

Study On Delta Between Primes And Triangular Numbers

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Abstract

There are several interesting properties of triangular numbers and research work devoted to them. One of the categories is correlation between them and primes - there is hypothesis that between every two different triangular numbers >1 there is always a prime number. This paper is focused on detailed examination of such difference, mainly between triangular numbers and their closest (smaller or greater) primes (this difference is called in this work delta, δ_{TP}), including its extreme values, also in spirit of finding effective test to search for prime numbers.

1 Introduction

Triangular numbers T [1] are a sequence of natural numbers representing the sum of consecutive integers from 1 to n , forming a system of points in the shape of an equilateral triangle. i -th triangle number T_i can be calculated using the formula: $T_i = \frac{i(i+1)}{2}$ (which may be proven using mathematical induction). Prime numbers P are natural numbers >1 that are not a product of two smaller natural numbers >1 .

Both T and P are infinite sets. This work is devoted to study differences between triangular numbers and the closest primes, also in spirit of selecting (potentially) triangular number as a starting point for effective prime number search. Framework [2] is used in all experiments.

2 Types of delta

Let's denote difference between triangular number T_i and its closest prime P as δ_{TP} . δ_{TP} can be positive ($\delta_{TP} = +N$, when the closest prime is bigger than triangular number), negative ($\delta_{TP} = -N$, when the closest prime is smaller than triangular number) or $\delta_{TP} = 0$ (triangular number is also a prime). Special case is when triangular number is an average of two primes and both are the closest - let's denote this case as $\delta_{TP} = \pm N$.

When comparing specific numbers T and P we can also write: $\delta_{T,P}$, where T and P will be exact integer values, for instance: $\delta_{1,3} = +2$, $\delta_{6,3} = -3$.

3 When triangular number is a prime

There is only one triangular number that is also a prime number: 3, thus $\delta_{TP} = 0$ happens only once: $\delta_{3,3} = 0$.

Lemma 1. $\delta_{TP} = 0$ if and only if $T = 3$.

Proof. $T_i = \frac{i(i+1)}{2}$ is the formula to calculate i -th triangular number. It gives $T_2 = 3$ for $i = 2$ and 3 is a prime.

Triangular numbers lesser than 3 are two: $T_0 = 0$ and $T_1 = 1$ - neither of them is prime, so let's focus on triangular numbers greater than 3 ($i > 2$). i and $i + 1$ are two consecutive integers, thus one of them is even, thus either i or $i + 1$ is divisible by 2. If $i > 2$, then half of even number (either i or $i + 1$) is >1 . This means that $T_i, i > 2$ (based on its formula) is a multiplication of two integers >1 and cannot be prime by definition. \square

4 Delta = ± 1

If $\delta_{TP} = \pm 1$, then T_i is odd and P is even, or vice-versa.

Let's analyze trivial cases first where the closest prime is 2. For $T_0 = 0$ the closest prime is 2: $\delta_{0,2} = 2 > 1$. For $T_1 = 1$ the closest prime is also 2: $\delta_{1,2} = +1$ and for $T_2 = 3$ the closest prime is also 2: $\delta_{3,2} = -1$. For $T_3 = 6$ and next triangular numbers, the closest prime is >2 .

All primes greater than 2 are odd. In that case, if $\delta_{TP} = \pm 1$, then T_i needs to be even. Triangular numbers >0 are in infinite repeated sequence: "odd-odd-even-even", so 50% of candidates (odd) are eliminated by nature. Executed experiments show that $\approx 7\%$ results are with delta ± 1 (see Figure 1).

5 Delta = ± 2

If $\delta_{TP} = \pm 2$, then both T_i and P are either even or odd.

Let's analyze trivial case first. For $T_0 = 0$ we have $P = T_0 + 2 = 2$ and for $T_1 = 1$ we have $P = T_1 + 2 = 3$ - both 2 and 3 are primes and δ_{TP} cannot be negative here (because all primes $>T_1$).

