What lies behind it?
-An attempt from matter field to quantum gravity-

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Abstract

We present an algebraic framework demonstrating that the spacetime manifold is an emergent structure from the field module rather than a pre-existing background. By employing a Clifford-valued gauge construction, we show that Minkowski spacetime is rigorously identified through the algebraic consistency of the field module network. In this framework, the spin connection is integrated as a dynamical messenger field on the same footing as the standard gauge bosons. Furthermore, this construction naturally leads to a Yang-Mills type gravity sector, which is known to be 1-loop renormalizable and asymptotically free, thereby providing a consistent UV completion. This framework offers a structural resolution to the cosmological constant problem by ensuring that vacuum energy does not contribute to the field equations, providing a consistent foundation for integrating quantum gravity with the Standard Model without an a priori manifold.

1 Introduction

The formulation of a consistent and conceptually transparent framework for quantum gravity remains the most formidable challenge in contemporary theoretical physics. Currently, the field is dominated by two major paradigms: string theory [1], which seeks unification by replacing point particles with extended objects in higher-dimensional backgrounds, and loop quantum gravity [2], which attempts a non-perturbative, background-independent quantization of geometry itself. While these approaches have provided profound insights, they often encounter persistent conceptual difficulties regarding the recovery of a smooth spacetime manifold at low energies or the maintenance of strict background independence.

Alternative perspectives suggest that the gravitational interaction should be understood through the lens of gauge theory. The discovery of the Ashtekar variables [3] and the comprehensive operator formalism developed by Nakanishi [4, 5] have established that general relativity can be profoundly reformulated as a gauge theory of connections. This viewpoint, rooted in the pioneering work of Utiyama [6], suggests that gravity may be treated on the same footing as other fundamental forces, provided that the underlying connection structure is properly identified within the matter sector.

Einstein's operationalist perspective [7, 8] provides the essential clue for this identification. He emphasized that space and time are not *a priori* backgrounds but are emergent structures

grounded in the behavior of physical processes. This theme, shared by Schrödinger [9] and Wheeler [10], leads to the hypothesis that the spacetime manifold is a manifestation of field relations. The present work evolves this lineage by extending the program initiated by Sogami [11, 12], in which field indices are treated as internal degrees of freedom from the outset. In this framework, Minkowski spacetime is not an assumed stage but is recognized through the algebraic consistency of the field module network.

Crucially, the generalized covariant derivative employed here leads naturally to a Yang-Mills type gravity sector. Unlike the non-renormalizable Einstein-Hilbert action, such theories are known to be 1-loop renormalizable and asymptotically free [13, 14], offering a robust UV completion. Furthermore, this framework provides a structural response to long-standing paradoxes. The cosmological constant problem [15] is addressed by a mechanism that decouples vacuum energy from the emergent field equations, while still allowing for the large vacuum energy densities required to initiate cosmic inflation [16, 17]. Additionally, the emergent nature of spacetime at the Planck scale suggests a resolution to the black hole information loss paradox [18] by replacing classical singularities with the underlying algebraic consistency of the field module.

By integrating these features with the concept of "little gauge theory" [19], this work resonates with modern emergent gravity programs [20, 21] and the noncommutative geometry of Connes [22, 23]. Mathematically grounded in fiber bundle theory [24], the present framework derives the effective spacetime structure directly from the internal symmetries of matter field modules, providing a background- independent pathway toward reconciling local field theory with gravitational phenomena.

2   Matter field and Field module

We consider a matter field $||\Psi\rangle\rangle$ expanded in a 4-component vector $|\Psi_I\rangle$ and a 4-dimensional internal basis $|e^I\rangle$, where the index $I = 1, 2, 3, 4$ labels the internal components:

$$||\Psi\rangle\rangle = |\Psi_I\rangle|e^I\rangle, \tag{1}$$

with summation over repeated internal index I understood. The basis is equipped with an internal metric, where $(\gamma^0)^{IJ}$ denotes the metric of the internal space

$$\langle e^I|e^J\rangle = (\gamma^0)^{IJ}, \qquad \gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{2}$$

Under a general active change of the components

$$|\Psi_I\rangle \to |\Psi'_I\rangle = (T)_I{}^J|\Psi_J\rangle, \tag{3}$$

the internal basis transforms according to the inverse matrix T^{-1} of the corresponding transformation $\widehat{T}$:

$$|e^I\rangle \rightarrow |e'^I\rangle = \widehat{T}^{-1}|e^I\rangle = (T^{-1})_J{}^I|e^J\rangle, \tag{4}$$

where the hat symbol is used to distinguish the action on $|\Psi_I\rangle$ from that on $|e^I\rangle$. Consequently, the matter field $||\Psi\rangle\rangle$ remains invariant.

Consider an internal metric-preserving transformation $\widehat{U}$, related to the passive transformation $|e^I\rangle \rightarrow |e'^I\rangle = \widehat{U}|e^I\rangle = U_J{}^I|e^J\rangle$, which satisfies the pseudo-unitarity condition

$$U^{\dagger I}{}_K (\gamma^0)^{KL} U_L{}^J = (\gamma^0)^{IJ} \tag{5}$$

Unless otherwise noted, we can write this equation in matrix form as

$$U^\dagger \gamma^0 U = \gamma^0. \tag{6}$$

To construct matrices that satisfy the pseudo-unitary condition (6), it is natural to introduce a matrix $\mathcal{M}$ satisfying

$$\mathcal{M}^\dagger \gamma^0 = \gamma^0 \mathcal{M}. \tag{7}$$

The most natural basis for such matrices acting on the internal basis $\{|e^I\rangle\}$ is given by the gamma matrices $\widehat{\gamma}_a$, where the index $a = 0, 1, 2, 3$ is a Clifford index labeling the gamma matrices and the action of $\widehat{\gamma}^a$ on the internal basis $|e^I\rangle$ is defined by $\widehat{\gamma}^a|e^I\rangle = (\gamma^a)_J{}^I|e^J\rangle$. These matrices satisfy the Clifford algebra

$$\{\widehat{\gamma}^a, \widehat{\gamma}^b\} = 2\eta^{ab}\widehat{\mathbb{I}}, \qquad \eta^{ab} = \mathrm{diag}(1, -1, -1, -1), \tag{8}$$

where $\widehat{\mathbb{I}}$ is the identity operator. We stress that the object η^{ab} appearing in the Clifford algebra $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ is not a space-time metric. The Clifford indices a, b are merely labels for the generators of the algebra and are not subject to index raising or lowering. Therefore, no Einstein summation is assumed for Clifford indices unless explicitly stated.

