the unified substrate $\mathcal{G}_{\mu\nu}$ be described by a self-consistent, covariant field theory. Having identified the temporal scalar (\mathcal{G}_{00}) and vector (\mathcal{G}_{0i}) modes with macroscopic gravity and electromagnetism, the remaining dilatational degrees of freedom within the spatial tensor \mathcal{G}_{ij} —which we have identified as the thermo-entropic field—must also adhere to these principles.

For a massless or very light effective field emerging in the low-energy limit, Lorentz covariance and locality uniquely constrain the form of its kinetic action. The most general, non-trivial Lagrangian at the two-derivative level must be quadratic in a field-strength tensor derived from an underlying potential. *This is a structural necessity for any fundamental interaction.*

Therefore, the dynamics of the thermo-entropic sector must be describable by an effective field strength tensor, $\mathcal{F}_{\mu\nu}^{(GE)}$, constructed from the underlying dilatational modes of the substrate:

$$\mathcal{F}_{\mu\nu}^{(GE)} := \partial_\mu \mathcal{G}_\nu^{(T)} - \partial_\nu \mathcal{G}_\mu^{(T)} \quad (144)$$

where $\mathcal{G}_\mu^{(T)}$ represents the effective 4-potential of the thermo-entropic modes. The corresponding Lagrangian density for this sector is thus structurally fixed to the canonical gauge form:

$$\mathcal{L}_{GE} = -\frac{1}{4k_{GE}} \mathcal{F}_{\mu\nu}^{(GE)} \mathcal{F}^{(GE)\mu\nu} \quad (145)$$

where k_{GE} is the modal stiffness constant for the thermo-entropic sector.

The total energy density of the vacuum is the macroscopic expectation value of the sum of the Lagrangians of all field modes, $\Lambda = \langle \sum_i \mathcal{L}_i \rangle$. In a late-time vacuum dominated by thermo-entropic expansion, this simplifies to $\Lambda \approx \langle \mathcal{L}_{GE} \rangle$.

This provides a profound field-theoretic meaning to our geometrically derived result. The factor of 4 in the numerator of $\Lambda = 4(l_p c)^2 / L_{\text{ref}}^4$ is precisely the inverse of the canonical normalization factor (1/4) mandated by Eq. 145 for any stable gauge theory. This validates our geometric derivation of Λ as the exact ground-state energy of a vacuum which—when formalized into a consistent effective field theory—necessarily adopts the canonical normalization of a covariant gauge structure.

M.4 Gravity as an Emergent Thermodynamic Action

This field-theoretic identification leads to a striking conclusion regarding the nature of the Einstein-Hilbert action itself. The reduction of the standard Einstein-Hilbert action in a vacuum-dominated universe takes the form:

$$S_{\text{EH}} = \frac{c^4}{16\pi G} \int 4\Lambda\sqrt{-g} d^4x = \frac{c^4}{4\pi G} \int \Lambda\sqrt{-g} d^4x \quad (146)$$

Combining this with our strict identification of the cosmological constant as the effective Lagrangian density of the thermo-entropic field ($\Lambda = \langle \mathcal{L}_{GE} \rangle$), we obtain:

$$S_{\text{EH}} = \frac{c^4}{4\pi G} \int \langle \mathcal{L}_{GE} \rangle \sqrt{-g} d^4x \quad (147)$$

This result has fundamental implications for the nature of gravity. It demonstrates that the *Einstein-Hilbert action is, up to a geometric prefactor, mathematically identical to the effective action of the thermo-entropic field.*

The dynamics of macroscopic spacetime geometry are thus shown to be the direct, coarse-grained manifestation of the underlying thermodynamics of the vacuum. This provides a concrete, analytically derived physical basis for the emergent gravity paradigm, realizing the vision hinted at by Jacobson and Verlinde. The principle of least action for geometry ($\delta S_{\text{EH}} = 0$) is rigorously reinterpreted as a principle of *extremal entropic action*, with the coefficient $\frac{c^4}{4\pi G}$ acting as the fundamental geometric conversion factor between the thermodynamic and gravitational descriptions of the exact same elastic substrate.

N Formal Existence, Stability, and Scaling of Macroscopic Solitons (Black Holes)

This appendix formalizes the existence and stability of macroscopic defects (black holes) as *cavitated solitons* of the QEG substrate. The key point is that the relevant variational problem is *not* Derrick's source-free lump problem: a black hole carries a conserved macroscopic charge (the mass deformation) which fixes the long-range $1/r$ tail through a Poisson–Green law. Under fixed charge, the energy minimizer is a finite-radius cavitated configuration, and the mass–radius scaling is enforced by the yield matching to the universal Green kernel.

N.1 Exterior Poisson–Green law and the yield matching condition

Let $\phi(r)$ denote the isotropic scalar strain mode (trace/longitudinal invariant) extracted from $\mathcal{G}_{\mu\nu}$ in the static, spherically symmetric sector. In the IR-static regime the QEG closure implies a Poisson-type field equation for ϕ ,

$$-\kappa \nabla^2 \phi = \mathcal{J}, \quad (148)$$

where \mathcal{J} is the effective localized source associated with the mass deformation (ADM charge in the macroscopic

limit). For a compact source, the exterior region ($r > R$) is source-free and therefore harmonic:

$$\nabla^2 \phi = 0 \quad (r > R), \quad (149)$$

whose spherically symmetric solution is fixed by the 3D Green kernel,

$$\phi(r) = \frac{Q}{4\pi\kappa} \frac{1}{r}, \quad (r > R). \quad (150)$$

The constant Q is the conserved monopole charge measured at infinity by Gauss flux:

$$Q \equiv -\kappa \int_{S_\infty^2} \nabla \phi \cdot d\mathbf{S}, \quad (151)$$

which is invariant under smooth deformations of the interior. In QEG this monopole charge is the macroscopic mass deformation; we parametrize it as

$$Q \equiv \gamma M, \quad (152)$$

with γ fixed by the IR normalization of the gravitational channel (equivalently by matching to the Newtonian limit).

Yield matching (definition of the horizon). In QEG the interior enters a saturated non-linear phase when the scalar strain reaches a universal yield value $\phi_c = \mathcal{O}(1)$ (signature flip / saturation of volumetric rigidity). We define the horizon (phase interface) radius R by

$$\phi(R) = \phi_c. \quad (153)$$

Combining (150) and (153) gives immediately

$$R = \frac{Q}{4\pi\kappa\phi_c} = \frac{\gamma}{4\pi\kappa\phi_c} M. \quad (154)$$

Therefore the linear scaling $R \propto M$ is not an assumption: it is enforced by (i) the universal $1/r$ Green tail and (ii) a fixed yield threshold ϕ_c . This is the macroscopic analogue of soliton radii being set by the matching of a fixed-amplitude tail to a non-linear saturation scale.

N.2 Existence and energetic stability as elastic cavitation at fixed charge

We now show why this configuration is stable and why Derrick's theorem does not apply.

(i) Not a source-free lump. Derrick's theorem concerns source-free scalar lumps where one rescales the *entire* configuration $\phi(\mathbf{x}) \rightarrow \phi(\lambda\mathbf{x})$ with no conserved charge fixing a long-range tail. Here the macroscopic charge Q is fixed by the flux at infinity, hence the $1/r$ amplitude is *not* free to relax away under rescaling. The correct variational problem is minimizing the energy *under the constraint of fixed Q .*

(ii) Reduced energy functional at fixed Q . Consider the static energy

$$E[\phi] = \int d^3x \left[\frac{\kappa}{2} (\nabla\phi)^2 + V(\phi) \right], \quad V(\phi) = \frac{a}{2} \phi^2 + \frac{\lambda}{4} \phi^4, \quad (155)$$

and assume the interior saturates to $\phi \simeq \phi_c$ for $r < R$ (plastic phase), while the exterior follows the harmonic tail (150). Then the exterior gradient energy is universal:

$$E_{\text{ext}}(R; Q) = \int_R^\infty 4\pi r^2 dr \frac{\kappa}{2} \left(\frac{Q}{4\pi\kappa} \frac{1}{r^2} \right)^2 = \frac{Q^2}{8\pi\kappa} \frac{1}{R} \quad (156)$$

The interior contributes a finite core energy. For a saturated phase it is natural to parameterize it by an effective bulk energy density gap $\Delta\rho$ (or equivalently an effective

pressure/tension scale related to the Λ -sector in the main text)³:

$$E_{\text{core}}(R) \simeq \frac{4\pi}{3} R^3 \Delta\rho, \quad (157)$$

Thus, at fixed charge Q the effective one-parameter energy is

$$E(R; Q) = \frac{4\pi}{3} R^3 \Delta\rho + \frac{Q^2}{8\pi\kappa} \frac{1}{R} + 4\pi R^2 \sigma + \dots \quad (158)$$

(iii) **Finite-radius minimizer and stability.** The key point is that (158) has:

- a repulsive divergence $E_{\text{ext}} \sim Q^2/R$ as $R \rightarrow 0$ (preventing collapse),
- a growing core cost $E_{\text{core}} \sim R^3$ (preventing expansion to infinity).

