Relativistic Energy Conservation in Gravitational Motion Across Scales

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Abstract:

I present an empirically grounded, effective description of gravitational dynamics. In my view, a full quantitative description would require the formalism of general relativity, a closely related extension, or an appropriate replacement, which is beyond the scope of this work.

When a rubber ball is pressed against the floor, its internal energy—and consequently its mass—increases by $\Delta m = \Delta E/c^2$; a similar effect occurs when a body is lifted. Building on this principle, the gravitational formulation proposed here avoids singularities at the centers of black holes. In regions where r_s exceeds r, gravity becomes repulsive, which can be interpreted as an effective contribution similar to dark energy. The framework also provides a mechanism consistent with quark confinement, while all resulting equations remain closely aligned with those of general relativity.

1. Introduction

In this work, gravity is treated as a force.

Two identical rubber balls are initially at rest on the floor. Ball **a** remains on the floor, while ball **b** is lifted to a height $h$above the level of ball **a**. In lifting ball **b**, work must be done against the Earth's gravitational force. This work is transferred into an increase of the energy of ball **b**.

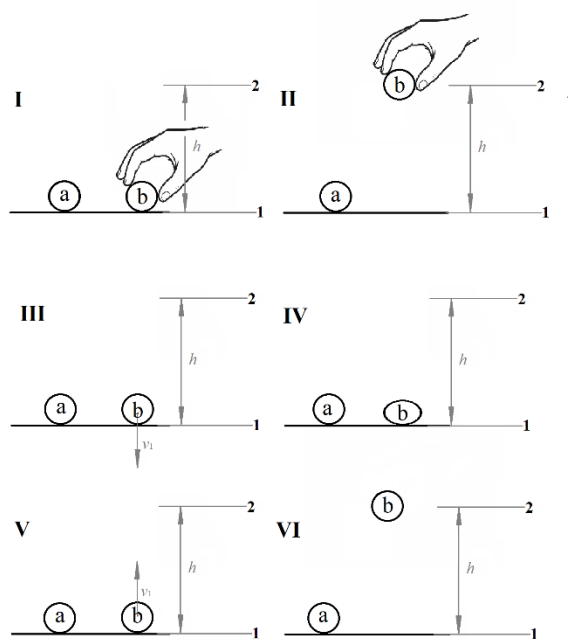
If the ideally elastic ball **b** is then released from height h, it will bounce indefinitely next to ball **a**, and at all times its energy will exceed that of ball **a** by exactly the amount of work invested in lifting it from the floor to height h.

When ball **b** is at height h, the entire energy difference is stored as gravitational potential energy relative to ball **a**. When it falls and reaches the floor, the entire excess energy appears in the form of kinetic energy.

As the motion continues downward after contact with the floor, the kinetic energy is fully converted into elastic energy associated with the deformation of the ball. At maximum compression, the entire excess energy is stored in the elastic deformation of ball **b** (a slightly smaller gravitational potential energy due to the lowering of the center of mass during deformation is neglected). At this point, the deformation stops and ball **b** begins to move upward, repeating its oscillatory motion.

To emphasize once more: at every moment of the motion, the energy of ball **b** exceeds the energy of ball **a** by exactly the amount of

work invested in lifting ball **b** to height h. This demonstrates that energy is conserved during free motion in a gravitational field.



2. Equations

II My work in lifting the ball to height $h$is equal to the difference in potential energy between the ball at levels 1 and 2:

$$W \approx m_2 gh = \Delta PE \qquad (1)$$

(Although a gravitational mass defect exists and is essential for the discussion of gravity, for this part of the calculation we take $m \approx m_1 \approx m_2$, where $m$is the rest mass of the ball, $m_1$is the rest mass of the ball on the floor, and $m_2$is the rest mass of the ball at height h.)

IV $m_2$is also the rest mass of the ball at maximum elastic compression, where all surplus of energy is stored in the form of elastic energy.

