Constituent Quarks of the First Generation

(Corrected Version)

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Abstract: This paper calculates the masses of up and down quarks within the proton and neutron. It is a corrected version of a paper previously published on the same platform. The correction concerns Formula (11) and the corresponding updates to the related numerical results.

For our analysis, we introduce a virtual particle, which we call the fundamental particle [1], whose mass m_f and radius r_f are given by:

$$m_f = 1.088621711 * 10^{-28}\ kg, \qquad r_f = 3.231308824 * 10^{-15}\ m, \tag{1}$$

These values satisfy the condition derived from the well-known form of the $\hbar/c$ relation, which follows from the standard definition of the **reduced Compton wavelength** ***(2)***:

$$ƛ = \hbar / (c*m) \tag{2}$$

valid for elementary particles, including the proton. Using the expressions from [2], we obtain:

$$r_f = (2\pi)^{1/3} * 2^{cy/9} * (m_{pl}/ m_p)^{-8/9} * ƛ_p = 3.231308824E\text{-}15 \tag{3}$$

$$m_f = (2\pi)^{-1/3} * 2^{-cy/9} * (m_{pl}/m_p)^{8/9} * m_p = 1.088621711E\text{-}28 \tag{4}$$

Here:

- m_{pl} – denotes the Planck mass,
- l_{pl} – the Planck length,
- m_p – the proton mass,
- $ƛ_p$ – the reduced Compton wavelength of the proton.

Relations ***(3)*** and ***(4)*** define the complementary scaling structure of the model, in which r_f and m_f are mutually paired quantities that together determine the invariant of the system. By multiplying expressions ***(3)*** and ***(4)***, the scaling factors are eliminated, yielding the invariant relation:

$$r_f * m_f = ƛ_p * m_p = \hbar/c \tag{5}$$

This complementarity represents a fundamental structural characteristic of the defined scale, rather than a derived property of the model.

Within the framework of this invariant, $\boldsymbol{r_f}$ defines an internal scale quantity of the model, which is not identified with the empirical scale of the order of 1 fm, but rather represents its structural equivalent within the defined scaling framework.

In this paper, the fundamental particle is regarded as a virtual boundary value, that is, a theoretical reference point analogous to the Planck scales, for which reason the term *fundamental boundary* is also introduced. In this way, the standard nuclear scale (~1 fm) acquires an internal interpretation through $\boldsymbol{r_f}$, as a phenomenological reference quantity of the same physical domain.

Within this framework, a structural quantum state around this virtual boundary is considered as the basis for describing mass increment, an idea indicated in different ways by various authors. Among them, two are particularly notable: **Steven Weinberg** in [3, p. 619, Eq. (11.4.2)] and **Stevan Bošnjak**, whose interpretation was closest to this concept and who states [4, p. 219]:

$$h/c = 2.202 \times 10^{-37}\ cm \cdot g. \qquad\qquad m \cdot \lambda = 2.202 \times 10^{-37}\ cm \cdot g.$$

Let us define the *noncohesion* limit of an elementary particle as:

$$r = r_f * (m/m_f)^{0.5} \tag{6}$$

Here, the state in the vicinity of the fundamental limit will be represented solely through the final dimensionless formula ***(7)*** for the **mass increment** relative to the previous one (index $_{n-1}$) [1].

$$m_n = m_{n-1} * \left(1 + \frac{\hbar / c}{r_{n-1} * m_{n-1}}\right) = m_{n-1} * \left(1 + \frac{r_f * m_f}{r_{n-1} * m_{n-1}}\right) \tag{7}$$

By substituting ***(5)*** and ***(6)*** into ***(7)***, we obtain:

$$m_n = m_{n-1} * \left(1 + \frac{m_f^{1.5}}{m_{n-1}^{1.5}}\right) = m_{n-1} + \sqrt{\frac{m_f^3}{m_{n-1}}} \tag{8}$$

If we define the dimensionless quantity $\boldsymbol{x}$ by reducing the mass with respect to $\boldsymbol{m_f}$,

$$x = m / m_f \tag{9a}$$

we obtain:

$$x_n = x_{n-1} + x_{n-1}^{-0.5} \tag{9b}$$

which is a dimensionless formula for the mass increment. At each step, a portion of the content smaller than ***ħ/c*** that lies within the *noncohesion* boundary is captured. Since for *up* and *down* quarks $\boldsymbol{m<m_f}$, they are therefore the only ones that can exist in a bound state, i.e., in mesons, protons, and neutrons, as so-called constituent quarks.