Therefore $E(R; Q)$ has a global minimizer at some $R = R_*(Q) > 0$. Moreover,

$$\left. \frac{d^2 E}{dR^2} \right|_{R_*} = 8\pi\Delta\rho R_* + \frac{Q^2}{4\pi\kappa} \frac{1}{R_*^3} + 8\pi\sigma > 0, \quad (159)$$

so the extremum is strictly stable.

Compatibility with the yield definition. The variational minimizer $R_*(Q)$ and the yield-defined radius R coincide in the macroscopic regime because the interface is precisely where the non-linear saturation is reached, i.e. where the harmonic tail matches ϕ_c . Thus the stable cavitation radius is the yield radius, and the configuration is physically anchored.

N.3 Linear mass–radius scaling and the Hawking

$T \propto 1/M$ law from Green + dimensional collapse

Equation (154) already yields the exact scaling

$$R \propto M, \quad (160)$$

without appealing to dimensional arguments: it follows from the universal Green tail and a universal yield threshold.

Thermal scaling (Hawking law) from the same mechanism.

In QEG the thermo-entropic (scalar) channel obeys an analogous Poisson–Green structure in the static limit; therefore its stationary profile is also $1/r$:

$$T(r) \propto \frac{1}{r} \quad (161)$$

Evaluating at the horizon (the only distinguished macroscopic radius) gives

$$T_H \sim T(R) \propto \frac{1}{R}. \quad (162)$$

Finally, using $R \propto M$ from (154),

$$T_H \propto \frac{1}{M}. \quad (163)$$

In particular, the $1/M$ scaling is a direct and universal consequence of: (i) Poisson-type closure, (ii) the 3D Green kernel, and (iii) horizon definition as the yield matching radius.

Role of α .

The universal damping parameter α does not set the scaling of T_H ; it governs the *dissipative flux* (evaporation rate) by controlling the leakage from coherent elastic strain into the incoherent channel in the near-yield regime.

Part IV: Topology and Non-Linear Regime in Quantum-Elastic Geometry

O Covariant Description of the Filamentary Order Parameter

While the main text uses a tube tangent t_i and a transverse projector $P_{ij} = \delta_{ij} - t_i t_j$ to obtain the canonical $3 \oplus 2$ shear decomposition, it is essential to demonstrate that this construction is fully covariant and does not introduce a preferred background axis (an “aether”) that would violate Lorentz invariance.

At the effective level, a filamentary configuration is characterized by an emergent local two-plane (the worldsheet tangent plane) embedded in spacetime. Let Φ denote the projected degrees of freedom that condense in the tube. We introduce a scalar amplitude measuring “core strength”, $\varphi(x) \equiv \sqrt{\Phi(x) \cdot \Phi(x)}$. In a tube phase, φ is localized around a worldsheet and varies rapidly in transverse directions. A fully covariant unit normal to constant- φ surfaces is:

$$n_\mu \equiv \frac{\nabla_\mu \varphi}{\sqrt{\nabla_\alpha \varphi \nabla^\alpha \varphi}} \quad (164)$$

To define a timelike unit vector u^μ in a coordinate-invariant way, we use the effective stress-energy tensor of the condensed sector, $T_{eff}^{\mu\nu}$, and define u^μ as its (future-directed) timelike eigenvector:

$$T_\nu^\mu u^\nu = -\rho u^\mu, \quad u^\mu u_\mu = -1. \quad (165)$$

Next, we define the spatial tangent t^μ as the principal spacelike stress direction within the hypersurface orthogonal to u^μ . Using the spatial metric $h_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu$ and the projected stress $\Sigma_{\mu\nu} \equiv h_\mu^\alpha h_\nu^\beta T_{\alpha\beta}$, the tangent is extracted covariantly via:

$$\Sigma_{\mu\nu} t^\nu = \tau t_\mu, \quad t^\mu t_\mu = +1, \quad t^\mu u_\mu = 0. \quad (166)$$

In a stable filamentary solution, the largest principal stress direction generically aligns with the tube tangent. Finally, the covariant projector onto the transverse 2D subspace is:

$$\Pi_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu - t_\mu t_\nu, \quad \Pi_{\mu\nu} u^\nu = 0, \quad \Pi_{\mu\nu} t^\nu = 0. \quad (167)$$

Replacing (t_i, P_{ij}) by $(t^\mu, \Pi_{\mu\nu})$ yields the fully covariant analogue of the tube-adapted decomposition. Crucially, u^μ and t^μ are not new fundamental fields nor background structures; they are dynamic order parameters extracted exclusively from the non-linear QEG state in the filamentary phase, fully preserving the background-free nature of the theory.

P Isotropy-safe emergence of the shear triplet and its topological label

P.1 No-go statement: why internal labels cannot be fixed spatial directions

A necessary consistency condition of QEG is that the vacuum remain statistically isotropic. In particular, an “internal” multiplet must *not* be realized by selecting three fixed spatial axes in the vacuum. Formally, suppose one postulates an internal index $a = 1, 2, 3$ by identifying it with three fixed orthonormal spatial vectors $\hat{e}_{(a)}^i$ and defines

$$Q^a(\mathbf{x}) \equiv S_{ij}(\mathbf{x}) \hat{e}_{(a)}^i \hat{e}_{(3)}^j \quad (\text{or any fixed-axis variant}). \quad (168)$$

Then the construction introduces explicit background tensors $\hat{e}_{(a)}^i$ into the effective functional, producing invariants that depend on preferred directions. Unless these background tensors are averaged over in the vacuum measure (which would trivialize the identification), the resulting EFT breaks $SO(3)$ at the level of the vacuum and is therefore inadmissible.

The resolution used in the main text is that the relevant “axis” is *not* a vacuum datum but a collective coordinate of a localized filamentary configuration: the tube

³Optionally one may include a surface term $4\pi R^2 \sigma$ if the interface has non-negligible membrane energy; it does not change the conclusion below

tangent $t_i(\mathbf{x})$ is defined *configuration-by-configuration* and can point arbitrarily in different realizations. Consequently, any tube-adapted internal labeling is attached to a localized excitation and does not define a global anisotropic order parameter. This is the same logical mechanism by which solitons in isotropic theories may carry a local frame without inducing spontaneous symmetry breaking in the vacuum.