III My work in lifting the ball to height $h$is equal to the difference in potential energy between balls **a** and **b**, and both are equal to the difference in kinetic energies of balls **a** and **b** when ball **b** touches the floor while moving downward with velocity v_1:

$$W \approx m_2 gh = \Delta PE = \Delta KE = \frac{m_1 v_1^2}{2} \qquad (2)$$

In accordance with the mass–energy relation, for our case, the following always holds:

$$W = \Delta PE = \Delta KE = \Delta mc^2 = (m_2 - m_1)c^2 \qquad (3)$$

where Δm is the real, physical, and literal change in the rest mass of the ball.

In other words, during free fall (using Einstein's expression for relativistic kinetic energy), the following relation holds:

$$W = \Delta PE = \Delta mc^2 = (m_2 - m_1)c^2 = \Delta KE = \frac{m_1 c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} - m_1 c^2 \qquad (4)$$

from which it follows:

$$m_2 c^2 = \frac{m_1 c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} \qquad (5)$$

Here, $m_1$is the rest mass of the ball on the floor, $m_2$is the rest mass of the ball at height h, and $v_1$is the velocity of the ball at the moment it touches the floor.

What does the rest mass $m_1$mean? It is the real, physical, and literal mass of ball **a**, which always remains on the floor. Likewise, if ball **b** is stopped at this level, it will have the identical mass m_1, and in that case my work will extract the excess energy of the ball that was moving with velocity v_1.

For small velocities, near the Earth's surface,

$$m_2c^2 = \frac{m_1c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} = m_1c^2\left(1+\frac{1}{2}\frac{v_1^2}{c^2}+\cdots\right) \approx m_1c^2 + \frac{m_1v_1^2}{2} \quad (6)$$

Example 1: If the rest mass of a body at the Earth's surface is $m_1 = 1$ kg, what will be the mass of that body $m_2$if I carry it to infinity?

According to Newton, the gravitational force acting on a mass $m$from mass $M$is:

$$\boldsymbol{F}(r) = -\frac{GMm}{r^2}\boldsymbol{e_r} \quad (7)$$

My work against the Earth's gravitational force (8):
$\mathbf{F}'$is my force, which I apply to overcome gravity and carry a body of mass $m$very slowly at constant velocity to infinity. The force has the same magnitude as the gravitational force but is directed oppositely, i.e., along increasing r. Therefore, the work of my force, which increases the energy of the body, is positive:

$$W = \int_r^\infty \boldsymbol{F'}\,d\boldsymbol{r} = \int_r^\infty \frac{GMm}{r^2}dr = -\frac{GMm}{r}|_r^\infty = -\left(0-\frac{GMm}{r}\right) = \frac{GMm}{r} \quad (8)$$

This is the amount by which the potential energy of mass $m$at infinity exceeds its energy at distance $r$from mass M. It is a magnitude larger than zero.

Combining equations (4) and (8), we get:

$$m_2 \approx m_1 + G\frac{Mm_1}{rc^2} \quad (10)$$

Thus:

$$m_2 \approx 1kg + \frac{6.67\cdot10^{-11}\frac{Nm^2}{kg^2}\cdot5.97\cdot10^{24}kg\cdot1kg}{6.38\cdot10^6m\cdot\left(3\cdot10^8\frac{m}{s}\right)^2} = 1kg + 6.94\cdot10^{-10}kg \quad (11)$$

This calculation is valid for small velocities, i.e., for

$$v \ll C$$

The escape velocity—the velocity required for a body to reach infinity from the Earth's surface—is equal to the velocity the body would have upon hitting the Earth's surface if it fell from infinity.

Using equations (2) and (8):

$$\frac{GMm_1}{r} = \frac{m_1v_1^2}{2} \quad (12)$$

Here, $m_1$is the mass launched with velocity $v_1$from distance $r$from mass Msuch that it escapes to infinity, or equivalently, $m_1$is the mass falling from infinity intercepted at distance $r$while moving with velocity v_1.