A complete basis for the 16-dimensional space of 4×4 operaters acting on the internal basis $|e^I\rangle$ is given by

$$\widehat{\mathbb{I}}, \qquad i\widehat{\gamma}^5, \qquad \widehat{\gamma}^a, \qquad \widehat{\gamma}_5\widehat{\gamma}^a, \qquad \widehat{\sigma}^{ab}, \tag{9}$$

where

$$\widehat{\gamma}^5 = i\widehat{\gamma}^0\widehat{\gamma}^1\widehat{\gamma}^2\widehat{\gamma}^3, \qquad \widehat{\sigma}^{ab} = \frac{i}{4}[\widehat{\gamma}^a, \widehat{\gamma}^b]. \tag{10}$$

The factor $i\widehat{\gamma}^5$ is introduced so that this basis satisfies the pseudo-unitary condition (7).

In the doubled-state formalism, the adjoint of the gamma operator $\widehat{\gamma}^a$ is defined with respect to the internal inner product, whose metric is γ^0. With this definition of the adjoint, the gamma operators satisfy

$$(\widehat{\gamma}^a)^\dagger = \widehat{\gamma}^a, \tag{11}$$

3

and are therefore self-adjoint as operators. This, however, does not imply that the component matrices γ^a are Hermitian in the usual sense. Their adjoint is instead determined by the internal metric:

$$(\gamma^a)^\dagger = \gamma^0 \, \gamma^a \, \gamma^0. \tag{12}$$

The 16 representation matrices form a linearly independent set, so that any 4×4 matrix obeying (7) can be expressed as a linear combination of them. Thus the general representation matrix $\mathcal{M}$ satisfying (7) can be expanded as

$$\mathcal{M} = \chi\mathbb{I} + \sum_{ab} \eta^{ab}\gamma^a\chi^b + i\gamma^5\xi + \gamma^5 \sum_{a,b} \eta^{ab}\gamma^a\chi_5^b + \sum_{a,b,c,d} \eta^{ac}\eta^{bd}\sigma^{ab}\lambda^{cd}, \tag{13}$$

with real parameters χ, χ^a, ξ, ξ^a and λ^{ab}. Thanks to the representation matrix $\mathcal{M}$, the corresponding pseudo-unitary transformation U acting on the internal basis can be written in exponential form:

$$U = \exp(i\mathcal{M}) = \exp\left(i(\chi\mathbb{I} + \sum_{ab} \eta^{ab}\gamma^a\chi^b + i\gamma^5\xi + \sum_{a,b} \eta^{ab}\gamma^5\gamma^a\chi_5^b + \sum_{a,b,c,d} \eta^{ac}\eta^{bd}\sigma^{ab}\lambda^{cd})\right) \tag{14}$$

which leads to

$$\widehat{U} = \exp(i\widehat{\mathcal{M}}) = \exp\left(i(\chi\widehat{\mathbb{I}} + \sum_{ab} \eta^{ab}\widehat{\gamma}^a\chi^b + i\widehat{\gamma}^5\xi + \sum_{a,b} \eta^{ab}\widehat{\gamma}^5\widehat{\gamma}^a\chi_5^b + \sum_{a,b,c,d} \eta^{ac}\eta^{bd}\widehat{\sigma}^{ab}\lambda^{cd})\right). \tag{15}$$

Remarkably, although U is only pseudo-unitary at the component level, $\widehat{U}$ turns out to be exactly unitary as an operator, which is a non-trivial consequence of the internal metric structure.

While the matter field is unchanged under both the active and passive transformations, covariance is discussed in terms of either active or passive transformation as stated above. For the sake of this purpose, matter field transformation is defined by

$$||\Psi\rangle\rangle \to ||\Psi'\rangle\rangle = |\Psi'_I\rangle|e^I\rangle \quad \text{with} \quad |\Psi'_I\rangle = (U^{-1})_I{}^J|\Psi_J\rangle. \tag{16}$$

We define a field module 4-component

$$J^a = \langle\langle\Psi||\widehat{\gamma}^a||\Psi\rangle\rangle = \langle\Psi_I|(\gamma^0)^{IJ}(\gamma^a)_J{}^K|\Psi_K\rangle, \tag{17}$$

which provides a mapping from $||\Psi\rangle\rangle$, composed of the internal 4-vector basis $\{|e^I\rangle\}$ and the field vector $|\Psi_I\rangle$. Note that $\widehat{\gamma}^a$, when acting on the internal basis $|e^I\rangle$, effectively reduces to the action of γ^a on the field vector $|\Psi_I\rangle$ after taking the internal inner product.

Consider the full pseudo-unitary transformation representation matrix U on the internal space. Let $\mathcal{U}$ denote the group of all pseudo-unitary transformations satisfying (6). Among

4

these transformations, we focus on a normal subgroup $\mathcal{G} \subset \mathcal{U}$ of the Clifford algebra that preserves the structure of the field module J^a. Element G in the subgroup $\mathcal{G}$ must satisfy

$$G\gamma^a G^{-1} = \sum_{bc} \eta^{bc}\Lambda^{ab}\gamma^c, \quad \Lambda^{ab} = (e^{2\lambda})^{ab} \tag{18}$$

so that under the transformation by G, the field module under the matter transformation transforms covariantly as

$$J^a \to J'^a = \langle\langle\Psi'\|\widehat{\gamma}^a\|\Psi'\rangle\rangle = \sum_{b,c} \eta^{bc}\Lambda^{ab}J^c. \tag{19}$$

Consequently, the G in such a subgroup $\mathcal{G}$ can be realized as

$$G = \exp(i\mathcal{M}) = \exp\left(i\chi I + i\sum_{a,b,c,d} \eta^{ac}\eta^{bd}\sigma^{ab}\lambda^{cd}\right) \in \mathcal{U}. \tag{20}$$

In this way, the choice of G is dictated by the requirement that the field module remains closed under its action, naturally selecting the subgroup generated by the identity and the spin generators σ^{ab}. All possible bilinear forms of the matter field, including the field module J^a, transform as follows:

- $\langle\langle\Psi\|\Psi\rangle\rangle$ and $\langle\langle\Psi\|i\widehat{\gamma}^5\|\Psi\rangle\rangle$ are Clifford index scalar invariants,

- $\langle\langle\Psi\|\widehat{\gamma}^a\|\Psi\rangle\rangle$ and $\langle\langle\Psi\|\widehat{\gamma}^a\widehat{\gamma}^5\|\Psi\rangle\rangle$ are Clifford index covariant vectors,

- $\langle\langle\Psi\|\widehat{\sigma}^{ab}\|\Psi\rangle\rangle$ is a Clifford index covariant tensor.

In this section, the space-time structure has not yet been introduced, and the index a appearing in $\widehat{\gamma}^a$ is merely an internal Clifford index. Its identification with space-time coordinates will be clarified in the following section. For clarity, these Clifford indices carry no geometric content and are not raised or lowered, and no Einstein summation is assumed unless explicitly indicated.