P.2 Shear sector decomposition induced by a filament tangent

Let S_{ij} be the symmetric traceless shear tensor defined by

$$\mathcal{G}_{ij} = \frac{1}{3}\delta_{ij}\text{tr}(\mathcal{G}) + S_{ij}, \quad S_{ij} = S_{ji}, \quad S_{ii} = 0, \quad (169)$$

so that S_{ij} carries 5 independent degrees of freedom. Given a tube configuration with a local unit tangent $t_i(\mathbf{x})$ (defined away from the core as a smooth field), introduce the transverse projector

$$P_{ij} \equiv \delta_{ij} - t_i t_j, \quad P_{ij} t_j = 0, \quad P_{ik} P_{kj} = P_{ij}. \quad (170)$$

Then the canonical tube-adapted splitting of the $\ell = 2$ shear sector is:

$$s \equiv t_i t_j S_{ij} \quad (m = 0, 1 \text{ dof}), \quad (171)$$

$$q_i \equiv P_{ik} S_{kj} t_j, \quad q_i t_i = 0 \quad (m = \pm 1, 2 \text{ dof}), \quad (172)$$

$$\tilde{S}_{ij} \equiv P_{ik} P_{j\ell} S_{k\ell}, \quad \tilde{S}_{\perp} \equiv P_{ij} \tilde{S}_{ij} = P_{ij} S_{ij}, \quad (173)$$

$$u_{ij} \equiv \tilde{S}_{ij} - \frac{1}{2} P_{ij} \tilde{S}_{\perp}, \quad u_{ij} t_j = 0, \quad u_{ii} = 0 \quad (m = \pm 2, 2 \text{ dof}). \quad (174)$$

The reconstruction identity used in the main text follows by direct substitution and tracelessness:

$$S_{ij} = s \left(t_i t_j - \frac{1}{3} \delta_{ij} \right) + (t_i q_j + t_j q_i) + u_{ij}, \quad (175)$$

which accounts for $1 + 2 + 2 = 5$ independent components.

P.3 Triplet extraction in a co-moving tube frame and the guaranteed symmetry

Choose a local orthonormal triad $\{e_{(1)}, e_{(2)}, e_{(3)}\}$ adapted to the tube,

$$e_{(3)} \equiv t, \quad e_{(\alpha)} \cdot t = 0 \quad (\alpha = 1, 2), \quad e_{(1)} \cdot e_{(2)} = 0, \quad |e_{(1)}| = |e_{(2)}| \quad (176)$$

Define the three real low-cost components

$$Q^1 \equiv s = S_{ij} t_i t_j, \quad Q^2 \equiv S_{ij} e_{(1)i} t_j, \quad Q^3 \equiv S_{ij} e_{(2)i} t_j. \quad (177)$$

Equivalently, in a transverse orthonormal basis one may write $Q^a = (s, q_1, q_2)$ with $Q^a Q^a = s^2 + q_i q_i$.

The tube geometry guarantees a local redundancy under rotations of the transverse basis:

$$(e_{(1)}, e_{(2)}) \mapsto (\cos \theta e_{(1)} + \sin \theta e_{(2)}, -\sin \theta e_{(1)} + \cos \theta e_{(2)}), \quad (178)$$

which is a local $SO(2)$ acting on (Q^2, Q^3) while leaving Q^1 invariant. Hence the *guaranteed* invariants in a tube EFT are $SO(2)$ -invariant combinations such as

$$(Q^1)^2, \quad Q_{\perp}^2 \equiv (Q^2)^2 + (Q^3)^2, \quad \text{and (optionally)} \quad (Q^1)^2 Q_{\perp}^2, \quad (179)$$

Any larger symmetry acting on $a = 1, 2, 3$ (e.g. an accidental $O(3)$) can only arise approximately from the Hessian of the restricted potential in the light subspace and is not enforced by geometry.

P.4 Topological sector label from the transverse doublet

The transverse pair (Q^2, Q^3) defines an effective complex field in the tube-adapted frame,

$$Q^2 + iQ^3 \equiv \rho_Q e^{i\vartheta}. \quad (180)$$

Assume a filamentary configuration in which outside the core $\rho_Q \rightarrow \rho_{\infty} \neq 0$ on transverse circles. Then the map from a transverse loop $\partial\Sigma_{\perp} \simeq S^1$ into the phase circle S^1 is classified by

$$n \equiv \frac{1}{2\pi} \oint_{\partial\Sigma_{\perp}} \nabla\vartheta \cdot d\ell \in \mathbb{Z}. \quad (181)$$

Fixing n prevents relaxation to the trivial vacuum sector everywhere and forces the existence of a localized core (or defect set) where either ρ_Q vanishes or ϑ becomes ill-defined. This provides the concrete endogenous realization of the ‘‘sector constraint’’ invoked in the flux-tube variational argument of Appendix Q.

Remark P.1 (Relation to \mathbb{Z}_2 projective sectors). *In the presence of director-like projectivization (e.g. an \mathbb{RP}^2 target for a higher-order order parameter), a minimal \mathbb{Z}_2 obstruction may coexist with the above \mathbb{Z} winding. Depending on the embedding of the complex transverse doublet into the full projective manifold, one may realize the \mathbb{Z}_2 class as $n \bmod 2$ or as an independent label carried by another component of the order parameter.*

P.5 Minimal low-energy hierarchy and the mass gap of the $m = \pm 2$ transverse sector

The decomposition $S_{ij} \rightarrow (s, q_i, u_{ij})$ naturally suggests a hierarchy: the $m = \pm 2$ sector u_{ij} controls transverse shape/biaxiality of the tube cross-section and generically acquires a large effective mass (gap) from the restricted curvature of $V(\mathcal{G})$ along these directions.

To formalize this shape gap, consider a minimal invariant potential depending on $I_2 \equiv S_{ij} S_{ij}$ and $I_3 \equiv S_{ij} S_{jk} S_{ki}$:

$$V_{eff}(S) = \frac{\alpha}{2} I_2 + \frac{\beta}{3} I_3 + \frac{\lambda}{4} I_2^2 + \dots \quad (\lambda > 0). \quad (182)$$

In the adapted basis, the quadratic invariant splits as $I_2 = c_s s^2 + c_q q_i q_i + c_u u_{ij} u_{ij}$ (with $c_{\bullet} > 0$). Expanding around a tube background $S^{(0)}$ that minimizes transverse anisotropy, the Hessian for the u -sector contains an effective mass term of the form:

$$m_u^2(\rho) \sim \left. \frac{\partial^2 V_{eff}}{\partial u^2} \right|_{S^{(0)}} \sim \alpha c_u + \lambda c_u I_2^{(0)}(\rho) + \dots \quad (183)$$

This demonstrates that u_{ij} deforms the tube cross-section and therefore costs substantial energy because it disrupts the transverse minimizer that defines the tension σ . A sufficient condition for a hierarchy is that V_{eff} weights anisotropic distortions more strongly than the light-sector distortions in the tube background, yielding $m_u \gg m_{light}$.

The dominant low-energy filament dynamics then cleanly resides in the triplet sector $Q^a = (s, q_1, q_2)$, while u_{ij} enters only through subleading renormalizations, allowing us to safely integrate it out to obtain the worldsheet EFT used in the main text.

Q Flux-Tube Energetics and Linear Confinement Scaling

A central claim of QEG is that confinement-like behavior emerges as a generic variational outcome in the non-linear medium. To formalize this, we evaluate the static energy functional for a minimal projected mode Q with a non-linear substrate potential $V(Q) = \frac{1}{2} m_Q^2 Q^2 + \frac{\lambda_Q}{4} Q^4$.

For a dispersed transmission of a sector datum Q_{sec} over a spherical region of radius R , the energy scales

as $E_{sph}(R) \sim c_1 Q_{sec}^2/R + c_2 \Delta V R^3$. In the non-linear regime, the volumetric cost ($\Delta V R^3$) heavily disfavors large R , preventing unconstrained spreading.

By contrast, if the datum is transmitted along a codimension-2 filament of transverse radius ρ^* , translational invariance along the filament axis z yields an energy strictly proportional to length, $E_{tube}(r) \simeq \sigma r$, where the string tension σ is the integrated transverse energy:

$$\sigma = \int d^2x_{\perp} \left[\frac{Z_f}{2} (\nabla_{\perp} f)^2 + \frac{Z_{\theta}}{2} f^2 (\nabla_{\perp} \theta)^2 + V(f) \right] \quad (184)$$

Under a fixed topological winding constraint $n = \frac{1}{2\pi} \oint \nabla \theta \cdot dl \in \mathbb{Z}$, constrained minimization dynamically selects an optimal finite core radius ρ^* and a constant string tension σ . A dimensional estimate balancing the winding gradient cost and the potential core cost ΔV rigorously yields:

$$\rho^* \sim \left(\frac{Z_{\theta} f_c^2 n^2}{\Delta V} \right)^{1/4}, \quad \sigma \sim |n| f_c \sqrt{Z_{\theta} \Delta V} \quad (185)$$

Therefore, linear scaling ($E \propto r$) is not a phenomenological postulate borrowed from QCD, but the mathematically unavoidable variational minimizer for a non-linear elastic substrate transmitting a conserved topological charge.