For this case, equation (12) gives:

$$v_1 \approx 11173\ \frac{m}{s} \ll 3\cdot10^8\frac{m}{s}$$

Since for small velocities

$$m_2c^2 \approx m_1c^2 + \frac{m_1v_1^2}{2} \quad (6)$$

derived from the general relation

$$m_2c^2 = \frac{m_1c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} \quad (5)$$

I assume that the corresponding small-velocity approximation

$$m_2 \approx m_1 + G\frac{Mm_1}{rc^2} \qquad (10)$$

originates from the general expression

$$m_2c^2 = \frac{m_1c^2}{\sqrt{1-\frac{2GM}{rc^2}}} \qquad (13)$$

In gravity, as in other systems, whenever there is an energy difference between positions, a force arises that acts opposite to the direction of energy increase. In other words, external work against a conservative force increases the energy of a body, bringing it to a higher energy level.

Interpretation of equations (13) and (5):

From equations (13) and (5), it follows that a body has finite mass $m_2$at infinity and can fall freely, conserving energy. If the body is stopped at distance $r$from mass M, toward which it is falling, its rest mass becomes m_1, and its velocity just before stopping is v_1. In extreme cases, velocity approaches $c$while mass approaches zero.

Consider the general case of a body of finite mass $m_2$at infinity, given a small push toward mass M. The following hold:

$$m_2c^2 = \frac{mc^2}{\sqrt{1-\frac{2GM}{rc^2}}} = const \qquad (14)$$

and

$$m_2c^2 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = const \qquad (15)$$

Differentiating equation (14) with respect to the coordinate r, we obtain:

$$0 = \frac{d}{dr}\frac{mc^2}{\sqrt{1-\frac{2GM}{rc^2}}} \qquad (16)$$

$$0 = \frac{\sqrt{1-\frac{2GM}{rc^2}}\frac{d}{dr}(mc^2) - mc^2\frac{1}{2\sqrt{1-\frac{2GM}{rc^2}}}\left(\frac{-2GM}{C^2}\right)\left(-\frac{1}{r^2}\right)}{\left(\sqrt{1-\frac{2GM}{rc^2}}\right)^2} \qquad (17)$$

Directly from (17):

$$F' = \frac{d}{dr}(mc^2) = G\frac{mM}{r^2}\frac{1}{1-\frac{2GM}{rC^2}} \qquad (18)$$

Equation (18) is the force F'required to move a body of mass $m$at small constant velocity against gravity toward infinity. In other words, it is the force with which the floor must hold the rubber ball to keep it at rest. The gravitational force has the same magnitude but opposite direction.

Thus, the gravitational force on mass $m \ll M$at distance $r$from mass $M$is:

$$\boldsymbol{F} = -G\frac{mM}{r^2}\frac{1}{1-\frac{2GM}{rC^2}}\boldsymbol{e_r} \qquad (19)$$

and the gravitational acceleration is:

$$ma_c = -G\frac{mM}{r^2}\frac{1}{1-\frac{2GM}{rC^2}}$$

$$a_c = -G\frac{M}{r^2}\frac{1}{1-\frac{2GM}{rC^2}} \qquad (20)$$

Equivalently:

$$\boldsymbol{F} = -G\frac{mM}{r^2}\frac{1}{1-\frac{r_S}{r}}\boldsymbol{e_r} \qquad (21)$$

$$a_c = -G\frac{M}{r^2}\frac{1}{1-\frac{r_S}{r}} \qquad (22)$$

where

$$r_s = \frac{2GM}{C^2} \quad (23)$$

is the Schwarzschild radius. Here it may carry a different connotation than usual, but is useful for our considerations and will be employed.

3. Testing

3.1. The equation holds at the surface of planet Earth

The gravitational acceleration at Earth's surface is known to be approximately $-9.81\frac{m}{s^2}$.