3 Matter field parameterization and Gauge connection

We introduce a new basis $\{\|q; \psi(q), e(q)\rangle\}$, in terms of which the local sections of the matter field $\|\Psi\rangle\rangle$ and its conjugate are expressed as [1]

$$((q; \psi(q), e(q)\|\Psi\rangle\rangle = \theta^\dagger \psi_I(q)\,\mathbf{e}^I(q), \quad \langle\langle\Psi\|q; \psi(q), e(q))) = \mathbf{e}^{\dagger I}(q)\,\psi_I^\dagger(q)\theta \tag{21}$$

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[1]This projection can be considered as a *local section* of the matter field. More precisely, at each parameter point q, the field is expressed in terms of a local 4-component vector $\psi_I(q)$ and a local internal basis $\{\mathbf{e}^I(q)\}$, which spans the internal Hilbert space at q. The collection of these local expressions over all q forms a sheaf-like structure, allowing the global matter field $\|\Psi\rangle\rangle$ to be reconstructed from its local sections.

where q denotes the bundled components q^a with Clifford indices $a = 0, 1, 2, 3$ and we emphasize that the parameter q^a is just an inert real parameter. Distances between points are defined in the Euclidean sense. Here θ and $\theta^\dagger$ are auxiliary global Grassmann-odd variables, introduced solely to adjust Grassmann parity. They satisfy the normalization condition $\theta\theta^\dagger = -\theta^\dagger\theta = 1$, and carry no q-dependence nor dynamical degrees of freedom. In addition, $\psi_I(q)$ is a four-component Grassmann-odd valued Dirac field, and $\{\mathbf{e}^I(q)\}$ is a vector Grassmann-even basis defined at each parameter point q, satisfying

$$\mathbf{e}^{\dagger I}(q) \cdot \mathbf{e}^J(q) = (\gamma_0)^{IJ}. \tag{22}$$

The new basis forms a complete set

$$\int ||q; \psi(q),\, e(q))) \,((q; \psi(q),\, e(q)|| \; d^4q \, \mathcal{D}\psi \, \mathcal{D}e = 1, \tag{23}$$

where the integration measures $\mathcal{D}\psi$ and $\mathcal{D}e$ should be understood as an aggregation over the local sections $\psi_I(q)$ and the local auxiliary degrees of freedom representing the gauge field, respectively, at each parameter point q. In this sense, it resembles a functional integral, collecting contributions from all local sections to represent the global matter field $||\Psi\rangle\rangle$. And the quantity q^a may be viewed as coordinates induced from the field module $J^a = \langle\langle\Psi||\widehat{\gamma}^a||\Psi\rangle\rangle$, but they remain inert under all field-module transformations in this stage.

With these identifications, the bilinear form $\langle\langle\Psi||\Psi\rangle\rangle$ can be expanded as

$$
\begin{aligned}
\langle\langle\Psi||\Psi\rangle\rangle &= \int \langle\langle\Psi||q; \psi(q),\, e(q))) ((q; \psi(q),\, e(q)||\Psi\rangle\rangle \, d^4q \, \mathcal{D}\psi \, \mathcal{D}e \\
&= \int \mathbf{e}^{\dagger I}(q)\, \psi_I^\dagger(q) \,\cdot\, \psi_J(q)\, \mathbf{e}^J(q)\, d^4q \, \mathcal{D}\psi \, \mathcal{D}e \\
&= \int \psi_I^\dagger(q)\, (\gamma^0)^{IJ} \psi_J(q)\, d^4q \, \mathcal{D}\psi \, \mathcal{D}e \\
&= \int \bar{\psi}(q)\psi(q)\, d^4q \, \mathcal{D}\psi \, \mathcal{D}e.
\end{aligned}
\tag{24}
$$

Similarly, the remaining bilinear forms are given by

$$\langle\langle\Psi||i\widehat{\gamma}_5||\Psi\rangle\rangle = \int \bar{\psi}(q) i\gamma^5 \psi(q)\, d^4q \, \mathcal{D}\psi \, \mathcal{D}e,$$

$$\langle\langle\Psi||\widehat{\gamma}^a||\Psi\rangle\rangle = \int \bar{\psi}(q) \gamma^a \psi(q)\, d^4q \, \mathcal{D}\psi \, \mathcal{D}e,$$

$$\langle\langle\Psi||\widehat{\gamma}^5\widehat{\gamma}^a||\Psi\rangle\rangle = \int \bar{\psi}(q) \gamma^5\gamma^a \psi(q)\, d^4q \, \mathcal{D}\psi \, \mathcal{D}e,$$

$$\langle\langle\Psi||\widehat{\sigma}^{ab}||\Psi\rangle\rangle = \int \bar{\psi}(q) \sigma^{ab} \psi(q)\, d^4q \, \mathcal{D}\psi \, \mathcal{D}e.$$

6

Here, $\widehat{\gamma}^a$, when acting on the internal basis $|e^I\rangle$, effectively reduces to the action of γ^a on the column vector $\psi(q)$ after taking the internal inner product, and the indices associated with the internal space are suppressed.

Any linear combination of bilinear forms can be expressed by inserting a matrix $\widehat{C}$ acting on the internal basis. The operator $\widehat{C}$ itself is a linear combination of the basis elements $\widehat{\mathbb{I}}$, $i\widehat{\gamma}^5$, $\widehat{\gamma}^a$, $\widehat{\gamma}^5\widehat{\gamma}^a$, and $\widehat{\sigma}^{ab}$, so it can be written as a linear combination of the corresponding bilinear forms $\langle\langle\Psi||\widehat{C}||\Psi\rangle\rangle$. Since the choice of the parametrization of q is arbitrary, we obtain

$$\delta\langle\langle\Psi||\widehat{C}||\Psi\rangle\rangle = \int \langle\langle\Psi||q'; \psi(q'), e(q'))((q'; \psi(q'), e(q')|\widehat{C}|\Psi\rangle\rangle \, d^4q' \, \mathcal{D}\psi \, \mathcal{D}e$$

$$- \int \langle\langle\Psi||q; \psi(q), e(q))((q; \psi(q), e(q)|\widehat{C}|\Psi\rangle\rangle \, d^4q \, \mathcal{D}\psi \, \mathcal{D}e = 0. \qquad (25)$$

In particular, the volume element d^4q is invariant under a constant translation $q'^a = q^a + c^a$, i.e. $d^4q' = d^4q$. Thus for an infinitesimal translation $q'^a = q^a + \epsilon^a$, the above equation becomes

$$\delta\langle\langle\Psi||\widehat{C}||\Psi\rangle\rangle = \int \left[\bar{\psi}(q+\epsilon)C\psi(q+\epsilon) - \bar{\psi}(q)C\psi(q) \right] d^4q \, \mathcal{D}\psi \, \mathcal{D}e$$

$$= \int \partial^a \left(\bar{\psi}(q)C\psi(q) \right) \epsilon^a \, d^4q \, \mathcal{D}\psi \, \mathcal{D}e = 0, \qquad (26)$$

which shows the translation invariance of the parameter q^a. Here $\partial^a = \frac{\partial}{\partial q^a}$, C is the representation matrix of $\hat{C}$. We emphasize again that the index a is merely a Clifford index with no metric or geometric meaning. On the other hand, noting that

$$\langle\langle\Psi||q; \psi(q), e(q))((q; \psi(q), e(q)|\widehat{C}|\Psi\rangle\rangle = \psi_I^\dagger(q)\mathbf{e}^{\dagger I}(q) \cdot C_K{}^J \psi_J(q)\mathbf{e}^K(q), \qquad (27)$$

we obtain

$$\delta\langle\langle\Psi||\widehat{C}||\Psi\rangle\rangle = \int \partial^a \left(\mathbf{e}^{\dagger I}(q)\psi_I^\dagger(q) \cdot C_K{}^J \psi_J(q)\mathbf{e}^K(q) \right) \epsilon^a \, d^4q \mathcal{D}\psi \, \mathcal{D}e = 0. \qquad (28)$$