R Computable Defect Spectrum and Sturm-Liouville Generational Structure

In the main text, the different particle generations (e.g., electron, muon, tau) are posited not as independent fundamental fields, but as discrete radial excitations of the same underlying topological defect. This appendix formally proves that such a discrete spectrum is a necessary consequence of the QEG solitonic framework.

We consider the minimal particle-like defect represented by the symmetric *hedgehog ansatz* for the projective order parameter:

$$\hat{n}(x) = \sin \alpha(r) \hat{r} + \cos \alpha(r) \hat{e}_3 \quad (186)$$

with boundary conditions $\alpha(0) = \pi$ and $\alpha(\infty) = 0$ fixing the topological charge $Q = \pm 1$. The static energy functional for this regularized core reduces to a one-dimensional integral over the radial profile:

$$E[\alpha, \chi, \mathcal{T}] = 4\pi \int_0^{\infty} dr \left[f(\chi, \mathcal{T}) (r^2 \alpha'^2 + 2 \sin^2 \alpha) + r^2 \left(\frac{1}{2} \chi'^2 + V(\chi) + \dots \right) \right] \quad (187)$$

A stationary ground state solution $\alpha_0(r)$ exists by minimizing this energy. To find the excitation spectrum, we consider small time-dependent perturbations about this stationary core:

$$\alpha(r, t) = \alpha_0(r) + \delta\alpha(r) e^{-i\omega t} \quad (188)$$

Linearizing the Euler-Lagrange equations around $\alpha_0(r)$ reduces the dominant sector to a classic *Sturm-Liouville* eigenvalue problem for the fluctuation $\delta\alpha$:

$$-\frac{d}{dr} \left(p(r) \frac{d}{dr} \delta\alpha \right) + q(r) \delta\alpha = \omega^2 w(r) \delta\alpha \quad (189)$$

where $p(r)$, $q(r)$, and $w(r) > 0$ are completely determined by the geometry of the stationary core.

Under standard regularity conditions at $r = 0$ and normalizability as $r \rightarrow \infty$, Sturm-Liouville theory guarantees that this operator admits a strictly discrete, well-ordered spectrum of eigenvalues:

$$0 < \omega_0 < \omega_1 < \omega_2 < \dots < \omega_{cont} \quad (190)$$

Each eigenvalue ω_k defines a distinct, normalizable vibrational state of the exact same topological defect, distinguished only by the number of radial nodes (k). Thus, the existence of distinct families ($k = 0 \rightarrow e$, $k = 1 \rightarrow \mu$, $k = 2 \rightarrow \tau$) is a rigorous theorem of differential equations applied to the non-linear QEG substrate, firmly eliminating the need to postulate generations as independent elementary species.

S Nuclear saturation as a quantitative consistency check: closing the quartic scale and an Unruh-type interpretation

This section provides a falsifiable numerical check of the single-substrate invariant potential paradigm stated in the Main paper. The logic is: (i) fix the *shape* of the substrate potential by covariant invariants and the single-potential principle; (ii) fix the overall scale by the vacuum energy density ρ_{vac} ; (iii) fix the quartic saturation scale λ by the QEG dimensional-collapse prescription; and (iv) demand that the resulting saturation pressure matches the empirical nuclear saturation density.

S.1 Invariant potential normalized by the vacuum energy density

We adopt the covariant minimal invariant completion (for the trace sector; the shear sector proceeds analogously by pullback)

$$V(G) = \rho_{vac} f(I_1, I_2^{sh}), \quad I_1 = \text{tr}(E), \quad E = \log(G_*^{-1} G), \quad (191)$$

so that I_1 and I_2^{sh} are dimensionless and f is a dimensionless scalar.

Restricting to the pure trace mode,

$$\phi := I_1 = \text{tr}(E) \quad (\text{dimensionless}), \quad (192)$$

the modal pullback takes the Landau–Ginzburg form

$$V_{\chi}(\phi) = \rho_{vac} \left[\frac{a}{2} \phi^2 + \frac{\lambda}{4} \phi^4 \right], \quad a, \lambda \text{ dimensionless.} \quad (193)$$

In this normalization the coefficients (a, λ) encode only the *shape* (curvatures and saturation) of the dimensionless potential, while ρ_{vac} sets the absolute physical scale.

S.2 Saturation density induced by the quartic completion

The quartic completion introduces a characteristic saturation energy density. In dimensionless Landau theory the quartic term controls the saturation pressure once the quadratic curvature is fixed. Accordingly, a natural estimate for the characteristic saturation density is

$$\rho_{sat} \sim \rho_{vac} \frac{a^2}{\lambda}, \quad (194)$$

where the ratio a^2/λ is invariant under rescalings of ϕ once the dimensionless normalization (193) is fixed by the geometric definition (192).

We now impose the empirical requirement that the non-linear saturated core associated with the confined regime has an energy density of nuclear order,

$$\rho_{sat} \approx \rho_{nuc}, \quad \rho_{nuc} \sim 10^{17} \text{ kg/m}^3. \quad (195)$$

Inserting (194) yields the dimensionless closure condition

$$\boxed{\frac{a^2}{\lambda} \approx \frac{\rho_{nuc}}{\rho_{vac}}}. \quad (196)$$

With $\rho_{vac} \approx 5.94 \times 10^{-27} \text{ kg/m}^3$ as derived in [190], one has

$$\frac{\rho_{nuc}}{\rho_{vac}} \approx \frac{2.14 \times 10^{17}}{5.94 \times 10^{-27}} \approx 3.6 \times 10^{43}. \quad (197)$$

where we adopt $\rho_{\text{nuc}} \approx 2.14 \times 10^{17} \text{ kg/m}^3$ as the nuclear saturation benchmark, which is consistent with the lower bound of interior densities measured in heavy nuclei as determined by electron scattering experiments [28, 79, 128, 200], and provides a unitary curvature coefficient $a \approx 1$ in our invariant potential.

S.3 Dimensional-collapse prescription for the quartic coupling

A central input of QEG is that the quartic saturation coefficient is not a free EFT parameter but is fixed by the dimensional-collapse prescription. Specifically, we take

$$\lambda := \frac{\hbar}{4\pi c L_{\text{ref}}^2}, \quad (198)$$

which is dimensionless in the QEG collapse bookkeeping. Choosing $L_{\text{ref}} = 1 \text{ m}$ as the reference scale gives the following.

$$\lambda \approx 2.8 \times 10^{-44}. \quad (199)$$

Combining (196), (197), and (199) yields

$$a \approx \sqrt{\lambda \frac{\rho_{\text{nuc}}}{\rho_{\text{vac}}}} \approx \sqrt{(2.8 \times 10^{-44})(3.6 \times 10^{43})} \approx 1.004 \quad (200)$$

Key outcome (naturalness check).

The invariant completion and the dimensional-collapse prescription imply that enforcing nuclear saturation does not require any fine tuning: the remaining dimensionless curvature coefficient is of order unity,

$$a \simeq \mathcal{O}(1) \quad (201)$$

This constitutes a nontrivial quantitative consistency check of the minimal invariant potential paradigm.

S.4 Unruh-type structural interpretation

The Unruh temperature is characterized by the universal action block

$$k_B T_U = \frac{\hbar}{2\pi c} a_{\text{proper}}. \quad (202)$$

Independently of whether one interprets the saturation mechanism as horizon-like, the dimensional-collapse prescription (198) exhibits the same structural block $\hbar/(2\pi c)$:

$$\lambda = \frac{\hbar}{4\pi c L_{\text{ref}}^2} = \frac{1}{2} \left(\frac{\hbar}{2\pi c} \right) \frac{1}{L_{\text{ref}}^2}. \quad (203)$$

Hence the quartic saturation strength may be viewed as an ‘‘Unruh-type’’ action quantum evaluated at an intrinsic geometric curvature scale L_{ref}^{-2} . This is not asserted as an identity with the Unruh effect itself, but as a structural resonance: the same universal factor $\hbar/(2\pi c)$ governing horizon thermality also controls the normalization of the non-linear saturation in the QEG collapse prescription.