For Earth, we compute the Schwarzschild radius:

$$r_s = \frac{2 \cdot 6.67 \cdot 10^{-11}\frac{Nm^2}{kg^2} 5.97 \cdot 10^{24} kg}{\left(3 \cdot 10^8 \frac{m}{s}\right)^2} = 8.85 \cdot 10^{-3} m \quad (24)$$

Now, using the modified gravitational acceleration equation:

$$a_c = -\frac{GM}{r^2}\frac{1}{\left(1-\frac{r_s}{r}\right)}$$

$$a_c = -\frac{6.67 \cdot 10^{-11}\frac{Nm^2}{kg^2} 5.97 \cdot 10^{24} kg}{(6370000m)^2} \cdot \frac{1}{\left(1-\frac{0.0085m}{6370000m}\right)}$$

$$a_c = -9.81\frac{m}{s^2} \cdot 1.000000001 = -9.81\frac{m}{s^2} \quad (25)$$

According to Newtonian gravity:

$$g = a_c = -\frac{GM}{r^2} = -\frac{6.67 \cdot 10^{-11}\frac{Nm^2}{kg^2} 5.97 \cdot 10^{24} kg}{(6370000m)^2} = -9.81\frac{m}{s^2} \quad (26)$$

The result obtained using the modified equation coincides with Newton's value, differing only at the 9th decimal place.

3.2 Dark Energy

Gravitational acceleration — and consequently gravitational force — is defined by equation (22), with equation (23) applying. This formulation allows for both attractive and repulsive regimes.

There are solutions in which the gravitational force is attractive (when $r_s<r$), but there also exist regions or conditions where the force becomes repulsive, i.e., where the sign of the acceleration changes when $r_s>r$.

Let us assume that the repulsive gravitational force is responsible for the observed acceleration of distant galaxies.

We now examine under which conditions this occurs.

Some estimates of cosmic acceleration are as follows:

- "At 1 Mpc it is about 10^{-15} m/s²."
- "At the edge of the observable universe (about 47 Gly), it is only about 10^{-11} m/s²."
  (See S. Carroll's blog and cosmological articles [4], or N. Wright's cosmology tutorial [5].)

In addition, several physicists have expressed views supportive of the idea that cosmic acceleration may be gravitational in origin, or that gravity itself may require modification:

"If general relativity is correct, then the repulsive force that causes the acceleration of the universe cannot be gravity in the traditional sense. It must be something different. Or gravity itself must be modified."
— L. Susskind [6]

“There’s a kind of anti-gravity out there, stretching the space between galaxies.”
— J. Levin [7]

“The acceleration of the expansion means that gravity, on cosmological scales, behaves differently than what Newton or even Einstein would expect.”
— M. Tegmark [8]

Let us now calculate the acceleration of a galaxy at the edge of the observable universe.

Estimated mass of the observable universe:

$M=10^{53}$ kg [4,5]

Distance to the edge of the observable universe:

$r=4.4\cdot10^{26}$ m [4,5]

Using the modified gravitational acceleration equation:

$$a_c = -\frac{GM}{r^2}\frac{1}{\left(1-\frac{2GM}{rC^2}\right)} \quad (22)$$

Substituting values:

$$a_c = -\frac{6.67\cdot10^{-11}\frac{Nm^2}{kg^2}\cdot10^{53}kg}{(4.40\cdot10^{26}m)^2}\frac{1}{1-\frac{2\cdot6.67\cdot10^{-11}\frac{Nm^2}{kg^2}\cdot10^{53}kg}{4.40\cdot10^{26}m\cdot(3\cdot10^8\frac{m}{s})^2}} \quad (27)$$

$$a_c = -3.445\cdot10^{-11}\frac{1}{1-0.336}\frac{m}{s^2} \quad (16)$$

$$a_c = -5.188\cdot10^{-11}\frac{m}{s^2} \quad (28)$$

This value represents the attractive acceleration a galaxy would experience toward us, according to the equation. However, this is not what is observed — galaxies at that distance are accelerating away.

Now, note that the equation also supports real solutions for the case where $r_s>r$.

