HOW NEWTON'S LAW (THE LAW OF TWO-BODY GRAVITATION) BECAME CALLED THE LAW OF UNIVERSAL GRAVITATION.

Mykola Kosinov
Ukraine
e-mail: nkosinov@ukr.net

Abstract. *This article attempts to dispel the biggest myth surrounding Newton's law of gravitation for over 300 years. The history of the term "law of universal gravitation" and the negative consequences of its unjustified application to the law of two-body gravity are revealed. This term was preceded by Robert Hooke's term "universal attraction," which referred not to the law of gravity between two bodies, but to the attraction between all bodies in the universe. In addition to the term "universal gravitation," Robert Hooke gave a verbal formulation of the future law of universal gravitation, which was radically different from Newton's verbal formulation. The term "law of universal gravitation" was not coined by Isaac Newton. It was not Newton, but popularizers who began to unjustifiably use the term "law of universal gravitation" in relation to the local law of gravity of two bodies* $F = GmM/r^2$*. The substitution of terms created the illusion that the law of universal gravitation had been discovered and that no other laws of gravity existed. From an ethical perspective, this substitution led to a historical injustice, as a result of which Hooke's contribution in the shadows for over three centuries. In science, this led to the true law of universal gravitation, as Robert Hooke envisioned it, never being discovered.*
Keywords: *Robert Hooke; Isaac Newton; astronomy; Newton's law of gravitation*

1. Introduction

Isaac Newton's discovery of the law of universal gravitation became the foundation of classical physics. The term "law of universal gravitation" is usually associated with Newton [1]. But Robert Hooke was the first to speak of "*universal attraction*" long before the publication of *Philosophiae Naturalis Principia Mathematica.* It is well known that Robert Hooke, with his remarkable intuition and broad outlook, served as the intellectual catalyst for Newton's discovery of the law of gravitation. Hooke viewed the force of universal gravitation as a gravitational force exerted on all bodies by all other celestial bodies.

In his letters to Newton in 1679–1680, Robert Hooke discussed the force of gravity, which decreases inversely with the square of the distance, leading to the planets' elliptical orbits. Hooke's emphasis on the connection between gravitational force and orbital motion was a true scientific breakthrough for the time. This was the first verbal formulation of the law of universal gravitation in the history of science. It preceded Newton's verbal formula. Hooke's verbal formulation was ignored and forgotten after Newton's law was published. It was Hooke's fateful 1679 letter that changed Newton's thinking and pushed him to discover the law of gravity.

In 1687, Newton presented his verbal formulation of the law of gravitation, which included the inverse-square law. However, the elliptical orbits discussed by Hooke were not mentioned in his verbal formula. Instead of orbital parameters, Newton introduced mass into his law. As a result, instead of a law of universal gravitation for all bodies in the universe based on the orbital motion of bodies, Newton proposed a law of two-body [2, 3, 4] gravitation based on the mass of the gravitating body. This radically differed from Hooke's verbal formula. Newton's law of gravitation did not take into account the additional attractive force of all other bodies in the universe.

The euphoria of Newton's supporters over the discovery of the law of gravitation was so great that they began to unjustifiably call the law of two-body gravitation the law of universal gravitation. Although this was Hooke's term (*universal attraction)*, it referred not to the law of gravitation between two bodies, but to the attraction between all bodies in the universe. This "substitution" of terms led to Hooke's contribution remaining in the shadows for a long time. But the most destructive consequence of this “substitution” of terms was that the ***real law of universal gravitation***, as Robert Hooke envisioned it, was never discovered.

2. Two different verbal formulas for the law of universal gravitation.

In the history of science, there have been two different verbal formulas for the law of universal gravitation. The first verbal formula for the law of universal gravitation in the history of science was given by Robert Hooke in letters to Newton [5]. This occurred in 1679–1680, seven years before the publication of Philosophiae Naturalis Principia Mathematica. In his verbal formula, Robert Hooke indicated that the law of universal gravitation must take into account the orbits of the planets and the inverse-square law. Hooke didn't mention mass in his verbal formula. It was a verbal formula with emphasis on both the inverse square law and the connection between the force of gravity and the orbital motion of bodies. Hooke knew the main thing: *the force of attraction that leads to the orbital motion of bodies is the force of universal gravitation.*

Newton's verbal formula was different. Newton pointed out the proportionality of force to the masses of bodies and the inverse-square law. The symbolic representation of Newton's law of gravity looks like this:

$$F_N \propto \frac{mM}{r^2} \qquad (1)$$

Where: F_N is the force, m, M are the masses of the bodies, r is the distance, ∝ is the proportionality sign.

Newton didn't assign any numerical value to the gravitational force. This wasn't a precise law of gravity, but a proportional relationship. In this unfinished form, Newton's law existed for more than 100 years! It became a precise equation in the form:

$$F=GmM/r^2 \qquad (2).$$

This happened only after the introduction of the gravitational constant G into formula (1) [6-9].

Formulas (1) and (2) describe the attraction of only one local source of gravity and do not take into account that bodies simultaneously gravitate toward all other bodies. Formulas (1) and (2) describe the local gravity and do not take into account the additional gravitational force that actually exists as a result of the gravitational action of all bodies in the Universe. The law of gravitation $F = GmM/r^2$ is the law of gravitation, applicable to closed, strictly elliptical orbits, which do not actually exist in the Universe. This led to the fact that instead of the law of universal gravitation, Newton discovered the law of gravity of two bodies, and popularizers unreasonably elevated it to the rank of the law of universal gravitation.

3. The inverse-square law is only part of Hooke's verbal formula.

When scholars of Robert Hooke's work discuss his contribution to the discovery of the law of universal gravitation, they traditionally mention the inverse-square law, which Hooke emphasized in his letter to Newton. However, they often overlook Hooke's second emphasis on the connection between gravity and the orbital motion of the planets. The inverse-square law is only part of Hooke's verbal formula. Another, no less important, part of Hooke's verbal formula concerned the connection between gravity and the trajectory of motion: "*...and consequently that the direction will be a curved line very near resembling an ellipsis...*"

Hooke made a revolutionary discovery that led him to conclude that the law of gravity must take into account the elliptical orbit: «...compounding the celestiall motions of the planetts of a direct motion by the tangent & an attractive motion towards a central body».(Hooke's letter to Isaac Newton, November 24, 1679). This discovery changed Newton's thinking and led him to discover the law of gravity.