From comparison with equations (26) and (28), it is natural to write the derivative of $\mathbf{e}^I(q)$ as

$$\partial^a \mathbf{e}^I(q) = i\mathcal{A}^a{}_J{}^I(q)\mathbf{e}^J(q), \qquad (29)$$

where a straightforward calculation leads to the compatibility condition

$$\mathcal{A}^{a\dagger I}{}_L(q)(\gamma^0)^{LK} = (\gamma^0)^{IL}\mathcal{A}^a{}_L{}^K(q), \qquad (30)$$

which can be written in matrix form as $\mathcal{A}^{a\dagger}(q)\gamma^0 = \gamma^0\mathcal{A}^a(q)$. In addition, the above equation ensures that the representation of $\widehat{\gamma}^a$ in terms of $\mathbf{e}^I(q)$ and $\mathbf{e}^{\dagger I}(q)$ is independent of the parameter

q within the local section. From the most general expansion in the Clifford algebra compatible with (30), the general form of $\mathcal{A}^a(q)$ can be written as

$$\mathcal{A}^a(q) = g_1 A^a(q)\mathbf{I} + g_2 \sum_{b,c} \Phi^{ab}(q)\eta^{bc}\gamma^c + ig_3 A_5{}^a(q)\gamma^5$$
$$+ g_4 \sum_{b,c} \Phi_5^{ab}(q)\eta^{bc}\gamma^5\gamma^c + g_5 \sum_{b,d,c,h} \eta^{bd}\eta^{ch}\omega^{abc}(q)\sigma^{dh}, \tag{31}$$

where the local fields $A^a(q), \Phi^{ab}(q), A_5^a(q), \Phi_5^{ab}(q), \omega^{abc}(q)$ are interpreted as Hermitian gauge fields. Note that $\mathcal{A}^a(q)$ itself is not a Hermitian field. Introducing an effective covariant derivative $\overrightarrow{\mathcal{D}}^a = \partial^a + i\mathcal{A}^a(q)$, the derivative of the field including the internal basis can be compactly written as

$$\mathbf{e}^{\dagger J}(q)\psi_J^\dagger(q) \cdot \partial_a((C_K)^I \psi_I(q)\mathbf{e}^K(q)) = \bar{\psi}(x)\overrightarrow{\mathcal{D}}^a C\psi(q). \tag{32}$$

Under simultaneous local active and passive transformations that preserve the internal metric,

$$\psi_I(q) \to \psi_I'(q) = (U^{-1})_I{}^J(q)\psi_J(q), \quad \mathbf{e}^I(q) \to \mathbf{e}'^I(q) = U_J{}^I(q)\mathbf{e}^J(q), \tag{33}$$

the theory remains invariant, where U is the local representation matrix of $\hat{U}$ in Eq. (15). In this sense, the gauge principle naturally emerges from the internal geometric structure, rather than being imposed as an independent postulate. In fact, using the relation $\partial_a\mathbf{e}'^I(q) = i\mathcal{A}'^a{}_J{}^I\mathbf{e}^J(q)$, the corresponding gauge transformations are

$$\psi(q) \to \psi_I'(q) = U^{-1}(q)\psi(q),$$
$$\mathcal{A}^a(q) \to \mathcal{A}'^a(q) = U^{-1}(q)\mathcal{A}^a(q)U(q) - iU^{-1}(q)\partial^a U(q),$$
$$C \to C'(q) = U^{-1}(q)CU(q), \tag{34}$$

where $C'(q)$ is a representation matrix of $\hat{C}$ acting on $\mathbf{e}'^I(q)$. Utilizing the local gauge degree of freedom, we henceforth adopt the basis $\{\mathbf{e}^I(q)\}$ such that the representation matrix C of $\hat{C}$ becomes independent of the parameter q.

4 Lagrangian Construction from Matter Module Operator, Dynamical Link Kernel and Field Strength

We define a unit operator $||\Psi\rangle\rangle\langle\langle\Psi||$ on the field module by

$$||\Psi\rangle\rangle\langle\langle\Psi|| = |\Psi_I\rangle |e^I\rangle\langle e^J| \langle\Psi_J| = 1. \tag{35}$$

The components $|\Psi_I\rangle$ and $|e^I\rangle$ satisfy

$$\langle\Psi_I|(\gamma^0)^{IJ}|\Psi_J\rangle = 1, \qquad \langle e^I|e^J\rangle = (\gamma^0)^{IJ}, \tag{36}$$

8

and

$$|\Psi_I\rangle(\gamma^0)^{IJ}\langle\Psi_J| = 1, \qquad |e^I\rangle\langle e^J| = (\gamma^0)^{IJ}. \tag{37}$$

This defines a completeness relation with respect to the indefinite inner product induced by γ^0. For the convenience, the upper and lower indices are related by

$$\psi^I = (\gamma^0)^{IJ}\psi_J, \quad \psi_I = (\gamma^0)_{IJ}\psi^J, \tag{38}$$

and similarly for the basis vectors $|e^I\rangle$ and $\langle e_I|$.

Associated with the field module, we define four related states through the action of the Dirac matrices,

$$||\Psi^{(a)}\rangle\rangle = \widehat{\gamma}^a||\Psi\rangle\rangle. \tag{39}$$

Using these states, we introduce the field module operators $\widehat{J}^a$ as

$$\begin{aligned}
\widehat{J}^a &= ||\Psi^{(a)}\rangle\rangle\langle\langle\Psi|| = \widehat{\gamma}^a||\Psi\rangle\rangle\langle\langle\Psi|| \\
&= (\gamma^a)_J{}^I |\Psi_I\rangle |e^J\rangle\langle e^K| \langle\Psi_K|.
\end{aligned} \tag{40}$$

The expectation value of $\widehat{J}^a$ for the matter field $||\Psi\rangle\rangle$ yields the corresponding field module,

$$\langle\langle\Psi||\widehat{J}^a||\Psi\rangle\rangle = J^a. \tag{41}$$

The adjoint operators are defined by

$$\widehat{J}^{\dagger a} = ||\Psi\rangle\rangle\langle\langle\Psi^{(a)}||, \tag{42}$$

which turns out the Hermite operator $\widehat{J}^{\dagger a} = \widehat{J}^a$. The field module operators satisfy the pseudo–unitary relations

$$\widehat{J}^a\widehat{J}^{\dagger a} = \widehat{J}^{\dagger a}\widehat{J}^a = ||\Psi^{(a)}\rangle\rangle\langle\langle\Psi^{(a)}|| = \eta^{aa} ||\Psi\rangle\rangle\langle\langle\Psi|| = \eta^{aa} \tag{43}$$

for each value of a.

