On the Microstructure of Black Hole Singularity

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Abstract

The singularity of a black hole with a quantum index b was previously shown to be consisted of b concentric spherical shells of a constant diametral step size A, corresponding to a fundamental *resolution interval* in nature. An interior shell j was found to be a condensation of a Primordial Stem Particles (PSP) of mass $\bar{m} = h/Ac$, at excitation state j and momentum $j\bar{m}c$. The spatial uncertainty of the constituents at this excitation state was found to be jA, therefore, confining the particles to the very surface of the shell of diameter jA. The number of constituents $\bar{N}_1 = A^2/4L_p^2$ of the innermost shell $j = 1$ was found to be the *quanta of particle count* in the singularity, such that particle count on a shell of index j was $\bar{N}_j = (3j^2 - 3j + 1)\bar{N}_1$. Invoking Tammes's conjecture, we have shown in this article that the microstructure of the black hole singularity is consisted of a distinct set of curved hexagonal lattices that are wrapped into the spherical shells. It is shown that the distance of the constituents on the innermost shell is $\bar{d}_1 = (8\pi/\sqrt{3})^{1/2}\, L_p$. The microstructure gets progressively finer towards the outer shells. In the outermost shell of the singularity of the supermassive black holes, the lower limit $\bar{d}_\infty$ is approaching to $(8\pi/3)^{1/2}\, L_p$. It is also noted that a *centered hexagonal tessellation* naturally emerges from $\bar{N}_j$ law which is a remarkable hint of consistency in the proposed theory of black holes.

1 Background

The combined theory of Special Relativity and Quantum Mechanics (c-SRQM) studies the state of an abstract particle under a definite momentum in absolute vacuum - isolated from any external influences [1]. The spacetime coordinate of such particle is shown to be describable by being in an infinite series of *boxes* that are aligned along the particle's trajectory, plus a periodicity condition that guarantees there are no *gaps* between any two walls of the adjacent boxes. It is postulated that the length of these boxes *must* obey Lorentz transformation among the inertial observers of the particle. The invariant of transformation, A, is shown to represent the boundary of a physical regime where the very notion of "localized particle" is transitioning into the notion of "field". The invariant A, therefore, is thought to be a fundamental physical constant which defines the *resolution interval* in nature. In other words, rest mass of a particle whose Compton wavelength is greater than A gets so feeble that it physically becomes *unresolvable*. Similar to the speed of light which is independent of the inertial observers' specific clocks and yardsticks, the resolution interval A is also an *inherent* physical constant, and therefore, *must* be independent of the inertial observers' vantage point.

Moreover, the invariant A is shown to correspond to the dimensional features of a pair of extreme objects in the universe: 1) the diameter of a black hole of mass $M_1 = Ac^2/4G$ named the Unit Black Hole (UBH) and 2) the Compton wavelength of a particle of mass $\bar{m} = h/Ac$ named the Primordial Stem Particle (PSP). The UBH and PSP represent the *least massive black hole* and the *least massive localized particle* ever possible in nature. Since the Compton wavelength and the spatial uncertainty of the PSP at the ground state $n = 1$ are both equal to the fundamental resolution interval A, the particle represents a physical boundary wherein the transitioning from particle to field is occurring. Having the resolution interval constant A defined, the c-SRQM then shows that there exists an upper limit to acceleration $a_u = c^2/A$ in nature, such that [2]:

$$\lim_{a \to a_u} \frac{d}{dt'}(\delta x') = c. \tag{1}$$

where t' is coordinate time, a is local acceleration and $\delta x'$ is the spatial uncertainty along the direction of the acceleration a. From Eqn 1 it is inferred that any acceleration higher than the limit a_u would result in a superluminal expansion of the spatial uncertainty $\delta x'$ of the accelerating object. This is in a direct violation of GR that prohibits superluminal phenomenon. From equivalence principle, the limit acceleration then must also represent the *gravitational strength at the surface of the physical singularity of the UBH*. This then leads to the UBH mass $M_1 = Ac^2/4G$, where the gravitational strength at the surface of physical singularity (or equivalently at the event horizon) is $a_u = c^4/(4GM_1)$. The UBH, in turn, plays a central role in quantizing the mass, singularity, size, entropy and radiation of larger black holes, as briefly described below.