Hooke's insight is admirable. Hooke used the formulation "*very near resembling an ellipse*", knowing that Kepler's laws describe perfect ellipses. He did not emphasize perfect ellipses, as Newton did. Hooke's insight here proved closer to reality: *perfect elliptical orbits do not exist in the universe. The actual orbits of the planets are not elliptical.* His formulation, "*very near resembling an Ellipsis*," is not the vague formulation of an insecure person, but an acknowledgement of the complexity of the real world. He had a profound understanding of gravity. He understood that nature is more complex than geometrically perfect ellipses on paper. He understood that complexity is introduced by the gravity of multiple bodies.

As for the connection between the force and the trajectory of motion, this part of Hooke's verbal formula was either not understood by Newton, or he deliberately distanced himself from Hooke's formulation. On this matter, Newton responded to Hooke as follows [32]: "*Your acute letter having put me upon considering thus far ye species of this curve, I might add something about its description by points quam proximè. But the thing being of no great moment, I rather beg your pardon for having troubled you thus far with this second scribble….*"

For Hooke, it was important that the future law of universal gravitation take into account both the inverse-square law and orbital motion, but for Newton it was: "*...But the thing being of no great moment...*" Indeed, Newton did not use orbital parameters in his law of gravity, but used masses in conjunction with the inverse-square law. Newton abandoned the use of orbital parameters in favor of masses.

The peculiarity of Hooke's discovery, presented in his verbal formulations, is that he combined two ideas into a single system:

1. The force of attraction decreases proportionally to the square of the distance $1/r^2$: *«...my supposition is that the Attraction always is in a duplicate proportion to the Distance from the Center Reciprocal...»*. (Letter from Hooke to Isaac Newton, January 6, 1680).
2. Under the influence of gravity, planets move in an orbit close to an ellipse: «... *and consequently that the Direction will be a Curve Line very near resembling an Ellipsis»*. (Letter from Hooke to Isaac Newton, January 6, 1680).

Following Hooke's letters, Newton also made a unification, but he did not use the orbital parameters along with the inverse-square law, but used the mass of the central body:

1. The force of gravity decreases proportionally to the square of the distance $1/r^2$
2. The force of attraction is proportional to the masses.

Newton proved that if a body moves along an elliptical orbit (parabola, hyperbola), then the force must be inversely proportional to the square of the distance ($1/r^2$). Judging by his response to Hooke's letter, Newton considered the orbital parameters to be secondary. For Hooke, the elliptical orbit was a primary, observable fact (thanks to Kepler). Hooke did not consider the parameter ($1/r^2$) to be a substitute for the orbital parameters. Hooke did not believe that the inverse-square law "absorbs" or replaces the geometric parameters of the ellipse itself. Hooke considered a parameter of the form ($1/r^2$) as an additional parameter to the elliptical orbit. In his letters to Newton, Hooke literally pushed him toward this synthetic model: *to combine a parameter of the form ($1/r^2$) with the geometry of the curve (ellipse).*

Hooke's intuition is astounding! Indeed, acceleration can be represented not only as a combination of the parameter $1/r^2$ with mass ($a = GM/r^2$), but also as a combination of the parameter $1/r^2$ with the orbital parameters (R^3/T^2) from Kepler's law ($a = R^3/T^2r^2$).

After the discovery of Newton's law of gravitation, Hooke's verbal formula, in its full formulation emphasizing the connection between the force of attraction and the orbital motion of the planets, was forgotten for more than 300 years, even though it held the key to the real and precise law of universal gravitation.

Underestimating and misunderstanding Hooke's emphasis that the force of universal gravitation should be determined using the inverse square law in conjunction with orbital parameters, rather than in conjunction with mass, cost science dearly. This led to the fact that instead of the law of universal gravitation, Newton discovered the law of two-body gravitation, and popularizers unreasonably elevated the law of two-body gravitation to the rank of the law of universal gravitation. The exaggeration of the status of the local law of gravity became more important than the truth and overshadowed the need to discover a true law of universal gravitation that takes into account the gravity of all bodies in the universe.

4. The real orbits of the planets are not elliptical.

Planetary orbits are not strictly closed in the sense of a perfect, unchanging ellipse, but form a complex shape reminiscent of a "spiral rosette" (a flower with many petals). In 1859, U. Le Verrier discovered an anomalous shift in Mercury's perihelion [10]. This shift in Mercury's perihelion could not be explained by Newton's law of gravitation. S. Newcomb discovered a shift in the perihelion not only for Mercury but also for other planets [11]. This indicates that some additional force is acting on the planets. U. Le Verrier strongly believed in the correctness of Newton's law and believed that the extra shift was caused by another unknown planet located even closer to the Sun. He even gave it a name: *Vulcanus*. By the beginning of the 20th century, it became clear that there was no planet inside Mercury's orbit, meaning that Newton's physics was failing somewhere. The discovery of planetary perihelion shifts demonstrated that the planets' actual trajectories are not closed. With the advent of General Relativity (1915), Einstein demonstrated that this is a fundamental property of gravity, making planetary orbits non-closed.

5. Why did Hooke emphasize the orbital motion of the planets?

Hooke persistently studied the orbital motion of the planets [5]. He discovered the main secret of gravity: *the orbital motion of the planets occurs under the action of gravitational forces from all bodies in the universe*. He believed that the orbital motion of the planets is determined by the force of universal gravitation and that this force of universal gravitation forms the real orbits of the planets and acts on all bodies from all other celestial bodies. Even before 1679, Hooke spoke of "*universal attraction*". He understood gravity as a general attraction of everything to everything. He distinguished between the local gravity of two bodies and universal attraction. Hooke knew that the force of universal gravitation should be determined by the orbital motion of the planets. Hooke shared this knowledge with Newton in 1679. Newton did not accept Hooke's hint [5]. Newton did not use orbital parameters in his law of gravitation; He was looking for a solution to the problem of two-body gravity and used mass as a parameter.

6. The main secret of gravity, which Hooke revealed in a letter to Newton.

Newton did not provide a complete law of gravitation in his 1687 verbal formula. This was a proportional relationship ($F_{N.} \propto mM/r^2$). Newton's verbal formula was supplemented after Newton and after 1803 became the complete law of two-body gravitation $F = GmM/r^2$ [5]. Newton's path did not lead to the law of universal gravitation.