Interpretive summary.

In the normalization (191)–(193), the vacuum energy density ρ_{vac} sets the absolute scale of the substrate potential, while the dimensionless coefficients (a, λ) fix its shape. The collapse prescription (198) closes λ , and nuclear saturation (195) then yields $a \simeq 1.004$. Therefore, the minimal covariant invariant completion produces a quantitatively consistent non-linear saturation without introducing sector-dependent free parameters or unnatural dimensionless numbers.

T Geometric Realization of Electroweak Symmetry Breaking from Volumetric Stiffness

In the Standard Model, the generation of mass and electroweak symmetry breaking are implemented through the ad-hoc introduction of an independent scalar doublet field acquiring a non-zero vacuum expectation value (VEV). In Quantum-Elastic Geometry (QEG), no additional fundamental scalar species is postulated. Instead, the Higgs-like mass generation mechanism emerges intrinsically from the isotropic volumetric trace mode of the spatial deformation tensor.

T.1 The Volumetric Trace Mode as Geometric Order Parameter

Let the spatial part of the unified deformation tensor be denoted by \mathcal{G}_{ij} . Its unique rotationally invariant scalar projection is the trace:

$$\chi(x) := \text{tr}(\mathcal{G}_{ij}(x)). \quad (204)$$

The field χ represents the pure isotropic volumetric strain (compression or dilation) of the spacetime substrate. It is crucial to distinguish this geometric field from the trace of the source tensor, $\Theta(x) := \text{tr}(J_{ij}(x))$, which corresponds to the thermo-entropic source density. The two are related through the unified compliance relation but are not identical.

For the elastic vacuum to remain globally stable—neither collapsing into a singularity nor expanding with infinite compliance—the effective potential restricted to the trace sector, $V_\chi(\chi) := V(\mathcal{G}_{ij})|_{\text{trace sector}}$, must admit a non-trivial global minimum. This requires:

$$\left. \frac{dV_\chi}{d\chi} \right|_{\chi=\chi_0} = 0, \quad \left. \frac{d^2V_\chi}{d\chi^2} \right|_{\chi=\chi_0} > 0 \quad (205)$$

The non-zero value χ_0 defines the spontaneous volumetric stiffness of the vacuum substrate. In this sense, χ_0 acts as a geometric order parameter.

Importantly, in the absence of volumetric locking ($\chi_0 = 0$), transport modes belonging to the same modal manifold are degenerate under internal rotations within the transport subspace. The selection $\chi_0 \neq 0$ lifts this degeneracy by fixing a preferred volumetric background. Thus, spontaneous stiffness selection reorganizes the modal spectrum in a way structurally analogous to spontaneous symmetry breaking.

T.2 Induced Vector Mass from Volumetric Stiffness

The transport (vectorial) modes correspond to the projected components of \mathcal{G}_{0i} . In the quadratic elastic regime, the unified action generically contains stiffness-weighted terms coupling the volumetric and transport sectors. Schematically:

$$\mathcal{L} \supset \frac{1}{2} \kappa \chi \mathcal{G}_{0i} \mathcal{G}^{0i} \quad (206)$$

Expanding the trace mode around its geometric VEV, $\chi(x) = \chi_0 + \delta\chi(x)$, the Lagrangian becomes:

$$\mathcal{L} \supset \frac{1}{2} \kappa \chi_0 \mathcal{G}_{0i} \mathcal{G}^{0i} + \frac{1}{2} \kappa \delta\chi \mathcal{G}_{0i} \mathcal{G}^{0i} \quad (207)$$

The first term generates an effective Proca-type mass for the transport modes ($m_V^2 \propto \kappa \chi_0$). The second term describes the dynamical coupling between volumetric fluctuations and vector bilinears. This perfectly reproduces the structural pattern of the Higgs mechanism for gauge bosons, where the propagating scalar excitation is the geometric trace fluctuation $\delta\chi$ itself.

T.3 Geometric Mass Generation for Topological Defects

In QEG, fermions are modeled as localized, finite-energy topological defects Φ . The energetic cost of sustaining a local defect is determined by the substrate's resistance to local gradients. The minimal covariant coupling between the defect's gradient field and the vacuum's volumetric stiffness takes the form:

$$\mathcal{L}_{int} \supset f(\chi) \|\nabla\Phi\|^2 \quad (208)$$

where $f(\chi)$ is a positive-definite stiffness modulation function. Expanding around the VEV, χ_0 , the interaction Lagrangian yields:

$$\mathcal{L}_{int} \supset f(\chi_0) \|\nabla\Phi\|^2 + f'(\chi_0) \delta\chi \|\nabla\Phi\|^2 + \dots \quad (209)$$

- The first term, evaluated at the uniform background stiffness $f(\chi_0)$, defines the rigid rest-mass energy of the defect (analogous to the Dirac mass term $m\bar{\psi}\psi$).
- The second term describes the dynamical coupling between the defect and the local fluctuations of the volumetric stiffness (analogous to the fundamental Yukawa coupling $y\bar{\psi}\psi H$).

Thus, the geometric stiffness of the QEG vacuum intrinsically generates both the rest mass of the topological defects and their interaction with the scalar trace field.

U Topological Interpretation: Spin, Chirality, and Emergent Gauge Structure

The non-linear defect program provides quantitative closure relations for mass scales. We now address three structural questions within the same geometric framework: the origin of spin, the emergence of chirality in the massive torsional sector, and the status of gauge symmetry. No additional *fundamental* matter fields are introduced. All statements below rely on: (i) a symmetric tensor substrate $G_{\mu\nu}$, (ii) finite-action boundary conditions, (iii) projective/orientational topology of the induced order parameters, and (iv) modal projection structure.

U.1 Spin from projective and orientational topology

Projective vacuum manifold (director structure).

Because $G_{\mu\nu}$ is symmetric, any principal-axis order parameter extracted from it is an unoriented axis (director),

$$\vec{n} \sim -\vec{n}, \quad (210)$$

so the minimal target for such orientational data is the projective space

$$\mathcal{M}_{\text{dir}} \simeq \mathbb{RP}^2 = S^2/\mathbb{Z}_2, \quad \pi_1(\mathbb{RP}^2) \cong \mathbb{Z}_2. \quad (211)$$

Where topology enters (finite-action sector).

On a fixed time-slice $\Sigma_t \simeq \mathbb{R}^3$, finite-action boundary conditions impose $G_{\mu\nu}(x) \rightarrow G_{\mu\nu}^{(0)}$, $\nabla G \rightarrow 0$ as $|x| \rightarrow \infty$, which compactifies spatial infinity:

$$\mathbb{R}^3 \cup \{\infty\} \simeq S^3.$$

Localized defects are classified by restricting the induced order parameter $\Phi(G)$ to *linking spheres/loops* around the defect core: S^2 for point-like cores and S^1 for line-like cores. In particular, transporting the director around a closed loop $\gamma \simeq S^1$ linking a core defines a \mathbb{Z}_2 homology class

$$\omega(\gamma) := [\Phi|_\gamma] \in \pi_1(\mathbb{RP}^2) \cong \mathbb{Z}_2. \quad (212)$$

The nontrivial class $\omega = 1$ implies that a 2π rotation corresponds to the nontrivial loop in the projective target (a sign flip in a lift to S^2), while a 4π rotation is trivial. This yields the defining *spinorial holonomy* property without postulating fundamental spinors.

Orientational refinement ($SO(3)$).

In the torsional sector one may refine the order parameter from a director to a full triad/frame, i.e. a map into $SO(3)$, for which

$$\pi_1(SO(3)) = \mathbb{Z}_2. \quad (213)$$

This is the same double-cover mechanism: $SU(2)$ is the universal cover of $SO(3)$ and the nontrivial \mathbb{Z}_2 loop encodes 4π periodicity.

Fermionic quantization (Finkelstein–Rubinstein criterion).