All primes greater than 2 are odd, which leads to the non-trivial case: δ_{TP} can be -2 or 2 if and only if T_i is odd too. Triangular numbers >0 are in infinite repeated sequence: "odd-odd-even-even", so 50% of candidates (even) are eliminated by nature. Executed experiments show that $\approx 13\%$ of results are with delta ± 2 (see Figure 1).

6 In search of the largest delta

The largest delta between triangular number and the closest prime to it found so far is $\delta_{TP} = +202$ ($T_{1817553} = 3303502498845$, $P = T_{1817553} + 202 = 3\ 303\ 502\ 499\ 047$). Other notable findings are listed in Table 1.

Figure 1 is presenting frequency of $|\delta_{TP}|$ for the first 10^7 triangular numbers - it is clearly depicting that in most of the cases the δ_{TP} is small. This observation is a foundation to the experimental Hypothesis 1.

Hypothesis 1. Search for prime numbers is more effective around triangular number than a random number.

Table 1: Big δ_{TP} found during experiments.

T_i	δ_{TP}	Prime(s)
61 237 225 666	+157	61 237 225 823
395 550 422 641	± 168	395 550 422 473, 395 550 422 809
429 137 662 096	+181	429 137 662 277
455 980 508 061	-170	455 980 507 891
479 887 920 721	+186	479 887 920 907
3 303 502 498 845	+202	3 303 502 499 047
13 626 817 955 751	+190	13 626 817 955 941
22 722 660 593 955	+188	22 722 660 594 143
25 477 724 120 586	-187	25 477 724 120 399
36 239 385 654 181	+196	36 239 385 654 377

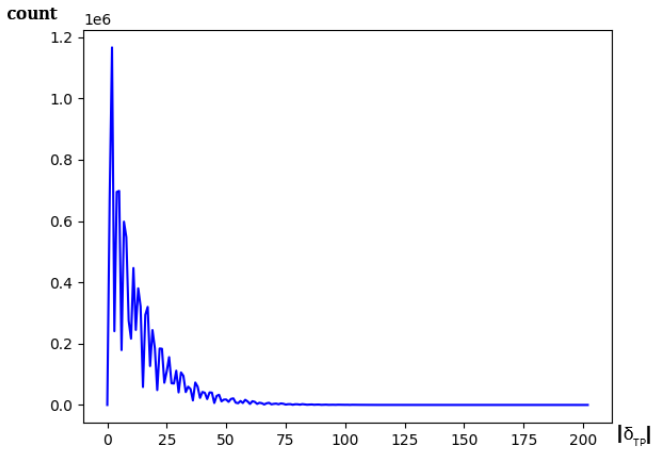


Figure 1: Frequency of $|\delta_{TP}|$ for the first 10^7 triangular numbers.

Figure 2 and Figure 3 show either positive or negative δ_{TP} recorded during experiments for consecutive T_i . Figure 4 is a concatenation of Figures 2 and 3 - it depicts that positive extreme δ_{TP} is rather more frequent than negative one (in other words, there is more red than blue on top of the concatenated range).

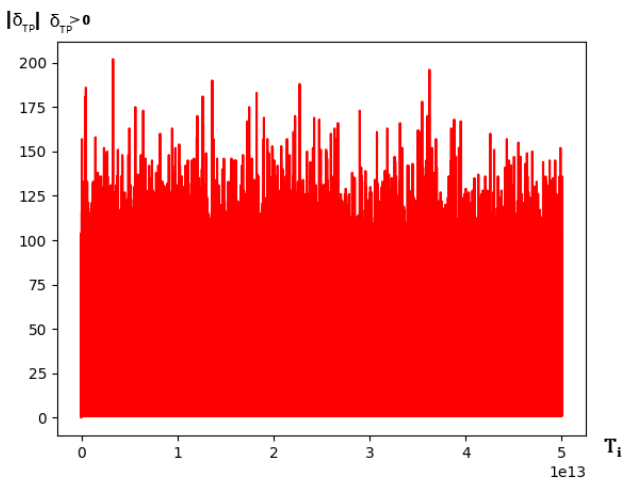


Figure 2: $\delta_{TP} > 0$ for the first 10^7 triangular numbers T_i .