Hooke knew something about gravity that Newton did not. Hooke knew that the answer to the law of universal gravitation should be sought in the trajectories of planets, not in masses. Hooke knew that the force of universal gravitation is "*encoded*" in the actual trajectories of planets. Trajectories unambiguously reflect the gravitational forces of all bodies in the universe, but masses introduce ambiguity. Kepler's laws were already known at that time. Kepler's laws were sufficient to incorporate orbital parameters into the law of gravity, along with the inverse-square law. But Newton rejected Hooke's key proposal, despite the fact that Hooke's verbal formula described the path to the actual law of universal gravitation.

7. Comparison of Hooke's and Newton's formulas.

There are similarities and differences between Hooke's and Newton's formulas. This is shown in Fig. 1. The common feature is the inverse square law. The difference is that Newton emphasized the masses in his verbal formula, while Hooke emphasized the orbits in his verbal formula. The following parameters follow from the verbal formulas: Newton's - (m, $1/r^2$, M), Hooke's - (m, $1/r^2$, Orbital parameters: R, T). Using the parameters (m, $1/r^2$, M), Newton's verbal formula led to the proportional dependence $F_N \propto mM/r^2$. This was not the law of universal gravitation. It was not even a complete law of two-body gravitation. The maximum expected result from Newton's verbal formula is the law of two-body gravitation, represented by masses ($F = GmM/r^2$). Proportional dependence $F_N \propto mM/r^2$ turned into the complete law of two-body gravitation $F = GmM/r^2$ only in 1875. In 1875, the Potential of Newton's verbal formula was fully realized.

Hooke's approach is more promising. Hooke's verbal formula allows one to derive both the complete law of two-body gravitation and the law of universal gravitation. The maximum expected result of Hooke's verbal formula is the actual law of universal gravitation. Hooke's verbal formula states: "very close to an ellipse." This points not to the gravitation of two bodies, but to the gravitation of all bodies in the universe. As an intermediate result, for an idealized elliptical orbit, Hooke's verbal formula leads to a new law of gravitation of two bodies, represented by the orbital parameters: $F = mR^3/T^2r^2$. For a real orbit close to an ellipse, Hooke's verbal formula leads to the law of gravitation of all bodies in the universe: $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ [21]. The deviation of the orbit from an ideal ellipse is caused by the additional gravitation of all other bodies in the universe F_{Cos}. In formula (4), this additional force is represented by the second component: $mc^2\sqrt{\Lambda}$.

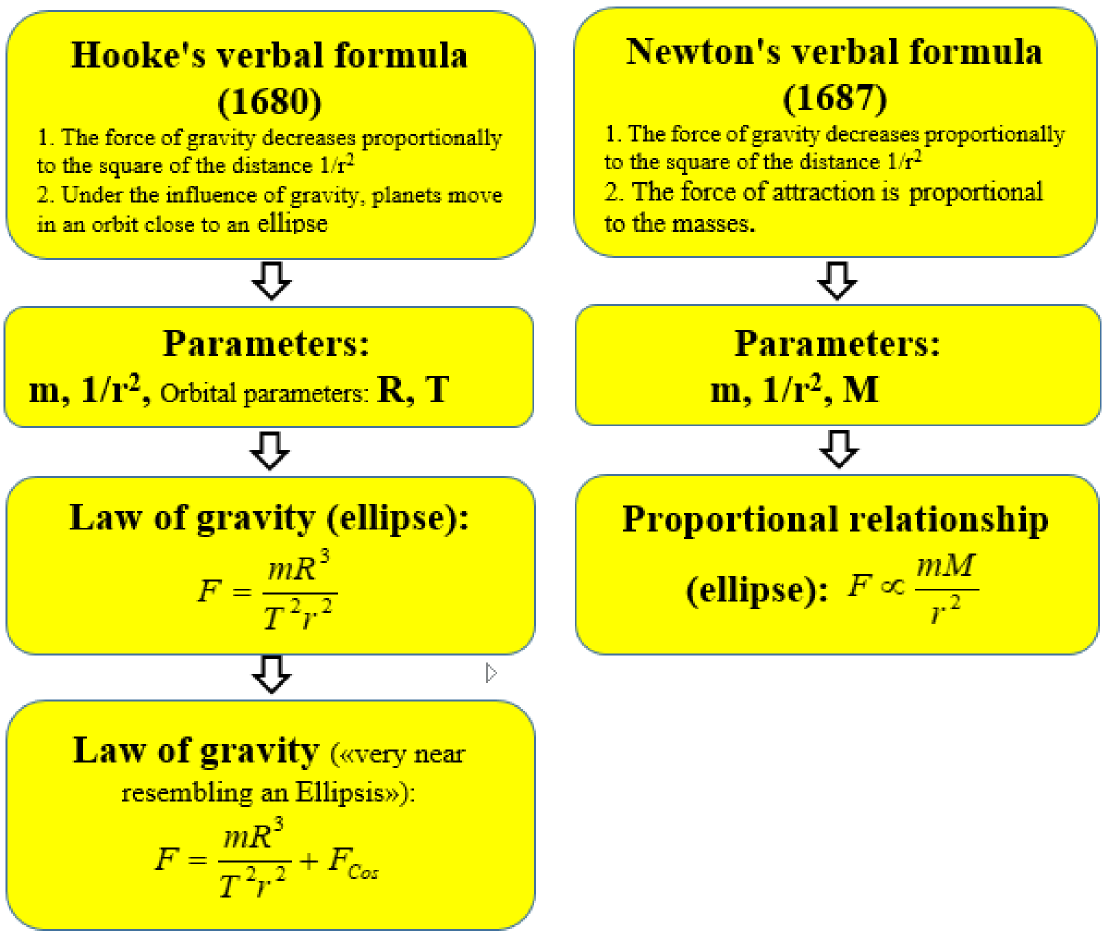

Fig. 1. Comparison of Hooke's and Newton's verbal formulas. Where: m, M is the mass of the bodies, R and T are orbit parameters, FCos is the cosmological force, r is the distance.