For an infinitesimal displacement from q to $q + dq^a$ in the a-direction, we now introduce an infinitesimal dynamical link kernel $K(q, q + dq^a)$ from a state $||q; \psi(q), e(q)))$ to a state $||q + dq^a; \psi(q + dq^a), e(q + dq^a)))$ defined in terms of the field module operator $\widehat{J}^a$ as

$$K(q, q + dq^a) = ((q + dq^a; \psi(q + dq^a), e(q + dq^a)||\widehat{J}^a||q; \psi(q), e(q))). \tag{44}$$

The midpoint evaluation for the dynamical link kernel $K(q, q + dq^a)$ leads to the following s:

$$
\begin{aligned}
&K(q; q + dq^a) \\
&= \theta^\dagger \psi_I(q + dq^a) \mathbf{e}^I(q + dq^a)(\gamma^{\dagger a})^J{}_K \mathbf{e}^{\dagger\, K}(q) \psi^\dagger_J(q)\theta \\
&= (\mathbf{e}^{\dagger\, K}(q + dq^a/2)\psi^\dagger_J(q + dq^a/2) - \partial^a(\mathbf{e}^{\dagger\, K}(q + dq^a/2)\psi^\dagger_J(q + dq^a/2))dq^a/2)(\gamma^{\dagger a})^J{}_K \\
&\qquad (\psi_I(q + dq^a/2)\mathbf{e}^I(q + dq^a/2) + \partial^a(\psi_I(q + dq^a/2)\mathbf{e}^I(q + dq^a/2))dq^a/2) + o(dq^a) \\
&= \psi^\dagger_J(q + dq^a/2)(\gamma^{\dagger a})^J{}_K(\gamma^0)^{KI}\psi_I(q + dq^a/2) \\
&\quad + \frac{1}{2}\Big(\psi^\dagger_J(q + dq^a/2)(\gamma^{\dagger a})^J{}_K(\gamma^0)^{KI}\partial^a\psi_I(q + dq^a/2) \\
&\qquad\quad - \partial^a\psi^\dagger(q + dq^a/2)_J(\gamma^{\dagger a})^J{}_K(\gamma^0)^{KI}\psi_I(q + dq^a/2)\Big)dq^a \\
&\quad + \frac{1}{2}\Big(\psi^\dagger_J(q + dq^a/2)(\gamma^{\dagger a})^J{}_K(\gamma^0)^{KL}i\mathcal{A}^a{}_L{}^I(q + dq^a/2)\psi_I(q + dq^a/2)\Big)dq^a \\
&\quad + \frac{1}{2}\Big(\psi^\dagger_J(q + dq^a/2)(\gamma^{\dagger a})^J{}_K i\mathcal{A}^{\dagger aK}{}_L(q + dq^a/2)(\gamma^0)^{LI}\psi_I(q + dq^a/2)\Big)dq^a + o(dq^a) \\
&= \bar\psi(q)\gamma^a\psi(q) + \frac{1}{2}\Big(\partial^a(\bar\psi(q)\gamma^a\psi(q)) + \bar\psi(q)\gamma^a(-\overleftarrow{\partial}^a + \overrightarrow{\partial}^a)\psi(q)\Big)dq^a \\
&\quad + \frac{1}{2}\Big(\bar\psi(q)\gamma^a i\mathcal{A}^a\psi(q) + \bar\psi(q)\gamma^a i\tilde{\mathcal{A}}^a\psi(q)\Big)dq^a + o(dq^a) \\
&= \bar\psi(q)\gamma^a\psi(q) + \Big(\bar\psi(q)\gamma^a\partial^a\psi(q) + \bar\psi(q)\gamma^a i\bar{\mathcal{A}}^a\psi(q)\Big)dq^a + o(dq^a) \\
&= \bar\psi(q)\gamma^a\psi(q) - i\bar\psi(q)\gamma^a i\mathcal{D}^{(eff)a}\psi(q)dq^a + o(dq^a),
\end{aligned}
\tag{45}
$$

where

$$
\begin{aligned}
\tilde{\mathcal{A}}^a(q) \;=\;& g_1 A^a(q)\mathbf{I} + g_2\sum_{b,c}\Phi^{ab}(q)\eta^{bc}\gamma^c - ig_3 A_5{}^a(q)\gamma^5 \\
& + g_4\sum_{b,c}\Phi_5^{ab}(q)\eta^{bc}\gamma^5\gamma^c + g_5\sum_{b,d,c,h}\eta^{bd}\eta^{ch}\omega^{abc}(q)\sigma^{dh},
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
\bar{\mathcal{A}}^a(q) \;=\;& \frac{1}{2}(\mathcal{A}^a(q) + \tilde{\mathcal{A}}^a(q)) \\
=\;& g_1 A^a(q)\mathbf{I} + g_2\sum_{b,c}\Phi^{ab}(q)\eta^{bc}\gamma^c \\
& + g_4\sum_{b,c}\Phi_5^{ab}(q)\eta^{bc}\gamma^5\gamma^c + g_5\sum_{b,d,c,h}\eta^{bd}\eta^{ch}\omega^{abc}(q)\sigma^{dh},
\end{aligned}
\tag{47}
$$

and the effective generalized covariant derivative $\mathcal{D}^{(eff)a}$ is obtained as

$$
\mathcal{D}^{(eff)a} \;=\; \partial_a + i\bar{\mathcal{A}}^a(q).
\tag{48}
$$

Note that the effective gauge field does not couple with the non-Hermitian axial gauge term $A_5^a(q_l)$.

After the construction of the basis $\mathbf{e}^I(q + dq^a/2)$, the dynamical link kernel is described in terms of the field $\psi(q)$ and the effective generalized covariant derivative $\overrightarrow{\mathcal{D}}^{(eff)a}$. We now consider a global active transformation $G^{-1} \in \mathcal{G}$ for the entire system. Under this transformation, the spinor field transforms as follows:

$$\psi(q) \to \psi'(q) = G^{-1}\psi(q). \tag{49}$$

However, a fundamental structural inconsistency arises within the kinetic-like term of the dynamical link kernel. The contraction between the Clifford basis and the covariant derivative transforms as:

$$\bar{\psi}\gamma^a \overrightarrow{\mathcal{D}}^{(eff)a}\psi \to \bar{\psi}G\gamma^a(\partial^a + i\bar{\mathcal{A}}^a)G^{-1}\psi = \bar{\psi}\left(\sum_b \Lambda^{ab}\gamma^b\right)(\partial^a + i\bar{\mathcal{A}}^a)\psi. \tag{50}$$

In this expression, while the Clifford basis has rotated into the new configuration $\gamma'^a = \sum_b \Lambda^{ab}\gamma^b$, the derivative $\partial^a = \partial/\partial q^a$ and the effective gauge field $\bar{\mathcal{A}}^a$ remain anchored to the static index a of the parameter space q^a.