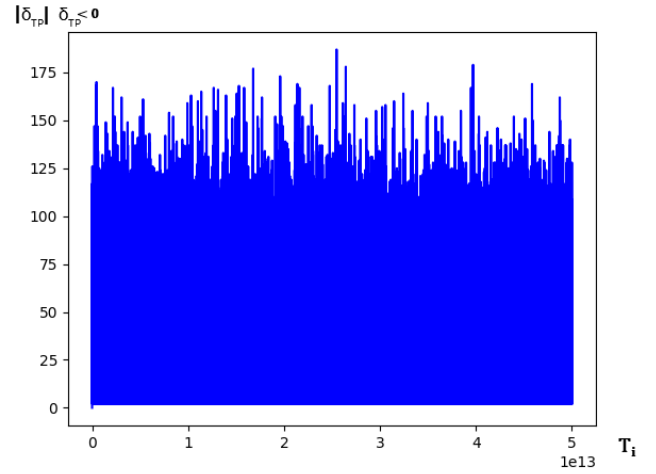


Figure 3: $\delta_{TP} < 0$ for the first 10^7 triangular numbers T_i .

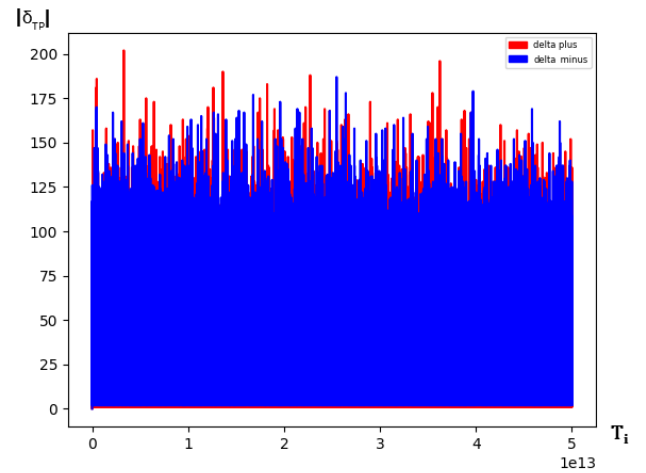


Figure 4: $|\delta_{TP}|$ for the first 10^7 triangular numbers T_i .

7 Experimental search for prime numbers

In order to verify Hypothesis 1 the following experiment was conducted: in examined interval $num_{min}; num_{max}$ it was compared how many iterations (± 1) from the starting point was required to find a prime number. Search for closest prime for conducted around triangular numbers, and, for comparison, around pseudo-random numbers and numbers of form $6 \times n - 1$.

Experiments were run in 4 series. In each series, for each type of number, focus was to search for the delta between it and its closest prime. Figures 5-12 depict the results with the two types of graphs: 1) frequency of the given delta, and 2) delta over consecutive numbers/runs. Additionally, every series ends with calculation of arithmetic average of the recorded deltas.

- Series #1: $num_{min} = 2500000; num_{max} = 3000000$
- Series #2: $num_{min} = 4000000; num_{max} = 4500000$
- Series #3: $num_{min} = 5000000; num_{max} = 5500000$
- Series #4: $num_{min} = 6000000; num_{max} = 6500000$

Results of series #1 (see also Figure 5 and Figure 6):

- Average delta - prime to triangular: 13.183896
- Average delta - prime to random: 6.226838

- Average delta - prime to $6n-1$: 5.649354

Results of series #2 (see also Figure 7 and Figure 8):

- Average delta - prime to triangular: 13.606476
- Average delta - prime to random: 6.3994
- Average delta - prime to $6n-1$: 5.649354

Results of series #3 (see also Figure 9 and Figure 10):

- Average delta - prime to triangular: 13.79982
- Average delta - prime to random: 6.552896
- Average delta - prime to $6n-1$: 5.649354

Results of series #4 (see also Figure 11 and Figure 12):

- Average delta - prime to triangular: 13.976484
- Average delta - prime to random: 6.648846
- Average delta - prime to $6n-1$: 5.649354

Executed experiments visualized that Hypothesis 1 is most probably false - more effective is either selection of a pseudo-random or number of form $6 \times n - 1$ as a starting point rather than a triangular number.