The mass M_b, event horizon D_b and singularity (core) C_b of larger black holes with the index b are quantized using those of the UBH, i.e M_1 and A, as follows: [3]:

$$\begin{aligned} M_b &= b^3 M_1 \\ D_b &= b^3 A \\ C_b &= bA \end{aligned} \tag{2}$$

Subsequently, the number of particles $\bar{N}_j$ on shell j of a black hole of index b was shown to be given by:

$$\bar{N}_j = (3j^2 - 3j + 1)\bar{N}_1 \qquad j = 1, 2, ..., b \tag{3}$$

where $\bar{N}_1$ is the number of PSP constituents on the innermost shell $j = 1$ given by :

$$\bar{N}_1 = \frac{M_1}{\bar{m}} = \frac{A^2 c^3}{4Gh} = \frac{1}{4}(\frac{A}{L_p})^2 \tag{4}$$

From Eqn 4 we noted that since *perfect squares of odd numbers are never divisible by* 4, the necessity of $\bar{N}_1 \in \mathbb{N}$ then demands that *the Planck length L_p must be an even divisor of UBH diameter A*. The number of total particles $\bar{N}_b$ in a black hole with index b is then given by:

$$\bar{N}_b = \sum_{j=1}^{b} (3j^2 - 3j + 1)\bar{N}_1 = \bar{N}_1\, b^3 \tag{5}$$

Knowing the event horizon diameter D_b from Eqn 2, the surface area H_b of the horizon is quantized as [4]:

$$H_b = \pi A^2 b^6 \tag{6}$$

Substituting for the latter in the Beckenstein-Hawking entropy $S = \kappa H/4l_p^2$, we then have the entropy of black holes quantized as:

$$S_b = \pi\kappa\,(\frac{A^2}{4l_p^2})\,b^6 \tag{7}$$

which leads to the quantization of the Hawking radiation as follows:

$$T_b = \frac{1}{4\pi^2}\,\frac{L_p}{D_b}\,T_p \tag{8}$$

Eqn 8 indicates that that the Hawking temperature is a function of the number of the units of Planck length across the diameter of the event horizon. It was also shown that the entropy of black holes can be alternatively expressed in terms of the number of their constituents as follows:

$$S_b = 2\pi^2\kappa\bar{N}_1 b^6 \tag{9}$$

Comparing the latter with the general definition of entropy $S_b = \kappa\ln\Omega_b$, the number of microstates Ω_b of a black hole with quantum index b will be:

$$\Omega_b = e^{2\pi^2\bar{N}_1 b^6} \tag{10}$$

The theory, therefore, offers an alternative and more intuitive answer to the fundamental question of *what is actually being counted by the entropy of a black hole*? In the following section we now describe the likely microstructure arrangement of the PSPs on the singularity shells.