In this way, the constituent quark "emerges" through a mass increment expressed by the ratio of Planck's constant ($\hbar$) to the speed of light (c), $\hbar/c$, which introduces reduced quantities that link mass and length independently of the choice of units.

The use of dimensionless reduced quantities significantly simplifies the treatment of elementary particles and enables a clearer analysis of their mutual relationships.

This is not a case of converting energy into mass (according to $E=mc^2$), but rather of a **structural quantum state** that forms around a virtual fundamental limit.

If the reduced Compton wavelength ***(2)*** is expressed relative to r_f and denoted by r_c, we obtain:

$$r_c = \lambda\!\!\!^{-} / r_f = (\hbar/cm)/r_f = (m_f * r_f /m)/r_f = m_f /m = 1/x \qquad \textbf{(10)}$$

which we interpret as representing Bošković's *cohesion* limit for particles [6]:

Formula ***(9b)*** is applied repeatedly to the reduced masses (Table 1), $x = m/m_f$. Its successive application yields the masses of mesons, then of the proton, and in the third step of the neutron, with up and down quarks modeled via non-cohesive boundaries. In doing so, it is observed that lighter quarks acquire a larger mass increment than heavier ones, which is a well–known fact.

In the domain of masses and radii smaller than that of the fundamental particle, we identify two "***bare***" first–generation quarks—*up* and *down*—obtained by heuristic means.

$$m_u = 8.373176145*10^{-30}\,kg, \qquad m_d = 4.185645769*10^{-30}\,kg \qquad \textbf{(11)}$$

Table – Quantum mass increment of quarks from (11) via formula (9b)

	$x_0 = m/m_f$	$x_1=x_0+x_0^{-0.5}$	$x_2 = x_1+x_1^{-0.5}$	$x_3 = x_2+x_2^{-0.5}$
d	0.07691539	3.68264687277	4.20374553917	4.69147814535
u	0.03844904	5.13829736663	5.57945153059	6.00280609395
		$m_{pr} = m_f*(2u_2+d_2)=$	1.67241128117E-27	2.10642520635E-31
			$m_{ne} = m_f*(u_3+2*d_3)=$	1.67492749791E-27

We have used the masses of bare quarks (with the note that other, and even imaginary, solutions are also possible). The second iteration yields the mass of the proton, understood as a system composed of two *up* quarks and one *down* quark, with a remaining mass of $m_{gl} = 2.10642543125*10^{\wedge-31}\,kg$, which may be attributed to the contribution of gluons.

The third step yields the mass of the neutron ***(13)***, which consists of two down quarks and one up quark. In expressions ***(12)*** and ***(13)***, the values for u and d are taken from Table 1, while $m_f = 1.08862171145*10^{-28}$ is given by (4). Thus, we obtain for the proton:

$$m_p = m_f*(2u_2+d_2) + m_{gl} = 1.67262192369 * 10^{-27}\text{kg} \qquad \textbf{(12)}$$

and for the neutron:

$$\boldsymbol{m_{ne} = m_f * (u_3 + 2d_3)} = 1.67492749787 * 10^{-27}\text{kg} \qquad \textbf{(13)}$$

These are the values:

$$m_p = 1.672\ 621\ 923\ 69(51) * 10^{-27}\ kg \quad ,$$

$$m_{ne} = 1.674\ 927\ 498\ 04(95) * 10^{-27}\ kg.$$

according to the CODATA 2018 reports [7]. According to this model, the neutron is most likely entirely composed of one *up* and two *down* quarks, since any possible gluonic remainder lies within the limits of numerical uncertainty; we therefore conclude that it is effectively absent in the neutron.

In other words, by employing the fundamental limit, we obtain constituent quarks that eliminate the need for the explicit inclusion of gluons and additional "exotic," pion–mediated interactions within the nucleon (with pions in this model already appearing after the first iteration, which is not discussed here in detail).

The *noncohesion* and *cohesion* limits, as well as the corresponding masses, can be expressed in the standard system of units [**m, kg**] by multiplying them by the values of $\boldsymbol{r_f}$ and $\boldsymbol{m_f}$ from ***(3)*** and ***(4)***. The constituent quark masses obtained from the second and third iterations of expression ***(9b)***, however, can be more conveniently used in dimensionless form for determining the forces within nucleons.

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