The main conclusions that follow from a comparison of Hooke's and Newton's verbal formulas are the following:

1. Hooke's verbal formula can be transformed into a law of universal gravitation. Hooke's model is a model of gravity for all bodies in the universe.

2. Newton's verbal formula does not lead to a law of universal gravitation; it leads to a law of two-body gravitation. The two-body model is a simplified model of gravity.

8. The two laws of two-body gravitation, derived from Hooke's and Newton's verbal formulas, complement each other.

In the two-body gravitation model, Hooke's and Newton's verbal formulas do not contradict each other. In this simplified model of gravitation, Hooke's verbal formula and Newton's formula complement each other. The two-body gravitation laws, $F = GmM/r^2$ and $F = mR^3/T^2r^2$, derived from these verbal formulas, also complement each other rather than replace each other. In the presence of an orbit, the gravitational interaction is more accurately described by the formula $F = mR^3/T^2r^2$. This is a convenient way to calculate the force when the mass of the central object is unknown. With a known mass of the central object, the formula $F = GmM/r^2$ describes the gravitational interaction with and without orbital motion. Here, the limiting factor is the low accuracy of the constant G.

In the all-bodies gravitation model of the universe, the picture is different. Newton's verbal formula, formulated for the two-body problem, is inapplicable to the gravity of all bodies, as confirmed by the unsolvable N-body problem. Hooke's verbal formula is applicable to the gravity of all bodies. It clearly indicates the deviation of the actual trajectory from a perfect ellipse. This is a direct indication of the presence of an additional force, in addition to the attractive force between the two bodies.

9. What is preferable in the law of gravity, orbital parameters or mass?

Using the two different verbal formulas for the law of gravitation, presented by Hooke and Newton, we see two different approaches. Each approach allows for a different definition of the same physical quantity: the acceleration caused by gravitational force. Gravitational acceleration can be represented in two different ways: using orbital parameters (Hooke's approach) or using the gravitating mass (Newton's approach) (Fig. 2).

$$a_{Hooke} = R^3/T^2r^2 \quad a_{Newton} = GM/r^2$$

Fig. 2. Two methods for representing gravitational acceleration that follow from the verbal formulas of Hooke and Newton.

Newton relied on masses in his approach to discovering the law of universal gravitation. Hooke relied on trajectories in his approach to discovering the law of universal gravitation. Hooke relied on Kepler's laws. Kepler's laws do not include masses. In work [5] it is shown that Hooke's approach leads directly to the complete law of two-body gravity $F = mR^3/T^2r^2$. Newton's approach did not allow him to obtain the complete two-body gravitational law. Only nearly 100 years later did the proportional relationship $F_N \propto mM/r^2$ begin to take the form of a complete law: $F = fM/r^2$, and it finally became the two-body law of gravity almost 200 years later, in 1885 ($F = GmM/r^2$), when the value of the constant G became known.

Despite their differences, both Hooke's and Newton's approaches were rational. They did not replace, but rather complemented, each approach. Each was valuable in its own way. They were two different paths to the same goal. If we compare what is preferable to use in the law of gravity, the orbital parameters or the mass, then the following follows.

Since force is defined in terms of acceleration, we cannot say with certainty whether we are dealing with a very heavy object far away or a light object close by. If we do not know the exact distance to an object, we can choose an infinite number of mass-distance pairs that will all yield the same trajectory. The same applies to density distributions. It is impossible to determine whether the mass is distributed uniformly over a vast volume or compressed into a point. Both will yield the same trajectory. This is the disadvantage of mass as a parameter in the law of gravitation compared to orbital parameters. Unlike masses, orbits "know" everything about gravity. The orbits of celestial bodies are formed as a result of the combined gravitational forces exerted by all bodies in the universe. This is the main difference between Hooke's and Newton's understanding of the law of universal gravitation. For this reason, the use of mass could not lead Newton to a law of universal gravitation that takes into account the actual gravity of all bodies in the universe.

Newton's use of masses as parameters in the law of gravitation leads to ambiguity. The actual trajectories of planets are formed by the gravitational forces of all bodies in the universe. The same trajectory can be explained by different combinations of masses and density distributions. There are an infinite number of force fields that can generate the same orbit. By emphasizing masses instead of orbits, Newton was doomed to develop a law of local gravitation rather than a law of universal gravitation. Only for the idealized local gravity of two bodies is there a clear relationship between mass and orbit. In the real universe, this is not the case.

In his letter to Newton, Hooke expected Newton to formulate a universal law of gravitation that takes into account the orbital motion of bodies. But instead of a law of universal gravitation based on the orbital motion of bodies, Newton proposed a proportional relationship $F_N \propto mM/r^2$, based on the mass of the gravitating body. As a result, instead of a law of gravitation that takes into account the gravity of all bodies, Newton formulated a law of gravitation that describes local gravity and is applicable only to idealized closed elliptical orbits.

The law of gravity $F = GmM/r^2$ still remains the law of gravity of two bodies [2, 3, 4]. It applies only to closed orbits, whereas real planetary orbits are not closed. This is precisely the point where Newton's mathematical justification of gravity collided with the harsh "*physical reality*" of the universe. Planets do not move in closed orbits.

10. Hooke's terminology as applied to universal gravitation.

Hooke first discussed gravity as a universal force that gives celestial bodies their spherical shape in 1665 in his book "*Micrographia*". In *Micrographia*, he does not yet use the precise formulation "*universal attraction*," but he lays the foundation for this idea by discussing gravity as a universal property of matter to aggregate into spheres. In his "*Observation VI*", Hooke makes a bold proposition for his time: "...*Attraction is common not only to the Earth, Sun, Moon, and Planets, but to every minute body in the Universe*..." [12].

In 1674, in his work "*An Attempt to Prove the Motion of the Earth from Observations*," Hooke already wrote about "*attraction which is universal*." He asserted that all celestial bodies exert a force of attraction toward their centers and that they also attract all other celestial bodies. Instead of "*universal gravitation*," he used the term "*universal attraction*." At that time, the word gravitas was used to describe the weight of an object. It was also used figuratively to describe a person's qualities: seriousness, dignity, importance, and authority.