Thanks to good side conversation with Bill McEachen (thank you, Bill, for great feedback!) and further experiments (see: Table 2), generally average values of $|\delta_{TP}|$ and $|\delta_{P_{6N-1}}|$ are increasing, still with pseudo-random approach showing some benefits - the average value of $|\delta_{PR}|$ stays more or less the same: ≈ 6.7 .

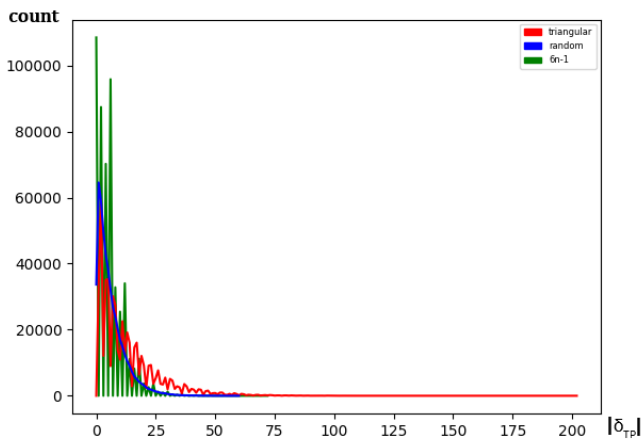


Figure 5: Comparison of frequency of $|\delta_{TP}|$ for three types of numbers (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set:

$$num_{min} = 2500000; num_{max} = 3000000.$$

8 Conclusions

The smallest possible δ_{TP} between triangular number and a prime is 0 but it happens only once. The largest delta between triangular number and the closest prime found during executed experiments is $\delta_{TP} = +202$. Based on executed experiments, triangular number is not the most effective starting point when looking for a prime using brute-force method (checking number ± 1 , one-by-one) - much more effective is selection of pseudo-random or $6 \times n - 1$ number instead.

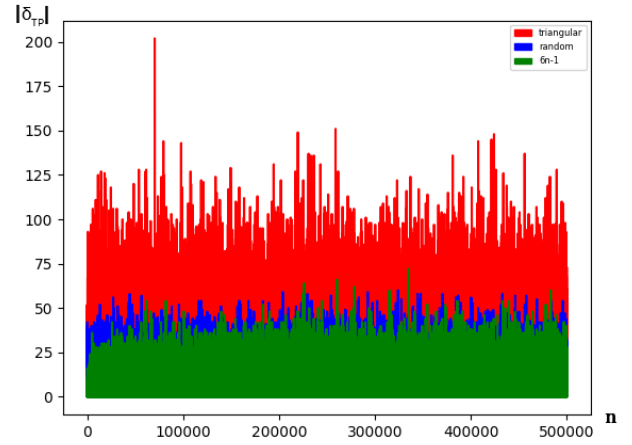


Figure 6: Comparison of $|\delta_{TP}|$ for three types of consecutive numbers n (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 2500000; num_{max} = 3000000$.