This reveals that the fixed labels of the parameter space q^a are no longer capable of tracking the dynamics of the matter field. As the Clifford basis γ^a rotates to a new direction due to the active transformation of $\psi(q)$, the directional indices of the derivative and the gauge field must also move to maintain their alignment with the field's internal frame. In other words, we must redefine our coordinate axes so that they always point in the same direction as the matter field's internal frame.

By introducing a new physical coordinate system x^a that is defined to move in tandem with the transformation of γ^a, we ensure that the operator:

$$\gamma^a(\partial_{x^a} + i\bar{\mathcal{A}}_{x^a}) \tag{51}$$

is always described in a frame synchronized with the field itself. The transition $q^a \to x^a$ is thus a requirement that the coordinate directions must be attached to the orientation of the matter field, rather than being fixed to an external, static parameter space.

By synchronizing the coordinates x^a with the internal rotation of the matter field, we successfully resolve the index mismatch. In this new coordinate system, the directional labels of the derivative $\partial/\partial x^a$ and the gauge field $\bar{\mathcal{A}}_{x^a}$ are always aligned with the rotated Clifford basis γ^a. As a result, the dynamical link from the original a-direction to the new a'-direction is consistently established without structural contradiction. This synchronization process marks the realization of the parameter space as a physical spacetime, yielding a well-defined, covariant kinetic term that reflects the true dynamical evolution of the system.

This synchronization implies that the coordinate system is no longer a mere mathematical background, but is fundamentally defined by the internal structure of the matter field. Through this process, we achieve the **recognition of physical spacetime**; the parameter space q^a is transformed into the physical spacetime x^a, where the Lorentz index a faithfully tracks the dynamical orientation of the field. In this sense, spacetime emerges as a consequence of the matter field's need for structural consistency under active transformations.

It should be emphasized that, from this point forward, the physical coordinates x^a will be employed as the fundamental variables for describing the dynamics, superseding the original parameter space q^a. To construct the total Lagrangian density from these individual transitions, we integrate the contributions of the dynamical links across all available directions. This summation is naturally expressed by adopting the Einstein summation convention for the index a:

$$\mathcal{L}_0 = \bar{\psi}\gamma^a i \left(\partial_{x^a} + i\bar{\mathcal{A}}_{x^a}\right)\psi. \tag{52}$$

Here, the repeated index a signifies that the matter field's kinetic term is the collective result of its links in every physical direction of the recognized spacetime. The resulting Lagrangian density $\mathcal{L}_0$ is thus manifest Lorentz invariant as a direct consequence of the synchronization between the Clifford basis and the physical coordinate axes. This invariance is not merely a formal requirement, but a structural property arising from the fact that our coordinate system x^a is intrinsically locked to the matter field's internal orientation.

Furthermore, the field strength $\bar{\mathcal{F}}_{ab}$ is naturally obtained from the "space-time rotation" of the connection-specifically, the holonomy around an infinitesimal closed loop in the x^a coordinates. This curvature reflects how the matter fields are twisted as they span across the recognized spacetime. By evaluating the non-commutativity of these transitions, we arrive at Sogami's generalized definition of the field strength:

$$\bar{\mathcal{F}}_{ab} = -i[\overrightarrow{\mathcal{D}}_{x^a}^{(eff)}, \overrightarrow{\mathcal{D}}_{x^b}^{(eff)}], \tag{53}$$

with the effective generalized covariant derivative $\overrightarrow{\mathcal{D}}_{x^a}^{(eff)}$

$$\begin{aligned}
\overrightarrow{\mathcal{D}}_{x^a}^{(eff)} &= \partial_{x^a} + i\bar{\mathcal{A}}_a(q) \\
&= \partial_{x^a} + i\left(g_1 A_a \mathbf{I} + g_2 \Phi_a{}^b \gamma_b + g_4 \Phi_{5a}{}^b \gamma^5 \gamma_b + g_5 \omega_{abc}\sigma^{bc}\right).
\end{aligned} \tag{54}$$

In this unified form, each component of the generalized gauge field $\bar{\mathcal{A}}_a$ represents a specific sector of the dynamical link, now rightfully projected onto the recognized spacetime axes x^a. The generalized field strength $\bar{\mathcal{F}}_{ab} = -i[\overrightarrow{\mathcal{D}}_a, \overrightarrow{\mathcal{D}}_b]$ is derived by explicitly calculating the commutators of the Clifford-valued connection $\bar{\mathcal{A}}_a$. The complete expression, sorted by the Clifford basis $\{\mathbf{I}, \gamma_a, \gamma^5\gamma_a, \sigma_{ab}\}$, is given as follows: The generalized field strength $\bar{\mathcal{F}}_{ab} = -i[\overrightarrow{\mathcal{D}}_a, \overrightarrow{\mathcal{D}}_b]$ is fully

expanded as:

$$\bar{\mathcal{F}}_{ab} = g_1 F_{ab}\mathbf{I}$$
$$+ g_2 \left(\nabla_a \Phi_b{}^c - \nabla_b \Phi_a{}^c\right)\gamma_c$$
$$+ g_4 \left(\nabla_a \Phi_{5b}{}^c - \nabla_b \Phi_{5a}{}^c\right)\gamma^5\gamma_c$$
$$+ g_5 R_{ab}{}^{cd}\sigma_{cd}$$
$$+ 4\left(g_2^2 \Phi_a{}^c \Phi_b{}^d + g_4^2 \Phi_{5a}{}^c \Phi_{5b}{}^d\right)\sigma_{cd}$$
$$- 2g_2 g_4 \left(\Phi_a{}^c \Phi_{5b}{}^d - \Phi_b{}^c \Phi_{5a}{}^d\right)\epsilon_{cd}{}^{ef}\sigma_{ef}, \tag{55}$$

where

$$\nabla_a \Phi_b{}^c = \partial_a \Phi_b{}^c + 2g_5 \omega_a{}^c{}_d \Phi_b{}^d, \tag{56}$$

and

$$F_{ab} = \partial_a A_b - \partial_b A_a, \quad R_{ab}{}^{cd} = \partial_a \omega_b{}^{cd} - \partial_b \omega_a{}^{cd} + g_5(\omega_a{}^c{}_e \omega_b{}^{ed} - \omega_b{}^c{}_e \omega_a{}^{ed}). \tag{57}$$

To ensure the stability of the kinetic terms, the Lagrangian Density with γ^5 insertion [2] can be obtained as:

$$\mathcal{L}_1 = -\frac{1}{4g^2}\mathrm{Tr}\left(\bar{\mathcal{F}}_{ab}\gamma^5 \bar{\mathcal{F}}^{ab}\gamma^5\right) + c\,\mathrm{Tr}\left(\bar{\mathcal{F}}_{ab}\sigma^{ab}\right). \tag{58}$$