Newton found a solution that only approximately describes the actual gravitational force of the entire universe. The force of gravitational interaction between two bodies is only a part of the force of universal gravitation. In the universe, there is no isolated gravitational interaction between two bodies. All bodies in the universe participate in gravitational interaction. This discovery is due to Robert Hooke. The gravitational force of two bodies always has an "*addition*." This "*addition*" is the gravitational force of all other bodies in the Universe. The Law of gravity $F = GmM/r^2$ "*doesn't see*" these "*additions*." Therefore, the two-body model is only a useful approximation in physics. The law of gravitation $F = GmM/r^2$ is applicable only on small scales, for small masses, where the contribution of the force F to the total gravitational attraction significantly exceeds the gravitational force of all other bodies in the Universe.

Thus, the problem of discovering the real law of universal gravitation, as set out by Hooke in his verbal formula, remains unsolved. In the minds of scientists and laymen, the formula for the gravitational force of all bodies in the universe has been replaced by the formula for the gravitational force of two bodies: $F = GmM/r^2$. This reinforced the illusion *that the law of universal gravitation had been discovered and there was nothing more to discover*.

All of the above shows that calling the law of gravity $F = GmM/r^2$ the law of universal gravitation is an exaggeration. This "*replacement*" of terms was made by popularizers who had little understanding of gravity.

If the universe consisted of only two bodies, it would be the law of universal gravitation. And then the orbit would be closed, elliptical. But in the universe, bodies do not move along closed trajectories. And in the universe, there are not two bodies, but many more.

11. To what law of gravity did Hooke's verbal formulation lead?

To what law of gravitation could Hooke's verbal formulation, taking into account the orbital motion of bodies, lead? Based on Hooke's verbal formulation, it must be a completely different law, unlike Newton's law. It must not include a gravitating mass. It must include the orbital parameters.

To implement Hooke's verbal formulation, it is sufficient to introduce the orbital parameters into the law of gravity together with the inverse square law, and not the mass. Hooke's verbal formulation, when applied to a closed Keplerian orbit, leads to a new law of gravitation [5]:

$$F = mR^3/T^2r^2 \qquad (3)$$

Where: m is the mass of the body, R and T are the orbital parameters, and r is the distance.

Equation (3) is a more precise law of gravity than Newton's law. The new law of gravity does not include the constant G. The insufficient accuracy of the constant G ceases to be a limiting factor in gravity. The law of gravity $F = mR^3/T^2r^2$ does not replace, but rather complements, the formula $F = GmM/r^2$. Each of these formulas has its own limits of applicability..

12. The N-body gravitational problem as an attempt to provide a realistic description of the gravity of the Universe.

The N-body gravitational problem is an attempt to provide a realistic description of the gravity of the Universe by moving from the two-body problem to a real system where each body attracts all others. The N-body gravitational problem is based on Newton's law of gravitation, $F = GmM/r^2$. This is a vicious circle: an attempt to find a realistic description of the gravity of the Universe using the two-body law as a basis. A mathematical impasse arose immediately for $N \geq 3$. A complete description of gravity requires taking into account the gravitational influence of billions of bodies in the Universe. Henri Poincoré proved that the N-body gravitational problem for $N \geq 3$ has no analytical solution. Attempts to provide a realistic description of the gravity of the Universe using the N-body gravitational problem have been unsuccessful.

Other attempts to give a realistic description of the gravity of the Universe have been based on modifications of Newton's law [13–19]. No modifications to the law of gravity $F = GmM/r^2$ have yielded results Neither the N-body gravitational problem nor modifications of Newton's law of gravitation led to the discovery of a Law of Gravitation that takes into account the gravity of all bodies in the Universe.

In [20] the problem of finding the law of universal gravitation based on the integral parameters of a system of N bodies was first formulated. This is the inverse N-body problem. Unlike the unsolvable direct N-body problem, the inverse N-body problem has a solution. All solutions to the inverse N-body problem are given in [20]. One solution to the inverse N-body problem is the law of universal gravitation without the gravitational constant G [21]:

$$F_U = \frac{mR^3}{T^2 r^2} + mc^2\sqrt{\Lambda} \qquad (4)$$

Where: m is the mass of the body, R and T are the orbital parameters, and r is the distance, **c** is the speed of light in vacuum, **Λ** is the cosmological constant.

The law of universal gravitation without the gravitational constant G has two components [14]. The additivity of the forces allows us to obtain the resulting force of universal gravitation as the vector sum of all individual forces. The first component describes the attraction between two bodies and is represented by the Hooke-Kepler law of universal gravitation [5]. The second component describes the attraction of all other bodies in the universe using the integral parameter of the universe—the cosmological constant Λ [14].

The law of universal gravitation (4) describes the real, non-closed orbits of bodies, which corresponds to the real motion of bodies in the universe. The path to this real law of universal gravitation was outlined by Robert Hooke in his verbal formula as early as 1679.

13. How the law of gravity of two bodies came to be called the law of universal gravitation.

To Newton's credit, he did not call his proportional relationship $F_N \propto mM/r^2$ the law of universal gravitational attraction. Moreover, Newton believed that it was impossible to discover the law of universal gravitation, which takes into account orbital motion and takes into account the gravity of all bodies in the universe. Here is what Newton wrote in 1684 in a text [22] known as the "Copernican Scholium":"*...the planets neither move exactly in ellipses nor revolve twice in the same orbit. Each time a planet revolves it traces a fresh orbit, as in the motion of the Moon, and each orbit depends on the combined motions of all the planets, not to mention the action of all these on each* *other. But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind*."

Therefore, instead of searching for a realistic description of gravity, instead of using the hint given to him by Hooke, Newton solved a simplified problem: the two-body gravity problem [2, 3, 4]. Using mass «M» as a parameter in the law of two-body gravity is equivalent to considering the motion of planets along a closed trajectory, which does not correspond to the actual motion of planets. How then did the laws of two-body gravity ($F_N \propto mM/r^2$, $F = GmM/r^2$), which do not take into account the gravity of all bodies in the Universe, come to be called the laws of universal gravitation?