Let us determine the mass M for which r_s slightly exceeds r:

$$r = 4.4\cdot10^{26}m \quad (29)$$

$$r_s = 4.41\cdot10^{26}m \quad (30)$$

From:

$$r_s = \frac{2GM}{C^2} \quad (23)$$

we obtain:

$$M = \frac{4.41\cdot10^{26}m\cdot(3\cdot10^8\frac{m}{s})^2}{2\cdot6.67\cdot10^{-11}\frac{Nm^2}{kg^2}} = 2.975\cdot10^{53}kg \quad (31)$$

The resulting acceleration is:

$$a_c = -\frac{6.67\cdot10^{-11}\frac{Nm^2}{kg^2}\cdot2.975\cdot10^{53}kg}{(4.40\cdot10^{26}m)^2}\frac{1}{1-\frac{4.41\cdot10^{26}m}{4.40\cdot10^{26}m}}\frac{m}{s^2} \quad (32)$$

$$a_c = -1.024\cdot10^{-10}\frac{1}{1-\frac{4.41\cdot10^{26}m}{4.40\cdot10^{26}m}}\frac{m}{s^2} \quad (33)$$

$$a_c = -1.024\cdot10^{-10}\frac{1}{1-1.002}\frac{m}{s^2} \quad (24)$$

$$a_c = 5.12\cdot10^{-8}\frac{m}{s^2} \quad (34)$$

This result corresponds to a repulsive gravitational force that accelerates the galaxy away from us — a candidate explanation for dark energy.

Now let us calculate the average density of matter in the observable universe under these conditions:

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3} \cdot r^3 \pi}$$

$$\rho = \frac{2.975 \cdot 10^{53} kg}{\frac{4}{3}(4.40 \cdot 10^{26} m)^3 \cdot 3.14} = 8.342 \cdot 10^{-28} \frac{kg}{m^3} \quad (35)$$

To summarize:

- $r > r_s$ gravity is an attractive force
- $r < r_s$ gravity is a repulsive force
- $r = r_s$ acceleration becomes undefined

(For practice: if $r = r_s/2$, then $M = 5.397 \cdot 10^{53} kg$, and $a_c = 2.045 \cdot 10^{-9} \frac{m}{s^2}$, etc.)

3.3. The Strongest Force in Nature

Let us now generalize the gravitational acceleration — and hence the gravitational force — for two objects with comparable masses. Specifically, the acceleration given by equation:

$$a_c = -\frac{GM}{r^2} \frac{1}{\left(1 - \frac{2GM}{rC^2}\right)} \quad (20)$$

is valid only under the condition m ≪ M. If we attempt to use this expression for objects of comparable mass, it would violate Newton's third law.

We therefore generalize the equation for magnitude of gravitational interaction between two point masses m and M as:

$$F_{mM} = F_{Mm} = \frac{GmM}{r^2} \frac{1}{\left(1 - \frac{2G(m+M)}{rC^2}\right)} \quad (36)$$

This leads to the symmetry of action and reaction:

$$a_m \cdot m = a_M \cdot M = \frac{GmM}{r^2} \frac{1}{\left(1 - \frac{2G(m+M)}{rC^2}\right)} \quad (37)$$

The magnitude of the acceleration of mass m toward (or away from) M is:

$$a_m = \frac{GM}{r^2} \frac{1}{\left(1 - \frac{2G(m+M)}{rC^2}\right)} \quad (38)$$

And the magnitude of the acceleration of M toward (or away from) m is:

$$a_M = \frac{Gm}{r^2} \frac{1}{\left(1 - \frac{2G(m+M)}{rC^2}\right)} \quad (39)$$

Let us now calculate the distance between two point-like electrons at which the gravitational attractive force equals the electric repulsive force [9]:

$$F_e = F_g \quad (40)$$

$$1 = \frac{F_g}{F_e} = \frac{\frac{Gmm}{r^2} \cdot \frac{1}{\left(1 - \frac{2G(m+m)}{rC^2}\right)}}{\frac{Kq^2}{r^2}} = \frac{Gm^2}{Kq^2} \cdot \frac{1}{1 - \frac{4Gm}{rC^2}} \quad (41)$$