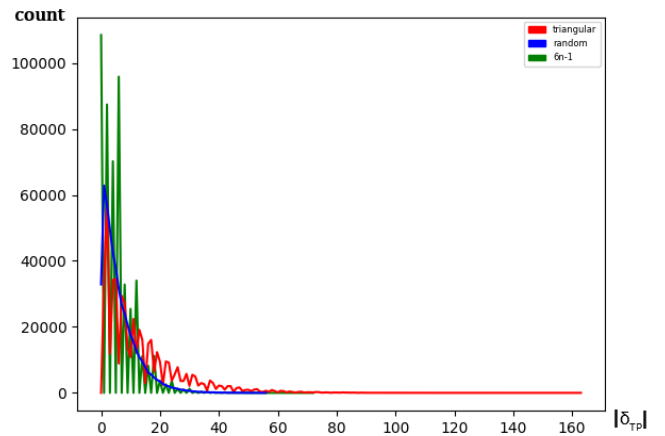


Figure 7: Comparison of frequency of $|\delta_{TP}|$ for three types of numbers (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set:

$$num_{min} = 4000000; num_{max} = 4500000.$$

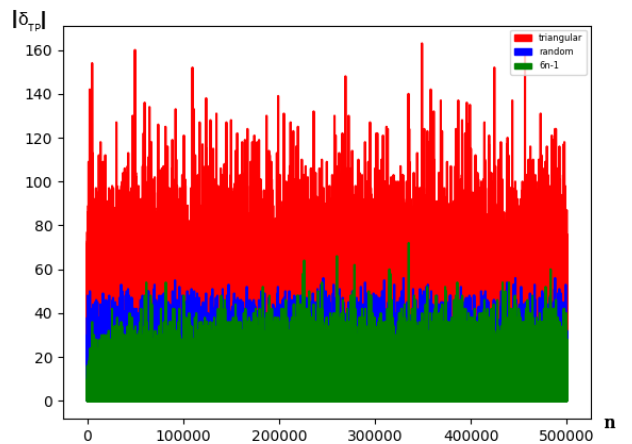


Figure 8: Comparison of $|\delta_{TP}|$ for three types of consecutive numbers n (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 4000000; num_{max} = 4500000$.

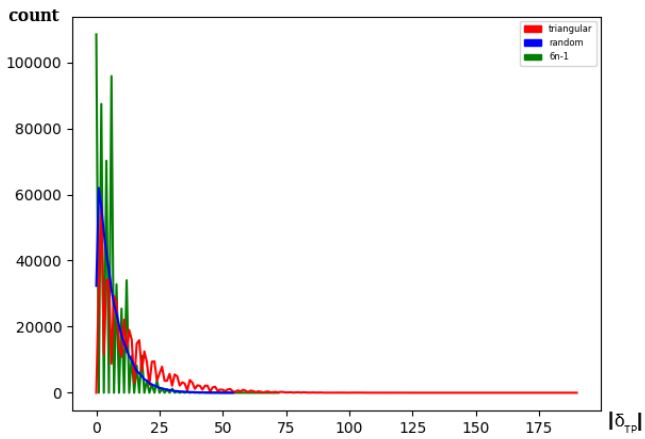


Figure 9: Comparison of frequency of $|\delta_{TP}|$ for three types of numbers (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set:
 $num_{min} = 5000000; num_{max} = 5500000$.

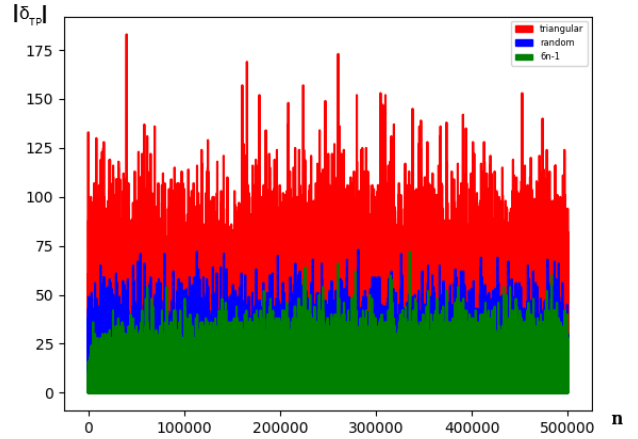


Figure 12: Comparison of $|\delta_{TP}|$ for three types of consecutive numbers n (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set:
 $num_{min} = 6000000; num_{max} = 6500000$.