Newton didn't do this. Others did. Newton, having solved the idealized two-body problem, was well aware of the limitations of his method. Roger Cotes, the editor of the second edition of "Philosophiae Naturalis Principia Mathematica" (1713), did. In the preface, he emphasizes the universality of Newton's law of gravitation. Voltaire, in his Philosophical Letters (1734), explained Newton's ideas using Hooke's concept of "*universal attraction*." Thus, the law of two-body gravity began to acquire an unjustifiably exaggerated status. It turned out that Hooke's term "*universal attraction*" was not used in its original sense and for its intended purpose. Voltaire failed to mention that "*universal attraction*" is Robert Hooke's term, and it refers not to the law of gravitation between two bodies, but to the attraction between all bodies in the universe. Thus, Voltaire also authored the historical injustice that kept Hooke's contribution in the shadows for over three centuries.

Calling the law of gravity, $F = GmM/r^2$, the law of universal gravitation, we are repeating not the words of Newton, but the words of Roger Coates, who, being a great admirer of Newton, unjustifiably called the law of two-body gravitation the law of universal gravitation.

But what about the real law of universal gravitation? This unjustified renaming of the law of two-body gravity to the law of universal gravitation did not advance scientific progress. This created the illusion that the law of universal gravitation had been discovered and that no other law of gravity existed.

14. Consequences of silencing Robert Hooke's contribution and of the "innocent" substitution of terms.

The unjustified assignment of the name "*law of universal gravitation*" to Newton's law had negative consequences. This seemingly "*innocent*" substitution of terms cost science dearly. First, it resulted in a historical injustice, as a result of which the contribution of Hooke, England's Leonardo, remained in the shadows for more than three centuries [23, 24].

Even greater harm was done to physical science. The unjustified designation of the law of gravitation $F = GmM/r^2$ as the law of universal gravitation gave rise to a blind faith in its omnipotence. This seemingly "*innocent*" substitution of terms gave rise to the illusion that the sought-after law of universal gravitation had been discovered and should be used to explain observations. The consequence of such blind faith and illusion was the emergence of false ideas and false concepts in science.

Mercury was the first to "*rebel*." He "refused" to obey Newton's law of gravity. U. Le Verrier, relying on Newton's law of universal gravitation, attempted to explain the anomalous precession of Mercury's perihelion [25]. He proposed the existence of an invisible planet, *Vulcanus*, located closer to the Sun, which turned out to be a mistake. V. Le Verrier's conviction in the infallibility of Newton's law of gravity led him to *Vulcanus*.

The history of the emergence of dark matter in science is similar to the story of *Vulcanus*. In the 1930s, Fritz Zwicky noticed that galaxies were moving too fast—as if there was much more mass there than we could see. A dilemma arose: either the galaxy was "wrong," or Newton's law was incorrect. Newton's law, $F = GmM/r^2$, was traditionally understood, in accordance with its name, as the law of universal gravitation. The change of terminology played a destructive role. Therefore, Fritz Zwicky, believing in the infallibility of Newton's law, applied it to the galaxy and confidently called the invisible mass dunkle Materie [26]. Fritz Zwicky made this conclusion based on Newton's law of gravity, considering it the law of universal gravitation.

Even earlier, in 1904, blind faith in Newton's law as the law of universal gravitation failed Lord Kelvin [27]. While attempting to estimate the mass of the Milky Way, he concluded that there exist "*dark bodies*" that we cannot see but that create additional gravity. The existence of these "*dark bodies*" was inferred from Newton's law of universal gravitation to balance the calculations. French mathematician and physicist H. Poincare criticized Kelvin's assumption. His verdict was: "*There is no dark matter, or at least not as much of it as visible matter*" [28].

Vera Rubin's research in the 1970s showed that the outer parts of spiral galaxies rotate too rapidly, contrary to Newton's laws of mechanics [29]. Vera Rubin concluded that dark matter exists. This conclusion was reached by the same attempt to apply the law of gravity, $F = GmM/r^2$, to the galaxy, beyond its applicability. Once again, the substitution of terminology played a destructive role. Vera Rubin was confident that Newton's law, $F = GmM/r^2$, was the law of universal gravitation and therefore applicable to the galaxy. Newton's law $F = GmM/r^2$ is not as "universal" as its name suggests.

15. Conclusions.

1. In the history of science, there have been two different verbal formulas for the law of universal gravitation. The first verbal formula for the law of universal gravitation was given by Robert Hooke seven years before the publication of Philosophiae Naturalis Principia Mathematica. Hooke emphasized the orbital motion of bodies. Newton's verbal formula differed from Hooke's. Newton emphasized the masses of bodies.

2. Hooke discovered the main secret of gravity: *the orbital motion of the planets occurs under the influence of gravitational forces from all bodies in the universe*. He believed that the orbital motion of the planets is determined by the force of universal gravitation and that this force of universal gravitation shapes the actual orbits of the planets and acts on all bodies from the side of all other celestial bodies. Hooke's verbal formula contained the key to the real law of universal gravitation.

3. Newton solved the problem of two-body gravity and pointed out the proportional relationship between force and mass. Newton's verbal formula was further developed and, after the introduction of the gravitational constant G, became the law of gravitation $F = GmM/r^2$, which became known as the "*law of universal gravitation*." It so happened that Hooke's term "*universal attraction*" was applied to the law of gravitation between two bodies. Hooke's term "universal attraction" was misused. Hooke used it not to refer to the attraction between two bodies, but to refer to the attraction between all bodies.

4. The formula $F = GmM/r^2$ describes the local gravity and does not take into account the additional gravitational force that actually exists as a result of the gravitational action of all bodies in the universe. The law of gravitation $F = GmM/r^2$ applies to closed, strictly elliptical orbits, which do not actually exist in the universe. The actual orbits of the planets are not closed.

5. Calling the law of gravitation $F = GmM/r^2$ the law of universal gravitation is an exaggeration. Newton did not call his law of gravitation the law of universal gravitation. This was done by his popularizers, who were too far removed from understanding of gravity.

6. Attempts to provide a realistic description of the gravity of the Universe by modifying Newton's law and by moving from the two-body problem to the N-body gravitational problem did not lead to the discovery of a new law of gravity that accounts for the gravity of all bodies in the Universe.

7. The unfounded designation of the law of gravity $F = GmM/r^2$ as the law of universal gravitation gave rise to a blind belief in its omnipotence and had its negative consequences. Attempts were made to apply it on a universal scale. This resulted in the emergence of false ideas and false concepts in science.

8. The unjustified renaming of Newton's law of gravity by popularizers as the law of universal gravitation did not contribute to scientific progress. This created the illusion that the law of universal gravitation had been discovered and that no other law of gravity existed.

