A Geometric Proof of Pi's Irrationality

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Abstract

Using two simple TI-84 programs we demonstrate how lines with rational ticks marks when wrapped around a circle can't locate the vertical radius at $\pi/2$. This implies that $\pi/2$ is irrational. The programs generate illustrations that yield a geometric proof.

Introduction

There have been many proofs of the irrationality of π [1, 3, 5]. The first is attributed to Lambert. It's long and complicated. In 1947 Niven greatly shortened the proof to half a page. Still his proof made various unacknowledged (hence obscure) references to the techniques of Hermite in his transcendence of e proof [6]. In both proofs the natural connection of π to the circle is quite remote.

The holy grail of such proofs is to reconnect such proofs with the circle. Here we give a geometric proof that seems to fit the bill. It seems to get at why π is irrational: lines aren't circles; when you wrap a line with rational tick marks around a circle you'll never hit (cover with a tick mark) $\pi/2$, a vertical from the origin radius; that's the essence of the proof given here. We give pictures with accompanying programs both using a TI-84 calculator: these reveal concretely the idea.

Geometric proofs of irrationality suffer from not being algebraic; they are attractive, to the point of being *cute*. Thus Sondow's geometric proof of the irrationality of e [9], Hardy's of the square root of five [4] and others are perhaps thought of as curiosities, not destined for standard analysis textbooks. But, I suggest, π's origins in geometry might make a geometric proof of its irrationality more natural and attractive (classy) to students and mathematicians.

Of course all these words are premised on the proof being correct. It may suffer from the same maladies as *proofs without words*, another genre of mathematical esoterica. If you don't see it in the illustrations provided (no words implies illustrations only), you're kind of stuck. We do provide words here, but the illustrations tell the story best. I see it, but frequently, I confess, I'm seeing things: supposing what I'm trying to prove. I hope that's not the case here: here goes something or nothing [2].

A Line With Tick Marks

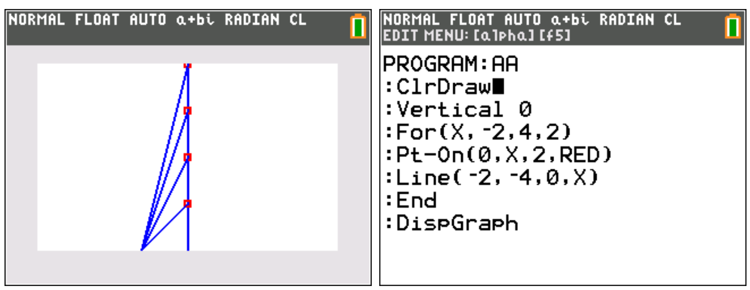

Figure 1: a) From any $1/Q$ a vertical line with such tick marks can be constructed; b) Various scaling ideas are revealed in the code.

In Figure 1 a) a vertical line with tick marks at $.5$, 1, 1.5, and 2 and b) the TI-84 program used for the illustration are shown. Lines are drawn that connect these dots (or tick marks) to a point left of the line. This point is to be understood as the center of a circle. If one extended the ruler vertically out to infinity with more of these $.5$ increments and generated more such lines, the slopes of the lines would approach infinity; that is the lines get closer and closer to becoming vertical.

A Circle

The tick marks in Figure 1 can get arbitrarily close to defining a vertical line, but can never generate a vertical line. Thus when we wrap this vertical line with its tick marks around the circle, as in Figure 2, the intersection of

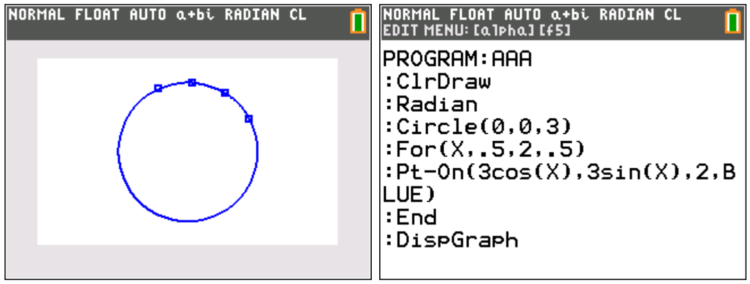

Figure 2: a) A circle with the line of Figure 1 wrapped around it; b) The code for generating this illustration. Scaling ideas are used but the idea does generalize.

the vertical y-axis with the circle will fall between tick marks; in this case, between 3 and 4. We know $\pi/2 = 1.57$ and that is between 1.5 and 2. But this is true for arbitrary $1/Q$, so no P/Q arc length can be equal to $\pi/2$: $\pi/2$ and hence π is irrational.

Conclusion

A few more words might be in order. For any $1/Q$ we can enter into the interval containing $\pi/2$ because the line of Figure 1 can get arbitrarily close to a vertical. Once it is in this arc interval, the next tick mark will be more than $\pi/2$ away. Do you see it? That is $\pi/2$ can't occur at any arc interval's endpoints. We can use any denominator in this same argument used for a denominator of 2.

References

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