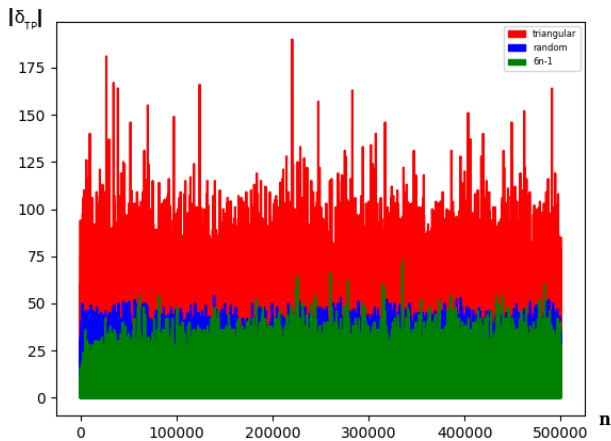


Figure 10: Comparison of $|\delta_{TP}|$ for three types of consecutive numbers n (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set:
 $num_{min} = 5000000; num_{max} = 5500000$.

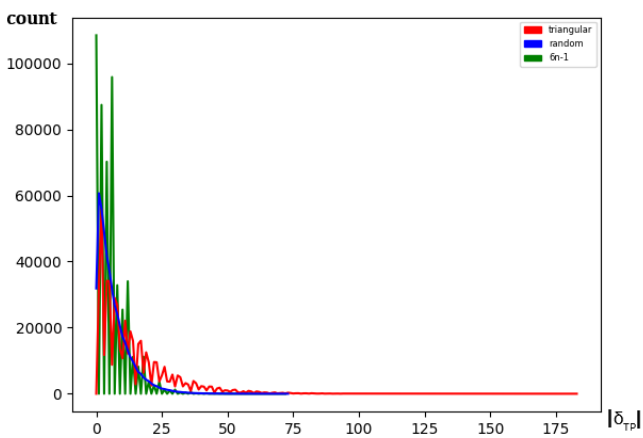


Figure 11: Comparison of frequency of $|\delta_{TP}|$ for three types of numbers (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set:
 $num_{min} = 6000000; num_{max} = 6500000$.

References

- [1] *Weisstein, Eric W.* "Triangular Number." From MathWorld – A Wolfram Resource. <https://mathworld.wolfram.com/TriangularNumber.html>
- [2] *Library for various operations on primes.* <https://github.com/mbarylsk/primes>

Table 2: Follow-up observations on averages of $|\delta|$ for $num_{min} = 0; num_{max} = 2 \times 10^6$ over time (checkpoint every 10^5 iterations) and examined types of numbers: PT - Prime to triangular, PR - prime to pseudo-random, P6N-1 - prime to number of form $6 \times n - 1$.

Checkpoint	Avg $ \delta_{PT} $	Avg $ \delta_{PR} $	Avg $ \delta_{P6N-1} $
100000	9.02326	6.70626	4.83723
200000	9.68797	6.704515	5.186985
300000	10.07056	6.703313	5.41249
400000	10.33746	6.707915	5.543198
500000	10.551448	6.703474	5.649354
600000	10.726043	6.703688	5.749425
700000	10.872434	6.705531	5.823096
800000	11.00382	6.710701	5.886694
900000	11.115558	6.711011	5.938134
1000000	11.226858	6.707796	5.992291
1100000	11.31836	6.714556	6.03671
1200000	11.39733	6.718638	6.078671
1300000	11.471386	6.717348	6.120755
1400000	11.543679	6.714992	6.155496
1500000	11.615045	6.713589	6.189762
1600000	11.67529	6.709936	6.219768
1700000	11.735036	6.708338	6.251105
1800000	11.790726	6.707284	6.275476
1900000	11.840858	6.708253	6.304371
2000000	11.892727	6.70967	6.328365