Figure 1: An Icosahedron with 20 faces, 12 vertices and 5 edges per vertex

2 Uniform particle distribution on spherical surfaces

In the mathematical literature, the problem of finding a configuration of N particles on a spherical shell wherein the minimal pairwise distance between the particles is maximized is known as *Tammes's problem* [6]. The infimum condition imposed on the sought configuration comes from the fact the potential energy of a set of mutually repelling or attracting particles will be at a minimum only *when each particle is equidistant from its immediate neighbors.* Such maximum uniform spacing among the particles leads to an isotropic distribution, therefore, putting all the particles on an equal footing relative to each other. In flat surfaces, the arrangement in which the pairwise distances between particles are maximized is found to be a *hexagonal lattice*. In the case of a spherical surface, if the number of particles is sufficiently large then the local packing density will be sufficiently high to make the local curvature of the sphere be nearly flat. Therefore, a hexagonal lattice also emerges on the spherical surfaces if the particle count $N \to \infty$. Interestingly enough, it also known that a spherical surface cannot be tiled using the hexagons solely; and in fact precisely 12 pentagons are needed to perturb the hexagonal pattern to fully fit onto a spherical surface. To see the reason for this [5], consider a situation where an Icosahedron (an awesome Platonic solid with 20 equilateral triangular faces,12 vertices and 5 edges per vertex), as shown in Fig 1, is intersected with a sphere which is infinitesimally smaller than its circumsphere. In that situation, there will be 12 infinitesimal pentagons generated on the intersecting sphere (ie. one pentagon for each vertex). Now the original 20 equilateral triangular faces of the Icosahedron can be subdivided

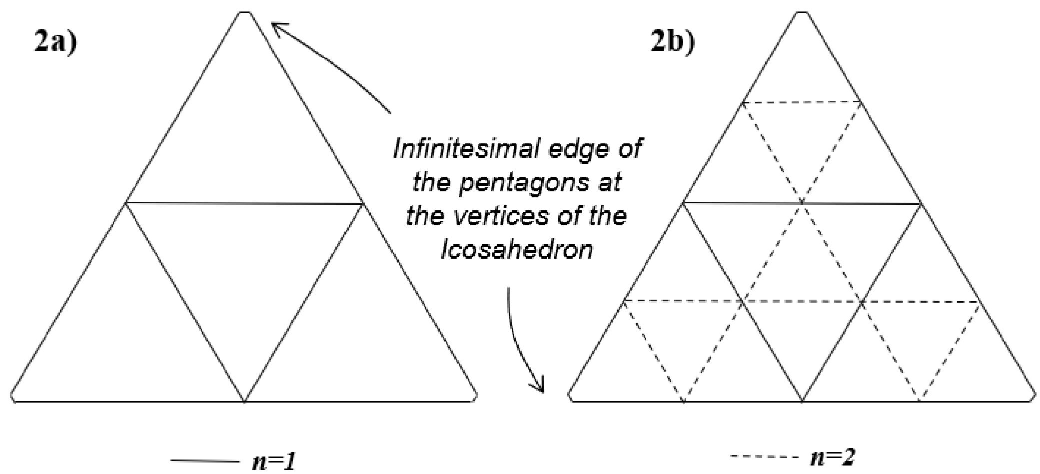


Figure 2: Equidistant points generated from successive subdivision of 20 equilateral faces

into 4 smaller equilateral triangles through joining their mid points, as shown in Fig 2a. The centers of the 4 newly generated equilateral triangles are then projected radially outward onto the intersecting sphere to generate a set of $20 \times 4 = 80$ optimally distributed points on the intersecting sphere. The process can be successively continued n times generating 20×4^n points with a hexagonal arrangement among them, except at the vertices where 12 pentagons are located [5]. Fig 2b shows the case of $n = 2$ for a second subdivision which generates 4^2 triangles. Note that an arbitrarily large n will produce infinitesimal hexagonal lattice everywhere except the 12 pentagons at the vertices. An example of this arrangement is the traditional soccer ball, where we encounter 20 hexagons (white facets) plus 12 pentagons (black facets). Another famous example is the Buckminsterfullerene C_{60} molecule which also has 20 hexagons and 12 pentagons. In C_{60}, the Carbon molecules, however, occupy the vertices of the lattice, that is why the number of polygons do not add up to 60.