Substituting known constants:

$$1 = 2.405 \cdot 10^{-43} \cdot \frac{1}{1 - \frac{2.7 \cdot 10^{-57} m}{r}} \quad (42)$$

After rearranging and applying a series expansion [10], we find:

$$r = (2.7 \cdot 10^{-57} + 6.49 \cdot 10^{-100}) m \quad (43)$$

$$r \approx 2.7 \cdot 10^{-57} m \quad (44)$$

The corresponding density under these conditions is:

$$\rho = \frac{2m}{V} = \frac{2m}{\frac{4}{3} \cdot \left(\frac{r}{2}\right)^3 \pi} = 1.77 \cdot 10^{140} \frac{kg}{m^3} \quad (45)$$

This (43) gives the distance at which gravitational attraction equals electric repulsion for two point-like electrons.

In addition, there are real solutions of the same equation in which the gravitational force exceeds the electric force by many orders of magnitude. For example:

$$r = (2.7 \cdot 10^{-57} + 6.49 \cdot 10^{-150})m$$

corresponds to the distance at which gravity becomes 10^{50} times stronger than the Coulomb force between two point-like electrons.

It is important to note that, according to standard physics, the gravitational force between two electrons is vastly weaker than the electric force — by a factor of approximately 10^{43} [9]. This is the universally accepted result, derived from classical laws.

However, equation (42) shows a crucial difference when the modified gravitational expression is used: the ratio becomes unity at extremely short distances, and vastly exceeds unity at shorter distances—a consequence of the nonlinear correction in the denominator.

Let me clearly emphasize: It is not proposed here that the electron is a point-like particle. On the contrary, I assume that it has internal structure and finite size. Nevertheless, the present calculation assumes point-like symmetry for simplicity, in order to demonstrate a key consequence of the proposed gravitational formulation.

This calculation is meant to illustrate the central claim: that gravity—as described by my modified expression—is in fact the strongest force in nature under appropriate conditions. Results in this part of the calculations are purely mathematical; they do not correspond to real physical objects, closeness, or densities. The main purpose is to demonstrate possibilities, namely to allow us to consider gravity as the force that can densely pack and deform electromagnetic objects such as quarks.

4. Further Analysis and Speculative Implications of the Modified Gravitational Equation

4.1. The Case r = r_s: Divergence and Physical Interpretation

Equation (22) predicts a divergence in gravitational acceleration when the radial coordinate approaches the Schwarzschild radius $r=r_s$, as the denominator tends to zero:

$$a_c = -\frac{GM}{r^2}\frac{1}{\left(1-\frac{r_s}{r}\right)}$$

Mathematically, the function becomes undefined at this point.

If I were to offer a physical interpretation, it would go along the following lines:

The gravitational interaction between two macroscopic bodies is, in essence, the sum of gravitational interactions among all of their elementary constituents. When two extended bodies approach one another to the point where $r=r_s$, the outcome may be determined by a single fundamental particle — for example, an electron — which, at one instant, maintains local conditions such that $r>r_s$, and in the next moment ceases to exist at that precise location and reappears slightly elsewhere, establishing a region where $r<r_s$.

Such a process may manifest as extreme compression followed by rapid or even explosive expansion. It may be relevant to physical phenomena such as supernova collapse and rebound, where transitions through critical densities involve nontrivial quantum-gravitational effects.

What I want to emphasize is this: macroscopic bodies are not truly continuous media. They are composed of quantum oscillators — particles with all of their quantum properties intact, regardless of scale. Even in extreme conditions such as $r \approx r_s$, those quantum attributes do not vanish; instead, they may play a decisive role in determining the system's evolution through the transition.

