The Compton effect is a relativistic effect

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Abstract

In this work, the formula for the Compton effect will be derived using the conservation of electron energy and relativity.

Keywords: Compton, electron energy, doppler shift, frequency, light, relativity

1. Introduction

Although Albert Einstein had already proposed the photoelectric effect in 1905, many scientists still doubted the corpuscular nature of light. Compton's experiment was the "irrefutable proof" because it allowed scientists to calculate exactly how much the wavelength changed, $\Delta\lambda = \lambda' - \lambda$ using the mass of the electron and Planck's constant.

2. Conservation of electron energy and the Compton effect

This is the experimental Compton array [1]

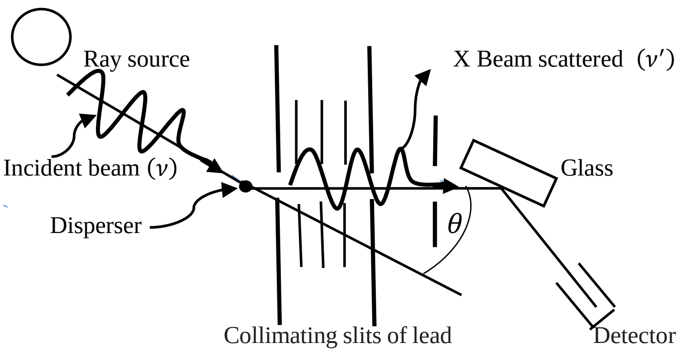

Fig. (1)

Adapted from R. Eisberg and R. Resnick, Fisica Cuantica: Atomos, moleculas. Solidos, nucleos y particulas, Limusa, Mexico 1979

Figura (2)

Simplified diagram of the Compton Experiment but only for frequencies.

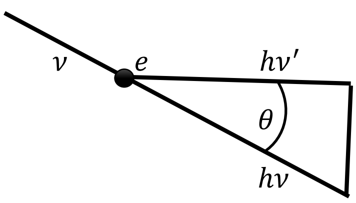

Fig. 2

The incident beam has a frequency ν which forms an angle θ with the scattered beam ν'. Therefore:

$$\cos\theta = \frac{\nu'}{\nu} \tag{1}$$

X-rays that strike graphite only have frequency; they have no mass, because they are electromagnetic waves.

This frequency, when combined with the action of the electron, gives it initial energy $h\nu$. And this energy becomes $h\nu'$ plus the kinetic energy of the electron K

$$h\nu = h\nu' + K \tag{2}$$

De (2) y Como $K = \frac{c^2 m_0}{\sqrt{1-\frac{v^2}{c^2}}} - c^2 m_0$ [2] the following is obtained:

$$h\nu\left(1 - \frac{\nu'}{\nu}\right) = c^2 m_0\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$h\frac{c}{\lambda}\left(1 - \frac{\nu'}{\nu}\right) = c^2 m_0\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$h\frac{1}{\lambda}(1 - \cos\theta) = c m_0\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$\frac{h}{c m_0}(1 - \cos\theta) = \left(\frac{\lambda}{\sqrt{1-\frac{v^2}{c^2}}} - \lambda\right)$$

$$\frac{h}{c m_0}(1 - \cos\theta) = \frac{\lambda}{\sqrt{1-\frac{v^2}{c^2}}} - \lambda \tag{3}$$

From the relativistic transverse Doppler effect of light:

$$\nu' = \nu\sqrt{1 - \frac{v^2}{c^2}} \qquad \nu' = \frac{c}{\lambda'} \qquad \nu = \frac{c}{\lambda} \implies \lambda' = \frac{\lambda}{\sqrt{1-\frac{v^2}{c^2}}}$$

Then (3) is:

$$\lambda' - \lambda = \frac{h}{c m_0}(1 - \cos\theta) \tag{4}$$

3. Planck constant

The angular momentum of the electron in the hydrogen atom is:
$l = v_e r_b m_e$ Where v_e is the speed of the electron, r_b is the Bohr radius and m_e is the mass of the electron.

But l is constant, therefore I can express the moment in terms of the speed of light c, but then for it to still have the same value the Bohr radius must have another smaller value which I call r_c and since m_e is constant then

$$l = c r_c m_e \qquad \text{Pero} \quad r_c = \frac{\lambda_c}{2\pi}$$

$2\pi l = c\lambda_c m_e \quad l = \hbar \quad 2\pi\hbar = h = c\lambda_c m_e$ Hence:

$$\lambda_c = \frac{h}{c m_e} \tag{5}$$

Substituting (5) en (4)

$$\lambda' - \lambda = \lambda_c(1 - \cos\theta) \tag{6}$$

Which is the same formula that Compton found

4. conclusions

Light is not made up of photons as Compton supposed. What light gives to electrons is its frequency, which, combined with the electron's angular momentum, provides an increase in energy

The Compton effect is an effect that has a relativistic origin because the speeds of the electrons are large and, therefore, relativistic effects are noticeable.

5. References

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3. Krane, K. S. (2012). *Modern Physics* (3ra ed.). John Wiley & Sons.