3 Microstructure of the singularity shells

Invoking Tammes's conjecture on the optimum arrangement of the PSPs on the singularity shells, mathematically, the problem statement boils down to finding an optimum (most uniform) distribution of a staggeringly large number of particles $\bar{N}_j$ on the surface of a sphere of diameter jA, such that the minimum distance among the particles is maximized as follows:

$$\bar{d}_j := \max(\min |x_v - x_w|) \quad x_v, x_w \in S_j^2 \quad 1 \le v \le w \le \bar{N}_j \tag{11}$$

where S_j^2 is the surface area of a sphere of diameter jA embedded in Euclidean space $\mathfrak{R}^3$as follows:

$$S_j^2 = \{x \in \mathfrak{R}^3 \;\; : \;\; |\, x\,| = \frac{jA}{2}\} \tag{12}$$

At first glance, this may not appear to be a hard problem, but in reality, finding an optimum configuration for a large number of points on a sphere is a challenging problem, attracting attention of many mathematicians and researchers in various fields in the past. From the literature, it is known that when the number of particles is very large - which is in line with the situation in black holes singularity - the optimum configuration is arrived almost entirely in a hexagonal pattern, as shown in Fig 3. Considering the number of the PSP constituents

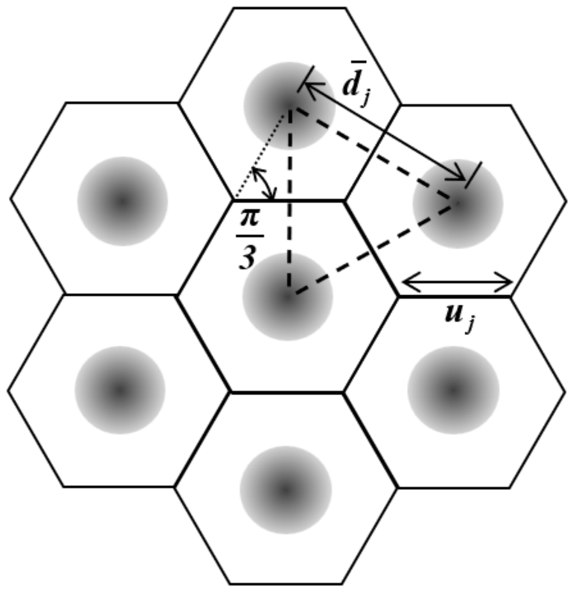


Figure 3: Hexagonal Voronoi cells of PSP constituents on shell index j

on shell j from Eqn 3, the surface area $\bar{\sigma}_j$ partitioned by each particle is then given as follows:

$$\bar{\sigma}_j = \frac{j^2}{3j^2 - 3j + 1}\, 4\pi L_p^2 \tag{13}$$

The PSP condensation, from one hand, is pulled towards the geometric center of the singularity under the gravitational force; and on the other hand, is forced to remain on the surface of singularity shell j to respect the quantum spatial uncertainty requirement $\bar{\epsilon}_j = jA$. As in Tammes's set up, the potential energy of the particle arrangement will be minimized when the partitioning generates a hexagonal lattice as discussed before. From Eqn 13 the smallest distance maximized among the constituents of shell j will be as follows:

$$\bar{d}_j = (\frac{8\pi}{\sqrt{3}})^{1/2} \, \frac{j}{\sqrt{3j^2 - 3j + 1}} L_p \tag{14}$$

From above it is evident the lower and upper bounds of the distance are given by:

$$(\frac{8\pi}{\sqrt{3}})^{1/2} L_p \;\leq\; \bar{d}_j \;<\; (\frac{8\pi}{3})^{1/2} L_p \tag{15}$$

The lower bound $\bar{d}_1 \approx 3.809\; L_p$ is the distance of the particles on the innermost shell $j = 1$ and the upper bound $\bar{d}_\infty \rightarrow 2.894\; L_p$ corresponds to that of the outermost layer of supper massive black holes. The finer lattice of the outer layers is in line with the finding in [4] that 50% of a black hole's entropy is stored at the top 11% layers of its singularity. From Eqn 15, since $\bar{d}_j > L_p$ it is evident that the gravitational pull between adjacent particles is less than the upper limit $a_u = c^2/A = G\bar{m}/L_p^2$.