In such situations, the phenomena are strongly size-dependent. Although the governing equation is the same, our observations will differ significantly for particles, stellar-scale objects, and the intergalactic medium. Therefore, when $r$is close to r_s, the outcome depends not only on the proximity to $r_s$but also on the relevant densities.

4.2. Are Elementary Particles Gravitationally Closed Objects?

The idea that elementary particles might be gravitationally self-contained or possess structures similar to black holes has intrigued physicists for decades. Rather than being a fringe speculation, it has been explored—each in their own way—by some of the most respected names in theoretical physics: Wheeler's concept of geons [11], Penrose's discussions of quantum gravity [12], Zel'dovich's early work on quantum black holes [13], and Hawking's contributions to quantum-scale gravitational collapse [14].

Interior of the d Quark

The possibility that elementary particles such as the electron or the quark may be understood as self-confined electromagnetic or photonic structures has been explored by several researchers. Williamson and van der Mark proposed a model of the electron as a photon trapped in a toroidal topology [20], while dos Santos and da Luz developed a similar model using classical electromagnetism [21]. Vassallo et al. constructed a classical model of both the electron and proton based on stable toroidal field configurations [22]. More recently, Maruyama et al. analyzed photon vortex generation in relativistic quantum settings, supporting the idea that photons can carry localized, structured field dynamics [23]. Gauthier also explored the notion of the electron as a charged photon, offering an alternative perspective grounded in wave mechanics [24]. These diverse approaches share the common hypothesis that photonic or electromagnetic field configurations may be at the heart of particle structure.

Building on this idea, I propose that the d quark may be understood as a gravitationally deformed and confined photon — with a core under extreme relativistic conditions, where gravitational attraction balances electromagnetic repulsion.

Consequently, it is proposed here that:

I do not refer to the classical notion of a photon as merely a point-like massless particle or a beam of electromagnetic radiation. Rather, I consider the photon to be a localized, quantized excitation of the vacuum field. Its role in pair creation processes and its dual electromagnetic nature suggest that it may carry more physical content than usually attributed to it in conventional quantum field theory [18, 19].

I think that most of us can agree that a photon is a pulse of electromagnetic radiation with energy:

$$E = hf = h\frac{C}{\lambda} = h\frac{1}{\lambda\sqrt{\mu_0\varepsilon_0}} \quad (46)$$

where $h$is Planck's constant, $f$is the photon frequency, $C$is the photon speed, $\lambda$is the photon wavelength, $\varepsilon_0 \approx 8.854 \times 10^{-12}$ F/m is the electric permittivity of free space, and $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic permeability of free space.

Building on this idea, I argue that, consistent with my earlier equations and the aforementioned models, deep inside the d quark relativistic gravitational conditions prevail, which hold it together and define its nature. I believe that the d quark is a photon bound and deformed by its own internal gravitational field. Only a small internal part of that photon (the d quark) is under conditions where $r \approx r_s$ and $r > r_s$, so that the gravitational attractive force is in equilibrium with the electric repulsive force and the magnetic repulsive force (32).

Reasons and justifications for such reasoning:

1. The d quark could be a photon, because during the annihilation of a particle and its antiparticle, only photons can be released. "Positron Annihilation," "Electron-Positron Pair Production" https://shorturl.at/I2qpe

2. There is a sort of gravitational redshift during annihilation:
By calculation [15],[16],
$mC^2 = hf = hC/\lambda$,
$\lambda = h / (mC) = (6.626\times10^{-34}\ \text{Js}) / (8.9\times10^{-30}\ \text{kg} \times 3\times10^{8}\ \text{m/s}) \approx 2.48\times10^{-13}\ \text{m}$ (46)
Equation (46) gives an approximate size of the d quark, while it is measured to be around 10^{-18} m [17]. This is what I expect, since 2.48×10^{-13} m is the length of the unconfined d quark, while the length is 10^{-18} m when it is bound and compressed by its own internal, extremely strong gravitational field.