Spinorial holonomy is a *classical* topological statement. Fermionic *statistics* is a property of quantization on the defect configuration space. Let \mathcal{C} denote the space of finite-action defect configurations in a fixed topological sector, modulo redundancies, and let $\gamma_{2\pi}$ be the loop in \mathcal{C} induced by a rigid 2π spatial rotation. If $\gamma_{2\pi}$ generates a \mathbb{Z}_2 factor in $\pi_1(\mathcal{C})$, the FR construction prescribes defining the wavefunctional on the universal cover and imposing the sign character

$$\Psi(\gamma \cdot \Phi) = \chi(\gamma) \Psi(\Phi), \quad \chi : \pi_1(\mathcal{C}) \rightarrow \{\pm 1\}, \quad \chi(\gamma_{2\pi}) = -1. \quad (214)$$

Under this standard condition, 2π rotations act as -1 and exchange of identical solitons (in the indistinguishable sector) acquires fermionic statistics. In QEG, the same protected \mathbb{Z}_2 structure that appears in the orientational sector provides the natural topological input for (214).

U.2 Gyromagnetic ratio from torsional kinematics

Projected torsional sector.

Define the torsional/transport potential by projection

$$A_\mu = \mathcal{P}_{\mu\alpha\beta}^{(V)} G^{\alpha\beta}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (215)$$

In a quasi-static rest frame,

$$\mathbf{B} = \nabla \times \mathbf{A}. \quad (216)$$

Half-curl identity (model-independent).

Continuum kinematics gives the universal relation between angular velocity and vorticity. For velocity v_i one has

$$\Omega_i = \frac{1}{2} \epsilon_{ijk} \partial_j v_k, \quad (217)$$

where the factor $1/2$ is purely kinematic (antisymmetric part of the velocity gradient).

Constitutive coupling and $g = 2$ at leading order.

Introduce a constitutive normalization (defect rest frame)

$$\mathbf{\Omega}_{\text{ext}} = \frac{\kappa_\Omega}{2} \mathbf{B}_{\text{ext}}, \quad (218)$$

where κ_Ω is fixed by how A_μ is normalized relative to $G_{\mu\nu}$. Assume the minimal interaction of a localized spin-carrying torsional defect with external rotation:

$$H_{\text{int}} = -\mathbf{S} \cdot \mathbf{\Omega}_{\text{ext}}. \quad (219)$$

Then, using (218),

$$\Delta E = -\boldsymbol{\mu} \cdot \mathbf{B}_{\text{ext}} \Rightarrow \boldsymbol{\mu} = \frac{\kappa_\Omega}{2} \mathbf{S}. \quad (220)$$

Comparing with the standard parametrization

$$\boldsymbol{\mu} = g \frac{q_{\text{eff}}}{2m_{\text{eff}}} \mathbf{S} \quad (221)$$

yields

$$g = \frac{\kappa_\Omega m_{\text{eff}}}{q_{\text{eff}}}. \quad (222)$$

Thus $g = 2$ follows at leading order provided the torsional normalization satisfies

$$\kappa_\Omega = 2 \frac{q_{\text{eff}}}{m_{\text{eff}}}, \quad (223)$$

which is the natural identification when the same torsional projection that defines A_μ also defines the defect's effective charge-to-inertia ratio. The factor of two originates in the universal kinematic $1/2$ in (217), not in Dirac algebra.

U.3 Quantized circulation and effective charge sectors

For finite-action localized configurations, single-valuedness of the effective torsional phase (or equivalently, integrality of holonomy in the reduced $S^1/U(1)$ transport phase) motivates the semiclassical circulation condition

$$\oint_{\gamma} A_{\mu} dx^{\mu} = 2\pi n\hbar, \quad n \in \mathbb{Z}, \quad (224)$$

on loops γ linking a defect core. In this sense, discrete charge sectors arise from closure/holonomy requirements of the projected transport mode.

U.4 Gauge redundancy as projection and frame freedom

Abelian gauge redundancy from projection.

Since A_{μ} is defined only through a projector acting on $G_{\mu\nu}$, its representative is not unique:

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\Lambda, \quad (225)$$

so gauge invariance is a redundancy of the projected description; physical observables are projector-invariant objects such as $F_{\mu\nu}$ and integrated fluxes.

Non-Abelian gauge structure from filamentary geometry.

In the non-linear filamentary regime, the shear sector admits an internal multiplet Q^a defined with respect to a local adapted frame along the filament worldsheet Σ . Local frame changes act as

$$Q^a(\sigma) \rightarrow Q'^a(\sigma) = R^a{}_b(\sigma) Q^b(\sigma), \quad R(\sigma) \in G_{\text{int}}, \quad (226)$$

where G_{int} is the induced internal rotation group (minimal completion determined by the retained modes). Because naive derivatives do not transform covariantly, consistent transport requires introducing a connection

$$D_{\alpha}Q^a = \partial_{\alpha}Q^a + \mathcal{A}_{\alpha}{}^a{}_b Q^b, \quad \mathcal{A}_{\alpha} \rightarrow R\mathcal{A}_{\alpha}R^{-1} - (\partial_{\alpha}R)R^{-1}, \quad (227)$$

with curvature

$$\mathcal{F}_{\alpha\beta} = \partial_{\alpha}\mathcal{A}_{\beta} - \partial_{\beta}\mathcal{A}_{\alpha} + [\mathcal{A}_{\alpha}, \mathcal{A}_{\beta}]. \quad (228)$$

A minimal gauge-invariant dynamical completion is the Yang–Mills–type term

$$\mathcal{L}_{\text{gauge}} = \frac{\kappa}{4} \text{Tr}(\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta}), \quad (229)$$

interpreted as the elastic cost of twisting/bending the internal frame along the filament. (The existence of the connection is forced by covariance; the specific choice of $\text{Tr}(F^2)$ is the minimal local invariant.)

U.5 Chirality from non-linear torsional structure

Parity symmetry holds in the linear homogeneous vacuum. In non-linear cores/tubes, a composite pseudoscalar chiral order parameter may be built from existing filament variables; a minimal geometric seed is the twist density

$$\Theta(\Phi) \sim \epsilon^{ijk} t_i \partial_j t_k + \dots, \quad (230)$$

which is parity-odd and localized where filament geometry varies.

A minimal covariant parity-odd completion of the torsional sector is

$$\mathcal{L}_{\text{odd}} = \frac{\xi}{4} \Theta(\Phi) F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}. \quad (231)$$

Varying yields the modified Proca equation

$$\partial_{\nu} F^{\nu\mu} + m_V^2 A^{\mu} = J^{\mu} - \xi(\partial_{\nu}\Theta)\tilde{F}^{\nu\mu}. \quad (232)$$

Hence parity-odd response is *inactive* when Θ is constant, and becomes active only where $\partial_{\mu}\Theta \neq 0$ (cores/tubes/walls), producing helicity-dependent dynamics (birefringence) in those regions. Strong-gradient profiles may in principle yield helicity-dependent attenuation or gapping, but establishing full chiral projection requires solving the corresponding mode-matching problem across a core/wall profile $\Theta(x)$.

U.6 Structural synthesis

Within QEG:

- Spinorial holonomy arises from \mathbb{Z}_2 topology of projective/orientational order parameters.
- Fermionic statistics follows *if* the relevant defect configuration space admits the FR \mathbb{Z}_2 loop.
- $g = 2$ arises at leading order from the universal half-curl identity plus the torsional constitutive normalization.
- Discrete charge sectors arise from circulation/holonomy closure of the transport phase.
- Abelian gauge redundancy is projection freedom; non-Abelian gauge structure is induced by filament-frame transport.
- Chirality admits a robust covariant seed via $\Theta(\Phi)F\tilde{F}$, localized to non-linear regions with $\partial\Theta \neq 0$.

All these features derive from a single symmetric tensor substrate with finite-action boundary conditions and modal decomposition.

V Topological Conjugation and Antimatter

V.1 Topological Defects of the Elastic Field

Within the Quantum Elastic Geometry (QEG) framework, particle-like excitations arise as localized topological defects of the deformation tensor field $\mathcal{G}_{\mu\nu}$. These defects correspond to non-trivial configurations of the elastic vacuum substrate and cannot be removed by continuous deformations of the field.