By expanding the first term, we obtain the kinetic part:

$$-\frac{1}{4g^2}\mathrm{Tr}\left(\bar{\mathcal{F}}_{ab}\gamma^5 \bar{\mathcal{F}}^{ab}\gamma^5\right)$$
$$= -\frac{g_1^2}{g^2}F_{ab}F^{ab} - \frac{g_5^2}{4g^2}R_{ab}{}^{cd}R^{ab}{}_{cd}$$
$$- \frac{g_2^2}{g^2}(\nabla_a \Phi_b{}^c - \nabla_b \Phi_a{}^c)(\nabla^a \Phi^b{}_c - \nabla^b \Phi^a{}_c)$$
$$- \frac{g_4^2}{g^2}(\nabla_a \Phi_{5b}{}^c - \nabla_b \Phi_{5a}{}^c)(\nabla^a \Phi_5{}^b{}_c - \nabla^b \Phi_5{}^a{}_c) \tag{59}$$

and the second term is evaluated by substituting the σ-component of $\bar{\mathcal{F}}_{ab}$. Using the trace

---

[2]Note that there are possible Lorentz invariant varieties, such as $\mathrm{Tr}(\bar{\mathcal{F}}_{ab}\bar{\mathcal{F}}^{ab})$ or $\mathrm{Tr}(\bar{\mathcal{F}}_{ab}\gamma_c\bar{\mathcal{F}}^{ab}\gamma^c)$ and *etc.*, *which could lead to different coupling ratios between fields.*

identity $\text{Tr}(\sigma_{cd}\sigma^{ab}) = (\delta_c^a \delta_d^b - \delta_c^b \delta_d^a)$, we obtain:

$$
\begin{aligned}
c\,\text{Tr}\left(\bar{\mathcal{F}}_{ab}\sigma^{ab}\right) &= c\left[g_5 R_{ab}{}^{cd} - 2\left(g_2^2 \Phi_a{}^c \Phi_b{}^d + g_4^2 \Phi_{5a}{}^c \Phi_{5b}{}^d\right)\right]\text{Tr}(\sigma_{cd}\sigma^{ab}) \\
&= 2cg_5(R_{ab}{}^{ab} - R_{ab}{}^{ba}) \\
&\quad - 4cg_2^2(\Phi_a{}^a \Phi_b{}^b - \Phi_a{}^b \Phi_b{}^a) \\
&\quad - 4cg_4^2(\Phi_{5a}{}^a \Phi_{5b}{}^b - \Phi_{5a}{}^b \Phi_{5b}{}^a) \\
&= 4cg_5 R - 4cg_2^2\left((\Phi_a{}^a)^2 - \Phi_a{}^b \Phi_b{}^a\right) - 4cg_4^2\left((\Phi_{5a}{}^a)^2 - \Phi_{5a}{}^b \Phi_{5b}{}^a\right) \quad (60)
\end{aligned}
$$

To ensure that the gauge sector corresponds to the standard Maxwell like action $-\frac{1}{4}F_{ab}F^{ab}$, the normalization requirement leads to a specific constraint on the coupling constants:

$$
\frac{g_1^2}{g^2} = \frac{1}{4}. \tag{61}
$$

In addition, to match the standard Yang-Mills normalization for the spin connection as a gauge field, the coefficient is constrained by:

$$
\frac{g_5^2}{g^2} = 1, \tag{62}
$$

and similarly, identifying these with the standard kinetic form for vector-valued scalar fields (where the normalization factor is typically 1/4 for the field strength-like part), we obtain the relative constraints:

$$
\frac{g_2^2}{g^2} = \frac{1}{4}, \quad \frac{g_4^2}{g^2} = \frac{1}{4}. \tag{63}
$$

However, it should be noted that these specific normalization ratios are derived from a minimal choice of the invariant trace. There are multiple possible Lorentz-invariant varieties, such as $\text{Tr}(\bar{\mathcal{F}}_{ab}\bar{\mathcal{F}}^{ab})$ or $\text{Tr}(\bar{\mathcal{F}}_{ab}\gamma_c\bar{\mathcal{F}}^{ab}\gamma^c)$ and others, which could lead to different coupling ratios between the fields.

The existence of such diverse invariant structures within the Clifford-valued gauge theory implies that the relative strengths of the gauge, scalar, and gravitational sectors can be modified depending on the detailed algebraic construction of the full Lagrangian. This provides the necessary theoretical flexibility to accommodate the observed hierarchy of fundamental interactions while maintaining the underlying algebraic consistency of the field module network. Moreover, it is naturally expected that these specific algebraic relations between the coupling constants can be modified by the renormalization group evolution. Even if these ratios are fixed at a high-energy unification scale by the algebraic consistency of the field module, the effective couplings will evolve independently towards lower energies. This renormalization-induced evolution provides a robust mechanism to explain the observed disparity in the strengths of the

fundamental interactions, allowing the unified framework to remain consistent with low-energy phenomenology.

Furthermore, it is possible to add the spin connection interaction term to the fermion part of the Standard Model Lagrangian. In this unified framework, the spin connection ω_{abc} is considered as the messenger to the Standard Model, acting on the same dynamical footing as the standard gauge bosons. We suppress the detailed analysis.

In this stage, the full Lagrangian density of the system is defined by the formal sum:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_{SM}. \tag{64}$$

Thus the action S takes the form

$$S = \int \mathcal{L} d^4 x, \tag{65}$$

where $d^4 x$ is the Minkowski invariant infinitesimal volume corresponding to the Euclidean invariant $d^4 q$. This construction provides a complete description of the interaction between the fermions and the messenger fields, including the spin connection, within the framework of a unified Clifford-valued gauge theory on Minkowski spacetime.

5 Network of Manifolds for Each Local Point

Although spacetime is fundamentally Minkowski, the non-uniqueness of coordinate choices allows one to introduce general coordinates locally. Such coordinates do not alter the underlying flat geometry, and they merely provide different local parametrizations attached to each point. Correspondingly, to ensure the local passive transformation $G^{-1}(x)$ in $\mathcal{G}$, one may introduce a local field $E^\mu{}_a(x)$ relating the global Minkowski coordinates x^a to the general coordinates X^μ via

$$dX^\mu = E^\mu{}_a(x) \, dx^a. \tag{66}$$

Here, $E^\mu{}_a(x)$ is non-dynamical and represents only the Jacobian of the coordinate transformation, not a dynamical tetrad field of gravity. Thus, the action $\mathcal{S}$ can be expressed in a generally covariant form with respect to the local coordinates X^μ, while the underlying Minkowski geometry remains unchanged. Any constant term in the Lagrangian carries no dynamical significance, *i.e.*, it does not contribute to the equations of motion. In particular, a cosmological constant term Λ is physically meaningless in this framework. This shows that the conventional cosmological constant problem is naturally avoided within this formulation.