9. None of the above diminishes the significance of Newton's discovery. In 1687, it was a monumental scientific breakthrough. Newton's discovery became the foundation of classical physics. Although the Law of Gravitation $F = GmM/r^2$ describes idealized closed orbits, it is applicable on small scales, for relatively small masses, where the contribution of the force F to the total gravitational attraction significantly exceeds the gravitational force of all other bodies in the Universe.

16. Final Remarks.

1. Scholars of Hooke and Newton's work have not given due consideration to the fact that in the history of science, there were two different verbal formulas for the law of universal gravitation. One was Hooke's, the other Newton's. Chronologically, Hooke's verbal formula was first.

2. Scholars of Robert Hooke's work have traditionally focused on the inverse-square law, but have ignored Hooke's emphasis on the connection between gravity and the orbital motion of the planets [5, 30, 31]. A dependence of the form $(1/r^2)$ is characteristic of an elliptical orbit, as Newton demonstrated. However, in the law of gravitational force, a parameter of the form $(1/r^2)$ does not replace the parameters of an elliptical orbit, but is an additional parameter, as Robert Hooke pointed out. By emphasizing the connection between gravity and the orbital motion of the planets, Hooke pointed the way to the real law of universal gravitation. To implement Hooke's verbal formulation, Newton only needed: *a) introduce the inverse square law into the law of gravity*; *b) introduce the orbital parameters instead of mass into the law of gravity*. What's surprising is that, despite Kepler's laws, Newton didn't do this. As a result, the second law of gravity F = mR3/T2r2, which is more perfect than Newton’s law, remained undiscovered [5].

3. Hooke's emphasis on the connection between the force of universal gravitation and orbital motion was a true scientific breakthrough for its time. Hooke's verbal formulation combined both the special case of a closed elliptical orbit (gravity of two bodies) and the general case of non-closed orbits (gravity of all bodies). In the particular case of an idealized closed elliptical orbit, Hooke's verbal

formula leads to a new law of gravitation: $F = mR^3/T^2r^2$. This is the second law of gravity, more accurate than Newton's law [5]. The same verbal formulation by Hooke for open orbits led to the real law of universal gravitation: $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$. The law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ takes into account the gravity of all bodies in the Universe and the accelerated expansion of the Universe. The law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ does not contain the gravitational constant G [21].

4. Newton used part of Hooke's verbal formula (the inverse-square law) and went to considerable lengths to obscure Hooke's contribution. Hooke's verbal formula, in its full formulation, was forgotten for more than 300 years, although it contained the key to the real and exact law of universal gravitation.

5. Newton, in his verbal formula of the law of gravitation in 1687, indicated that the force is proportional to the masses of the bodies and inversely proportional to the square of the distance: $F_N \propto mM/r^2$. This was not the law of universal gravitation. It was not even a complete law of two-body gravitation. Newton's verbal formula was refined and only many years later transformed into the complete law of gravity $F = GmM/r^2$ [2, 3, 4, 5]. This was done by Poisson (1803), Cornu and Baille (1873), K ̈onig and Richarz (1885), (Fig. 3) [5]. The path that Newton chose did not lead to the law of universal gravitation, it was the path to the law of gravitation of two bodies.

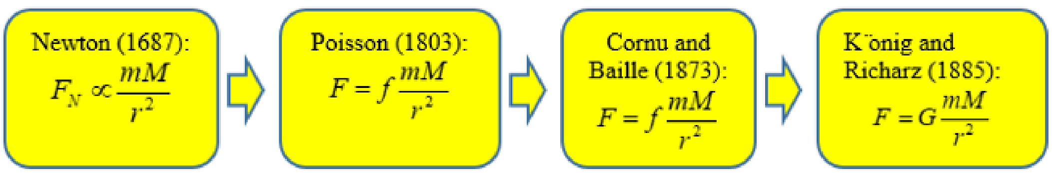

Fig. 3. The path from Newton's verbal formula to the law of two-body gravity.

6. In Proposition XV of Book I «Philosophiae Naturalis Principia Mathematica», Newton proved that if the force is inversely proportional to the square of the distance ($1/r^2$), then the squares of the revolution times in elliptical orbits are proportional to the cubes of their major axes. Mathematically, this means that the ratio R^3/T^2 is a constant for all bodies orbiting the same center of force. It remains a mystery why, with such a proof, Newton did not use the orbital parameters (R^3/T^2) in the law of gravitation along with the inverse square law ($1/r^2$). The law of gravitation $F = mR^3/T^2r^2$ could have been discovered as early as 1687! Essentially, the force had to be expressed using Kepler's law and the inverse square law. This is precisely what Hooke pointed out in his letter. It would not have taken more than 100 years for the proportional relationship $F_N \propto mM/r^2$ to evolve into the law of gravitation $F = GmM/r^2$, which, due to the low accuracy of the constant G, is still inferior to the law of gravitation $F = mR^3/T^2r^2$. Perhaps Newton was hampered by competition with Hooke, given that the law of gravitation $F = mR^3/T^2r^2$ corresponds too closely to Hooke's verbal formula, and everything in it is Hookean. Perhaps Newton deliberately distanced himself from Hooke's formulations out of personal animosity and wanted the law of gravitation to be presented as his own achievement, rather than as an "improvement" on Hooke's hypotheses or Kepler's observations.

7. Newton's use of masses as parameters in the law of gravitation leads to ambiguity. The actual trajectories of planetary motion are formed by the gravitational forces of all bodies in the universe. The same trajectory can be explained by different combinations of masses and density distributions. There is an infinite number of force fields that can generate the same orbit. Hooke knew

that the force of universal gravitation was "*encoded*" in the actual trajectories of planetary motion. By emphasizing masses instead of orbits, Newton was doomed to produce a law of local gravitation, not a law of universal gravitation.

8. Hooke's verbal formula and Newton's verbal formula are not contradictory. They represent two different paths to the same goal. The laws of gravitation that follow from Hooke's and Newton's verbal formulas, although different, complement each other in practical applications. In the presence of an orbit, gravitational interaction is more accurately described by the law F = mR3/T2r2. This is a convenient way to calculate the force when the mass of the central object is unknown. With a known mass of the central object, the law $F = GmM/r^2$ describes gravitational interaction both with and without orbital motion.