Let \mathcal{M} denote the manifold of admissible configurations of the deformation field. The topological properties of \mathcal{M} determine the possible classes of stable defects. In particular, localized excitations correspond to non-trivial mappings of closed surfaces surrounding the defect core into the order-parameter manifold \mathcal{M} .

Such configurations behave as solitonic objects whose stability is protected by topological invariants rather than energetic barriers.

V.2 Fundamental Group and Defect Holonomy

The topology of the configuration space is characterized by the fundamental group

$$\pi_1(\mathcal{M}), \quad (233)$$

which classifies closed loops surrounding the defect core. For a closed path γ encircling a defect, the parallel transport of the internal elastic degrees of freedom defines a holonomy

$$h(\gamma) \in \pi_1(\mathcal{M}). \quad (234)$$

This holonomy measures the net torsional circulation accumulated around the defect and represents a discrete invariant of the configuration. Reversing the orientation of the loop corresponds to the inverse element

$$h(\gamma)^{-1}. \quad (235)$$

Therefore, defects can naturally appear in conjugate sectors related by inversion of the topological orientation.

V.3 Homotopy Charge of Elastic Defects

Stable localized defects are classified by the second homotopy group

$$\pi_2(\mathcal{M}), \quad (236)$$

which characterizes mappings from a closed surface surrounding the defect to the configuration manifold. The associated topological charge can be written formally as

$$Q \in \pi_2(\mathcal{M}), \quad (237)$$

and remains invariant under continuous deformations of the field. This charge can also be expressed through a

circulation invariant of the antisymmetric sector of the deformation field,

$$Q = \kappa \oint_{\partial\Sigma} \text{Rot}(\mathcal{G}_{\mu\nu}) \cdot d\mathbf{l}, \quad (238)$$

where Σ is a closed surface enclosing the defect core and κ is a normalization constant determined by the elastic coupling scale of the vacuum. The sign of Q encodes the orientation of the torsional circulation associated with the defect.

V.4 Topological Conjugation and Antimatter

Because the topological charge depends on the orientation of the defect circulation, the theory naturally admits conjugate configurations corresponding to

$$Q \rightarrow -Q. \quad (239)$$

If $\Psi(\mathcal{G})$ denotes a defect configuration carrying charge Q , the conjugate configuration $\bar{\Psi}$ is obtained by reversing the orientation of the torsional circulation of the deformation field. Importantly, the elastic energy density of the substrate depends only on the quadratic magnitude of the deformation tensor,

$$\mathcal{E}_{elastic} \propto \text{Tr}(\delta\mathcal{G}^2), \quad (240)$$

and is therefore invariant under orientation reversal. As a result, conjugate defects possess identical positive energy and thus identical inertial and gravitational mass. Within the QEG framework, matter and antimatter can therefore be interpreted geometrically as defects with opposite topological orientation but identical elastic energy.

V.5 Topological Annihilation

Consider two defects carrying opposite topological charges Q and $-Q$. When the defects approach each other, the combined deformation field can be written as

$$\mathcal{G}_{tot} = \mathcal{G}_{mat} + \mathcal{G}_{anti}. \quad (241)$$

Because the torsional components have opposite orientation, they cancel when the defect cores overlap. The configuration can then continuously relax toward the undeformed vacuum state

$$\mathcal{G}_{tot} \rightarrow \mathcal{G}_0. \quad (242)$$

The elastic energy stored in the localized deformation is released during this relaxation process and is converted into propagating oscillatory modes of the elastic substrate,

$$\Delta E = \int (\mathcal{G}_{mat}^2 + \mathcal{G}_{anti}^2) dV \rightarrow \sum_i \hbar\omega_i. \quad (243)$$

This provides a geometric interpretation of annihilation processes as the relaxation of localized elastic strain into propagating transverse modes.

Remark V.1 (Relation to CPT symmetry). *In relativistic local field theories satisfying the assumptions of Lorentz invariance and locality, the CPT theorem guarantees the existence of a conjugate state for every particle state. Within the QEG framework, this symmetry acquires a geometric interpretation. The CPT-related state corresponds to the topologically conjugate defect obtained by reversal of the torsional circulation of the deformation field. Thus, the existence of antimatter follows both from the general principles of relativistic quantum field theory and from the topological structure of the elastic deformation field.*

V.6 Possible Cosmological Origin of Matter–Antimatter Asymmetry

Although the field equations admit symmetric solutions with charges Q and $-Q$, cosmological evolution may dynamically select one orientation. During the early thermo-entropic expansion phase of the universe, the elastic substrate underwent highly non-linear deformations. Infinitesimal primordial fluctuations in the torsional sector could have biased the formation rate of defects toward one orientation.

This situation is analogous to symmetry-breaking phenomena in elastic media, where a symmetric configuration becomes unstable and the system selects a particular deformation channel. Once a dominant orientation emerges, the background deformation field may suppress the nucleation of defects with the opposite orientation. In this picture, the observed matter–antimatter asymmetry could result from a spontaneous topological selection during cosmological evolution.

A quantitative description of this mechanism requires a detailed analysis of the non-linear dynamics of defect nucleation in the expanding elastic substrate and is left for future work.

Part V: Foundational Quantum Mechanics of Quantum-Elastic Geometry

W Topological Non-Factorization and the Origin of Quantum Entanglement

Quantum entanglement is often presented in standard literature as a puzzling dynamical phenomenon: correlations apparently established between distant subsystems in a manner incompatible with classical separability. Within Quantum-Elastic Geometry (QEG), however, entanglement is not an anomaly, nor does it require "spooky action at a distance." It is a strict structural consequence of the fundamental ontology of the theory.

The foundational postulate of QEG is that physical reality is not built from independent local objects, but from a single continuous elastic substrate encoded by a unified deformation tensor field $G_{\mu\nu}(x)$. The physically meaningful quantity is not the local density alone, but its irreversible accumulation into action:

$$S[G; \Omega] = \int_{\Omega} \mathcal{L}(G, \partial G) d^4x \quad (244)$$

Quantum coherence is governed by the path-integral phase weight $\mathcal{Z} \sim \int \mathcal{D}G \exp(iS/\hbar)$. Because the action is intrinsically global and spans a continuous elastic hypervolume, its accumulation generically obstructs any exact mathematical decomposition into independent local subsystems. Entanglement is the direct geometric imprint of this continuous elastic obstruction.

W.1 The Heisenberg Precursor: The Minimal Coherent Action Cell

In QEG, quantum structure originates from the fact that the vacuum supports accumulation only in discrete, coherent geometric units. This provides a natural, mechanical reinterpretation of the Heisenberg uncertainty principle ($\Delta x \Delta p \geq \hbar/2$).

In QEG, localization to a spatial interval Δx corresponds to artificially confining the deformation support of an excitation, while Δp measures the unavoidable stress induced in the conjugate coherent transport channel (flux response). Thus, the Heisenberg bound expresses a fundamental elastic coherence constraint: no admissible history can be confined below the minimal coherent accumulation cell (the quantum of action). Exact geometric truncation of a coherent history immediately implies the loss of boundary closure, destroying stable transport.

This one-body geometric obstruction dictates the multi-body generalization: if a coherent history cannot be sharply truncated at a single boundary, then global accumulation cannot generically factorize into independent regional accumulations.

W.2 Interface Obstruction: Why the Action Does Not Split

Let an observation domain $\Omega = A \cup B$ be partitioned with a mathematical interface $\Sigma = \partial A \cap \partial B$. Elastic propagation inherently requires spatial and temporal derivative terms in the Lagrangian $\mathcal{L}(G, \partial G)$. By mathematical integration by parts, any canonical kinetic contribution inevitably produces boundary/interface cross-terms. Schematically, for a quadratic elastic term:

$$S_{\text{kin}}[G; \Omega] \propto \int_{\Omega} (\partial G)^2 d^4x = \int_{\Omega} G \square G d^4x + \int_{\partial\Omega} G n \partial G d^3\Sigma \quad (245)$$

Therefore, upon splitting the domain, one generically obtains:

$$S[G; A \cup B] = S[G; A] + S[G; B] + \Delta S_{\Sigma}[G] \quad (246)$$

where $\Delta S_{\Sigma}[G] \neq 0$ generically. Vanishing ΔS_{Σ} requires exceptional decoupling boundary conditions (a singular limit, such as an infinite potential barrier). Hence, the accumulated elastic action is structurally non-separable.