The dynamical quantum behavior of the spin connection can, in principle, be described by the path-integral method. However, due to the highly nonlinear Yang-Mills type interactions, a detailed study of its quantum dynamics is extremely difficult and will not be pursued here. At the classical level, the spin connection is governed by the principle of least action. Although the

resulting classical equations are nonlinear and generally hard to solve analytically, their structure provides crucial insight into the underlying dynamics. q Torsion for each classical solution of the spin connection is defined by:

$$T^a{}_{\mu\nu} = \partial_\mu E^a{}_\nu(x) - \partial_\nu E^a{}_\mu(x) + \omega^a{}_{b\mu}(x)E^b{}_\nu(x) - \omega^a{}_{b\nu}(x)E^b{}_\mu(x). \tag{67}$$

In this framework, torsion is not a physical degree of freedom to be preserved. Instead, we introduce a local Lorentz-like transformation $\Lambda^a{}_b(x)$ to **scavenge** the torsion-free components from the dynamical ω, thereby defining the non-dynamical vierbein $e^\mu{}_a(x)$:

$$e^\mu{}_a(x) = E^\mu{}_b(x)\Lambda^b{}_a(x), \qquad dx^\mu = e^\mu{}_a(x)dx^a. \tag{68}$$

By imposing the torsion-free condition as a constraint for the emergence of classical geometry, the spin connection is forcibly reduced to a dependent function of the vierbein:

$$\begin{aligned}
\omega_\mu{}^{ab}(x) &= \frac{1}{2}e^{\nu a}(x)\left(\partial_\mu e_\nu{}^b(x) - \partial_\nu e_\mu{}^b(x)\right) \\
&\quad - \frac{1}{2}e^{\nu b}(x)\left(\partial_\mu e_\nu{}^a(x) - \partial_\nu e_\mu{}^a(x)\right) \\
&\quad - \frac{1}{2}e^{\rho a}(x)e^{\sigma b}(x)\left(\partial_\rho e_{\sigma c}(x) - \partial_\sigma e_{\rho c}(x)\right)e_\mu{}^c(x).
\end{aligned} \tag{69}$$

The vierbein $e^\mu{}_a(x)$ thus serves as a mere non-dynamical vessel that encodes only a partial, classical reflection of the spin connection 's full dynamical degrees of freedom. The essential information "discarded" during this extraction process—the non-geometric residue—constitutes the primordial debris that drives inflation and establishes the Planck scale through the symmetry breaking of Φ_{ab} and Φ_{5ab}.

In conclusion, the underlying spacetime is fundamentally Minkowskian. However, through the vierbein at each point, a structure emerges that effectively forms a network of local manifolds. This picture differs from the conventional interpretation of local spacetime in general relativity, yet it provides a framework that attempts to reconcile classical gravity with local field theories. In this framework, classical gravity can be treated without contradicting Einstein's theory, and the spin connection, representing the degrees of freedom of gravity, provides a natural pathway toward quantum gravity. Furthermore, this formalism establishes a rigorous and robust foundation for studying a quantum theory of gravity consistent with local field theories.

6 Discussions and Outlook

In this paper, we have demonstrated that the spacetime manifold is not a pre-existing background but a structure that is secondary to the matter field and the field module. The formalism reveals that what we perceive as "spacetime" is an emergent framework, rigorously constructed from

the dynamics of the local section for the matter field and its algebraic consistency within the field module network.

A crucial point of this construction is that the physical Minkowski coordinates x^a are identified through the matter-link sector. The emergence of spacetime corresponds to the identification of the conserved currents and the Clifford-valued connections that are required to maintain the covariance of the matter field. In this sense, the geometry of spacetime is a manifestation of the underlying matter field's interaction. The introduction of the non-dynamical field $E^\mu{}_a(x)$ does not signify a dynamical gravity, but reflects the flexibility of local coordinate identification derived from the matter field's distribution.

From this perspective, gravity is reinterpreted as the manifestation of the spin connection —a messenger field—acting within the network established by the matter fields. Since the geometry is a result of the matter field's configuration, the unified description ensures that all gravitational-like effects are primary to the algebraic interactions of the field modules.

The outlook of this work suggests that "spacetime" is a macroscopic consequence of the microscopic algebraic relations between matter fields. By quantizing the Clifford-valued connections that emerge from these fields, one can pursue a renormalizable theory of quantum gravity that is fundamentally and naturally integrated with the Standard Model $\mathcal{L}_{SM}$.

The outlook of this work suggests that spacetime is a consequence of the microscopic algebraic relations between matter fields. By quantizing the Clifford-valued connections that emerge from these fields, one can pursue a renormalizable theory of quantum gravity that is fundamentally and naturally integrated with the Standard Model $\mathcal{L}_{SM}$.

Moreover, this framework offers immediate insights into several fundamental problems in physics. First, as the theory is rigorously defined on the emergent Minkowski background, the zero-point energy of the vacuum does not gravitate. Since the constant terms in the Lagrangian density do not contribute to the field equations, the cosmological constant problem is naturally bypassed.

Second, a more detailed analysis reveals that the additional fields contained in the unified connection, namely the scalar Φ and the pseudoscalar Φ_5, induce spontaneous symmetry breaking. This mechanism generates the vacuum expectation values $\Phi_{ab} = v\,\eta_{ab}$ and $(\Phi_5)_{ab} = v_5\,\eta_{ab}$, which determine the structure of the scalar sector. The effective strength of gravity is determined by the term $\left[4cg_5 - \frac{12g_5}{g^2}(g_2^2 v^2 + g_4^2 v_5^2)\right] R$.

This structure suggests that the gravitational coupling arises from the interplay between the geometric trace contribution and the vacuum condensates of the scalar sector. Thus, the Planck scale is determined algebraically by the parameters of the unified connection rather than directly by the scalar vacuum expectation values. The scalar condensates instead satisfy a consistency relation among the coupling constants, reflecting the internal algebraic structure of the Clifford-valued gauge theory. In this way, the gravitational sector emerges naturally from the unified connection without introducing the Einstein-Hilbert term by hand.

Furthermore, this spontaneous symmetry breaking by the fields Φ and Φ_5 dictates the dynamics of the entire messenger sector. The emergence of these VEVs directly provides masses for the fermions ψ through their coupling within the field module network. Simultaneously, among the 24 components of the spin connection, 20 components acquire mass through these VEVs, while the remaining 4 components survive as massless degrees of freedom corresponding to the totally antisymmetric sector. The fields ψ emerge as candidates for dark matter, and the antisymmetric part of the spin connection carries the gravitational degrees of freedom.

Finally, the emergent nature of spacetime from the field module network provides a new perspective on the black hole information paradox. Since spacetime is a secondary structure constructed from the underlying matter fields, the information is fundamentally encoded in the algebraic consistency of the field modules. Therefore, even in the presence of strong gravitational effects, the information of the matter fields is never truly lost, as it remains rooted in the primary field module network from which the local manifold is identified.

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