4 Central hexagonal lattice growth

For the shell sequence $j \in \{1, 2, 3, 4, 5, 6, ...\}$, the population sequence obtained from equation $\bar{N}_j = (3j^2 - 3j + 1)\bar{N}_1$ is given by the sequence: $\bar{N}_j \in \{1, 7, 19, 37, 61, 91, 127, ...\} \times \bar{N}_1$. As shown in Fig 4, the sequence is known to generate a *central hexagonal pattern*, layer upon layer as j index increases. This is a remarkable hint of consistency in the proposed theory of black holes. Note that the population law of Eqn 3 is obtained directly from quantization of the black holes; completely independent from the hexagonal nature of the condensation lattice expected from Thomson/Tammes packing conjecture.

5 Discussion

The black hole singularity in our model is viewed as a condensate of a primordial quantum particle [3]. The source of entropy is shown to stem from the statistical ensemble of all permissible internal configurations in which the quantum constituents could organize themselves inside the singularity [4]. A *conceptually* similar idea is proposed in [7] in which the black hole singularity is viewed as a macroscopic quantum state analogous to a Bose-Einstein Condensate (BEC) of a quanta. In that model, similar to ours, the black hole is not considered as a classical spacetime geometric singularity but rather a many-body quantum system whose macroscopic properties arise from the collective behaviour of a large number of microscopic constituents. In their framework, the black hole constituents are *soft gravitons* forming a nebules of critical condensate, whereas in our framework the black hole constituents are *primordial stem particles* of mass $\bar{m} = h/Ac$ forming a layered condensate with a curved hexagonal microstructure that are wrapped onto spherical shells inside the singularity.

6 Conclusion

Under c-SRQM framework, singularity of the black holes can be viewed as a layered hexagonal lattice composed of a condensate of a primordial quantum particle of mass $\bar{m} = h/Ac$. The spherical layers are spaced by a fixed radial step size equal to a fundamental resolution interval A in nature. Each layer corresponds to a discrete quantum index which represents the collective state of excitation of the occupying constituents. The number of constituents per shell j follows the hexagonal lattice growth law $\bar{N}_j = (3j^2 - 3j + 1)\bar{N}_1$, reflecting an optimal packing on the spherical shells. The *quanta* of particle count $\bar{N}_1 = A^2/4L_p$ corresponds to the number of constituents in the innermost shell $j = 1$, the Unit Black Hole. In this model, the entropy of black holes arises from the number of possible microstates (degrees of freedom) that a condensed hexagonal lattice of the constituents generates on the singularity shells. Hawking radiation is shown to be a gradual depletion of the stored entropy in the singularity.

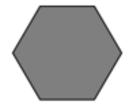
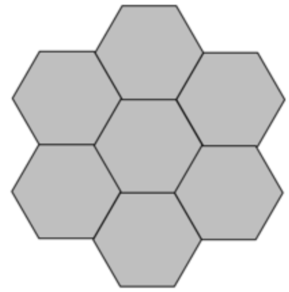
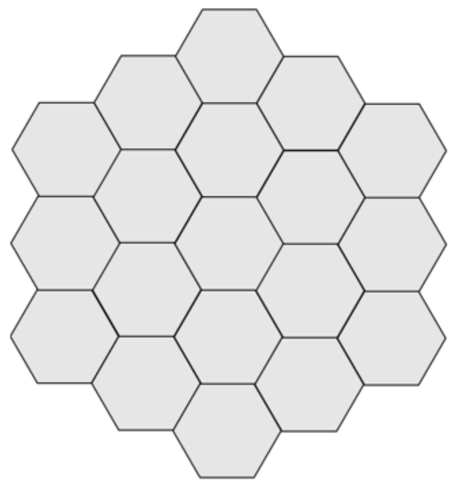
shell j	$N_j = (3j^2 - 3j + 1) \times N_1$	Pattern
1	$1 \times N_1$	
2	$7 \times N_1$	
3	$19 \times N_1$	

Figure 4: Emergence of the central hexagonal lattice growth from N_j

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