3. Deformed, because even if you are electromagnetically balanced (like a photon), existence and movement around your own relativistically dense interior most likely leads to an imbalance that manifests as a nonzero electric and magnetic field. In a way, this is similar to what causes a nonzero electromagnetic interaction between two conductors through which an electric current flows.
"How Einstein saved magnet theory", Fermilab, https://youtu.be/d29cETVUk-0?si=gcn7AbEjFdL3POu-&t=108

To briefly restate, the d quark is a self-confined particle whose small, densely packed inner core generates an intense gravitational field that relativistically deforms its exterior.

5. Formal Compatibility with Relativistic Equations: Photon Sphere and TOV

5.1 Photon Sphere Radius

In the vicinity of compact massive objects, such as black holes, gravity can bend light so strongly that photons may orbit the object on closed trajectories. This region, known as the photon sphere, defines the smallest radius at which massless particles can maintain a circular orbit. As described in classical treatments by Misner, Thorne, and Wheeler [1], and further explored in pedagogical accounts such as those by Carroll [2] and Levin [3], the photon sphere represents a critical threshold in the Schwarzschild geometry. For a non-rotating, uncharged black hole, the radius of the photon sphere is given by:

$$r_{ph} = \frac{3GM}{C^2} \quad (47)$$

where the Schwarzschild radius is:

$$r_s = \frac{2GM}{C^2} \quad (23)$$

The photon sphere radius can also be derived from **gravitational acceleration equation** (20):

$$a_c = -G\frac{M}{r^2}\frac{1}{1-\frac{2GM}{rC^2}} \quad (20)$$

The centripetal acceleration needed to hold a photon in a circular orbit is:

$$a_c = -\frac{C^2}{r_{ph}} \quad (48)$$

Equating the centripetal acceleration with my expression for gravitational acceleration:

$$-\frac{C^2}{r_{ph}} = -G\frac{M}{r_{ph}^2}\frac{1}{1-\frac{2GM}{r_{ph}C^2}} \quad (49)$$

$$C^2 = G\frac{M}{r_{ph}}\frac{1}{1-\frac{2GM}{r_{ph}C^2}} \quad (50)$$

$$r_{ph}\left(1-\frac{2GM}{r_{ph}C^2}\right) = \frac{GM}{C^2} \quad (51)$$

$$r_{ph} - \frac{2GM}{C^2} = \frac{GM}{C^2} \quad (52)$$

Hence, the photon sphere radius obtained from my modified gravitational acceleration equation is mathematically identical to the one predicted by general relativity:

$$r_{ph} = \frac{3GM}{C^2} \quad (47)$$

5.2 Gravitational Acceleration within the Tolman–Oppenheimer–Volkoff Equation

The Tolman–Oppenheimer–Volkoff (TOV) equation (48)[26] describes the condition of hydrostatic equilibrium of a spherically symmetric, self-gravitating fluid in general relativity.

$$\frac{dP}{dr} = -\frac{GM}{r^2}\frac{1}{1-\frac{2GM}{rC^2}}\left(1+\frac{P}{\rho C^2}\right)\left(1+\frac{4\pi r^3 P}{MC^2}\right)\rho \quad (48)$$

Here, $r$is the radial coordinate, $\rho(r)$is the density, $P(r)$is the pressure, and $M(r)$is the mass enclosed within radius r.

Using **expression for gravitational acceleration** (20), the TOV equation can be rewritten as:

$$\frac{dP}{dr} = a_c\left(1+\frac{P}{\rho C^2}\right)\left(1+\frac{4\pi r^3 P}{MC^2}\right)\rho \quad (49)$$

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• stylistic and grammatical refinement of the English text,
• restructuring of selected paragraphs for clarity and coherence,
• minor formatting adjustments to ensure consistency,
• verification of basic references and terminology where applicable.

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Conflict of Interest

The author declares that there is no conflict of interest

Data availability

No datasets were generated or analysed during the current study.

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For small x:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

used to extract approximate analytic solutions for equations involving singular denominators.

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