References.

1. Newton's Third Rule and the Experimental Argument for Universal Gravity
By Mary Domski. New York. 2021. DOI https://doi.org/10.4324/9781003184256
2. Massimi, Michela, *Perspectival Realism* (New York, 2022; online edn, Oxford Academic, 23 June 2022), https://doi.org/10.1093/oso/9780197555620.001.0001
3. Smith, George, "Newton's *Philosophiae Naturalis Principia Mathematica*", *The Stanford Encyclopedia of Philosophy* (Winter 2024 Edition), Edward N. Zalta & Uri Nodelman (eds.), URL = <https://plato.stanford.edu/archives/win2024/entries/newton-principia/>.
4. Roger Cotes - Natural Philosopher. Gowing, Ronald. ISBN 10: 0521237416 / ISBN 13: 9780521237413. Published by Cambridge University Press, 1983. Language: English
5. Kosinov, M. (2026). HOOKE-KEPLER LAW OF GRAVITATION: the rejected law of gravitation, which turned out to be more accurate and perfect than Newton's law. *Cambridge Open Engage*. doi:10.33774/coe-2025-53vlv-v7
6. S. D. Poisson, "Treatise on Mechanics", Vol 1, (1803), Graisberry and Gill, Dublin,pg. 377
7. A. K¨onig and F. Richarz, 'Eine neue Methode zur bestimmung der Gravitationsconstante', Annalen der Physik 260 (1885) 664-668.
8. Falconer , I J 2022 , ' Historical notes : the gravitational constant ' , Mathematics Today , vol. 58 , no. 4 , pp. 126-127 .
9. J. A. M. Pereira (2021). J. Phys.: Conf. Ser. 1929 012014. DOI 10.1088/1742-6596/1929/1/012014
10. U. Le Verrier (1859), (in French), "Lettre de M. Le Verrier à M. Faye sur la théorie de Mercure et sur le mouvement du périhélie de cette planète", Comptes rendus hebdomadaires des séances de l'Académie des sciences (Paris), vol. 49 (1859), pp. 379–383. https://en.wikipedia.org/wiki/Urbain_Le_Verrier
11. Newcomb S. The elements of the four inner planets and the fundamental constants of astronomy. Suppl. am. Ephem. naut. Aim. 1897. U.S. Govt. Printing Office, Washington, D. C., 1895.
12. Robert Hooke. Micrographia: or, Some physiological descriptions of minute bodies made by magnifying glasses. — London: J. Martyn and J. Allestry, 1665.

13. “Alexis_Clairaut” Wikipedia, Wikimedia Foundation, 14 April 2025, https://en.wikipedia.org/wiki/Alexis_Clairaut
14. “Inverse-square_law” Wikipedia, Wikimedia Foundation, 14 April 2025, https://en.wikipedia.org/wiki/Inverse-square_law.
15. A suggestion in the theory of Mercury // Astr. J. — 1894. — Vol. 14. — P. 49—51.
16. Haug, E.G. Different Mass Definitions and Their Pluses and Minuses Related to Gravity. *Foundations* 2023, *3*, 199-219. https://doi.org/10.3390/foundations3020017
17. Indranil Banik, Srikanth T Nagesh, Hosein Haghi, Pavel Kroupa, Hongsheng Zhao,Overestimated inclinations of Milgromian disc galaxies: the case of the ultradiffuse galaxy AGC114905, Monthly Notices of the Royal Astronomical Society, Volume 513, Issue 3, July 2022, Pages 3541–3548, https://doi.org/10.1093/mnras/stac1073.
18. J. W. Moffat; V. T. Toth (2007). "Modified Gravity: Cosmology without dark matter or Einstein'scosmological constant". arXiv:0710.0364
19. T.C. Scott. From Modified Newtonian Dynamics to Superfluid Vacuum Theory // Entropy. — 2023. — T. 25 (December). — C. 12. — doi:10.3390/e25010012
20. Kosinov, M. (2025). INVERSE N-BODY PROBLEM - THE KEY TO SOLVING GRAVITY PROBLEMS. *Cambridge Open Engage*. doi:10.33774/coe-2025-7w4k7-v4
21. Kosinov, M. (2026). LAW OF UNIVERSAL GRAVITATION WITHOUT GRAVITATIONAL CONSTANT G. *Cambridge Open Engage*. doi:10.33774/coe-2025-tcjtj-v8
22. Perturbation (astronomy). https://en.wikipedia.org/wiki/Perturbation_(astronomy).
23. Allan Chapman. England’s Leonardo: Robert Hooke and the Seventeenth-Century Scientific Revolution. (2004). ISBN 9780367393557. 330 Pages.
24. Stephen Inwood. The Forgotten Genius: The Biography of Robert Hooke, 1635–1703. MacAdam/Cage Publishing. (2003).
25. U. Le Verrier (1859), "Théorie du mouvement de Mercure" (Annales de l'Observatoire Impérial de Paris, 1859, Vol. 5, pp. 1–196).
26. Fritz Zwicky. On the Masses of Nebulae and of Clusters of Nebulae. The Astrophysical Journal (Vol. 86, pp. 217–246).
27. Kelvin, W. T. (1904). Baltimore Lectures on Molecular Dynamics and the Wave Theory of Light. London: C. J. Clay and Sons. Appendix H. «On Clustering of Gravitational Matter in any Part of the Universe».
28. H. Poincare. (1906). The Milky Way and the Theory of Gases. Popular Astronomy. (Vol. 14, pp. 475–481).
29. V. C. Rubin, N. Thonnard, W. K. Ford, Jr. Rotational properties of 21 Sc galaxies with a large range of luminosities and radii, from NGC 4605 (R=4kpc) to UGC 2885 (R=122kpc). The Astrophysical Journal. 1980.
30. Nicolae Sfetcu. Hooke's claim on the law of gravity. January 2019. DOI: 10.13140/RG.2.2.16867.81441
31. Nicolae Sfetcu. La controverse Newton-Hooke dans l'opinion des scientifiques contemporains. May 2019. DOI: 10.13140/RG.2.2.27734.40009
32. Nauenberg, M. Robert Hooke’s Seminal Contribution to Orbital Dynamics. Phys. perspect. 7, 4–34 (2005). https://doi.org/10.1007/s00016-004-0226-y