Because quantum path-integral weights are exponential in S , this non-zero interface term generates an unavoidable multiplicative coupling kernel across the boundary:

$$\exp\left(\frac{i}{\hbar} S_{AB}\right) = \exp\left(\frac{i}{\hbar} S_A\right) \exp\left(\frac{i}{\hbar} S_B\right) \underbrace{\exp\left(\frac{i}{\hbar} \Delta S_{\Sigma}\right)}_{:= \mathcal{K}_{\Sigma}[G]} \quad (247)$$

When evaluating the full quantum partition function $\mathcal{Z}_{AB} = \int \mathcal{D}G_A \mathcal{D}G_B \exp(iS_{AB}/\hbar)$, the coupling kernel $\mathcal{K}_{\Sigma}[G]$ explicitly prevents the functional measure from splitting into independent measures ($\mathcal{Z}_{AB} \neq \mathcal{Z}_A \mathcal{Z}_B$). The reduced state on $A \cup B$ is generically not a product state (nor a separable mixture of products). Entanglement is therefore the generic, mathematically inevitable outcome of global action coherence in a continuous elastic medium.

W.3 Physical Meaning and Emergent Locality

In QEG, the substrate cannot be "glued" across a mathematical cut without storing shared accumulated elastic action. In this continuous picture, correlated "subsystems" are simply spatial regions that still share a single, unbroken coherent deformation history (i.e., those for which S has not factorized due to external macroscopic boundary conditions).

Exact subsystem independence is therefore not fundamental. Standard locality arises only as an effective, emergent approximation when interface contributions are negligible, dynamically suppressed by distance, or completely coarse-grained away by a macroscopic thermal bath:

$$\Delta S_{\Sigma} \approx 0 \implies \mathcal{K}_{\Sigma} \approx 1 \implies \text{approx. factoriz. (eff. locality)} \quad (248)$$

W.4 Resolving Bell Non-Locality and the Born Rule

Bell Inequalities (Logical vs. Causal Non-Locality).

Bell-type inequalities assume that a complete physical state decomposes into local, independent hidden variables, such that conditional joint probabilities factorize:

$$P(a, b|x, y, \lambda) = P(a|x, \lambda)P(b|y, \lambda)$$

In QEG, *this foundational premise of statistical independence is structurally false*. The complete physical state is a continuous, global deformation history strictly constrained by elastic interface closure. Therefore, Bell factorization fails on purely structural grounds. This represents a form of *logical or configurational non-locality* (arising from a globally constrained, continuous phase-space), not causal non-locality. Because the elastic substrate itself still propagates strain through strictly hyperbolic covariant wave equations ($\square G_{\mu\nu} \propto J_{\mu\nu}$), entanglement cannot be leveraged to enable controllable superluminal signalling.

Wavefunction Collapse and the Born Rule.

Conditioning on an observed macroscopic outcome in region A corresponds, in the path-integral picture, to imposing a massive, irreversible boundary condition on the elastic network. Restricting the functional integral solely to histories compatible with that new physical constraint forces the entire globally connected deformation field to snap into a newly restricted stationary history. The "collapse" is thus a sudden topological reduction of the coherent phase-space, governed by global admissibility, rather than a physical dynamical signal traveling causally from A to B .

The associated probabilities of finding the substrate in a specific topological history must be positive, localizable, and invariant under global phase shifts. In a wave-bearing continuous medium, the unique mathematical density satisfying these structural constraints is strictly quadratic in the amplitude. Therefore, the Born weight ($P \propto |\Psi|^2$) emerges as a structural continuum-mechanics requirement of the elastic lattice, rather than an ad-hoc quantum axiom.

W.5 Synthesis

The structural chain defining quantum non-separability in QEG is summarized as:

$$\begin{aligned} \Delta x \Delta p \geq \frac{\hbar}{2} &\implies \text{No sharp truncation} \implies \\ S_{AB} \neq S_A + S_B &\implies \mathcal{K}_{\Sigma} \neq 1 \implies \text{Entanglement} \end{aligned} \quad (249)$$

In QEG, entanglement is therefore the geometric closure residue of an elastic vacuum whose primitive ontological object is accumulated global action, rather than pointwise localized extension.

X Physical Manifestation of the Constitutive Identity: The Photoelectric Effect as an Elastic Yield Point

The abstract equilibrium between static deformation and dynamic flux, formalized in the QEG Vacuum Constitutive Equation (see Sec. 5.1.5 of the main text), finds a direct physical manifestation in one of the cornerstones of quantum mechanics: the photoelectric effect. Traditionally interpreted as the proof of corpuscular light, this phenomenon can be rigorously reinterpreted in QEG as a macroscopic, local realization of the substrate's fundamental elastic yield threshold.

Recall the exact Electro-Quantum segment of the Vacuum Constitutive Equation derived in the main text:

$$\mu_0 e \equiv \frac{hc}{L_{\text{ref}}} \quad (250)$$

By defining the fundamental kinematic frequency of the macroscopic causal cell as $f_{\text{ref}} = c/L_{\text{ref}}$, this identity can be rewritten in a highly suggestive, dynamic form:

$$\mu_0 e \equiv hf_{\text{ref}} \quad (251)$$

Structurally, this is the exact photoelectric equation for the bare vacuum. We can map this fundamental equilibrium directly to the macroscopic phenomenon by treating photoemission as the topological unpinning of a defect under critical geometric strain:

- **The Bare Vacuum Yield Point** ($\mu_0 e \equiv hf_{\text{ref}}$): In the vacuum identity, the term $\mu_0 e$ represents the intrinsic static cohesive energy—the minimal geometric tension required to sustain the bare elemental charge defect against the baseline stiffness of the spacetime substrate (μ_0). The term hf_{ref} represents the dynamic transverse impulse of a fundamental causal wave. The identity dictates that, at the fundamental scale, releasing a bare charge defect requires a dynamic flux exactly equal to its static geometric tension.
- **The Macroscopic Cohesive Barrier (Work Function W)**: Within a macroscopic crystalline lattice (a metal), an electron defect is additionally bound by the emergent electrostatic topology of the material. The macroscopic energy required to unpin the defect from this lattice is the **work function**, W . In QEG terms, W represents the integrated local *elastic yield stress* of the modified substrate:

$$W \sim \int e \vec{E} \cdot d\vec{\ell}, \quad (252)$$

which replaces the bare vacuum tension ($\mu_0 e$) with an emergent, environment-dependent cohesive barrier.

- **The Dynamic Strain Impulse (Incident Photon)**: Rather than a corpuscular projectile, an incident photon is modeled in QEG as a localized, propagating torsional strain packet. Its dynamic impulse, conventionally written as $E_\gamma = hf$, corresponds strictly to the geometric quantum of action (h) scaled by the local kinematic rate of continuous deformation (f).
- **The Elastic Yield Condition** ($W = hf_{\text{min}}$): The famous threshold condition for photoemission,

$$W = hf_{\text{min}}, \quad (253)$$

is thus mathematically isomorphic to a macroscopic mechanical failure criterion. It marks the exact critical point where the impinging dynamic transverse flux (hf) overcomes the integrated static cohesive stress (W) binding the topological defect, causing it to unpin from the lattice and propagate freely.

Therefore, the photoelectric effect is revealed to be more than a simple energy exchange: it is a macroscopic mechanical manifestation of the substrate's static-dynamic equilibrium. The universal law governing the emission of electrons from a metal is a direct phenomenological scale-up of the $\mu_0 e \equiv hc/L_{\text{ref}}$ vacuum identity, grounding the abstract constitutive equivalence of QEG in one of the most robust experimental pillars of modern physics.

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