

METAMATHEMATICS

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INDEX

	Abstract
I	Spatial Dimensionality Via Metageometry
II	Infinity
III	Zero
IV	Nothing
V	Pure Geometry
VI	Curved Space

ABSTRACT

The following metamathematical analyses of the mathematical concepts and language "zero dimensional", "one dimensional", "two dimensional", "extra dimensional", "infinity", "zero", "nothing", "pure geometry", and "curved space", demonstrate that they are arrived at via one or more of conceptual dissociation, conceptual association, and variations of linguistic alteration, and that they, as such, cannot represent facets of the universe. (The above and below language that are quoted are so because their meanings are in question).

I

SPATIAL DIMENSIONALITY VIA METAGEOMETRY

I-I

- (1) Regarding a "one dimensional line", which, as such, is stated to have only length: It also has depth, because it rose into the pictorial medium, such as a graph (that is, it rose into space), via a pictorial medium, such as a writing instrument; and it also has height, because it is extended vertically on the pictorial medium, such as a graph, via a pictorial medium, such as a writing instrument. Regarding a "two dimensional plane", which, as such, is stated to have only length and height: It also has depth, because it rose into the pictorial medium, such as a graph (that is, it rose into space), via a pictorial medium, such as a writing instrument.
- (2) Therefore: A "line without height and depth" is the same as no line; and a "plane without depth" is the same as no plane.
- (3) Stating, via mathematic language and non-mathematic language, that a line does not have height, nor depth, does not mean that it does not. Stating, via mathematic language and non-mathematic language, that a plane does not have depth does not mean that it does not.
- (4) Moreover: A "heightless and depthless line", and "depthless plane", cannot be coherently conceived of (that is, per I-II below, non dissociatively conceived of), for the same reasons in (1) – (3). Stating that they have been conceived of via such language (that is, via the meaning of such language) does not entail that anything has been conceived of.
- (5) Moreover: If a "one dimensional line", and a "two dimensional plane", did not have depth, they would not appear when viewed from the side views, nor the top views, nor the bottom views. And there is nothing of the universe that is of this nature, nor could there be.
- (6) Moreover, per (1) – (5): If a line did not have height, there would be nothing to see; if a line did not have depth, there would be nothing to see; and a line's length cannot be produced without concurrently

producing height and depth. Moreover, per (1) – (5): If a plane did not have depth, there would be nothing to see; and a plane's length and height cannot be produced without concurrently producing depth.

(7) Per (1) – (6): The three dimensions of a graphical line and plane, and non-graphical line and plane, can be measured: The media that were used to produce both entails the production of depth via the media, which can be measured, as well as the production of (for the "one dimensional"), height, which can be measured.

(8) Therefore, stating via mathematic language and non-mathematic language that a line and plane have less than three dimensions is to, per I-II below, dissociate, from the line and plane, the dimensions that they are stated to not have.

I-II

(9) There can be only three spatial dimensions, namely length, height, and depth. Moreover, all three can only exist concurrently, such that there is no facet of the universe that does not have all three.

(10) The mathematic concepts "one dimensional" and "two dimensional" are produced by conceptually dissociating two of the components of the concept three dimensional from the concept three dimensional, namely, regarding "one dimensional", length, and regarding "two dimensional", length and height.

(11) That the three dimensional is an amalgam of three dimensions provides the opportunity for the aforementioned dissociation.

(12) The prevailing concept is that the graphical three dimensional is constructed from the sequential addition of length, height, and depth, and that, as such, the graphical three dimensional can be deconstructed into the "two dimensional" ("length and height only"), and "one dimensional" ("length only"). The third dimension, namely depth, cannot be dissociated from the three dimensional, because it is the final stage of the aforementioned additive sequence – that is, it is the final dimension. Dissociating from it would result in the dissociation of the three dimensional from the graph, which would result in the absence of any spatial dimensions.

(13) Mathematical and non-mathematical concepts and statement that are arrived at via dissociation cannot represent a facet of the universe, because the dissociation entails dissociating inextricably combined facets of the universe from one another. That is, dissociation entails ignoring what is the case.

(14) Therefore, the mathematic language and non-mathematic language 'one dimensional' and 'two dimensional' are conceptually meaningless. Moreover, they are only subtractive linguistic alterations (subtractive mathematic, and subtractive non-mathematic, linguistic alterations): They are only subtractive language of the meaningful language 'three dimensional', namely, for 'two dimensional', a subtraction of one from the 'three' of 'three dimensional', and for 'one dimensional', a subtraction of two from the 'three' of 'three dimensional'.

I-III

(15) Lisa Randall, regarding her discussion about the "two dimensionality" of book pages, states the following (Randall, 2006, p22):

"... for the pages to be truly two-dimensional they would have to be infinitely thin slices with no thickness at all in the third dimension. For now, though, two dimensions is a fine approximation for pages as thin as these."

Regarding, "infinitely thin slices with no thickness at all in the third dimension", the concept "infinitely thin" expresses that there is depth, namely X thin, yet she concurrently conceives of this thinness as "no thickness at

all". That is, her concept "two dimensional" is that the thinness that there is, is of no thinness. Aside from this being incoherent (namely, thinness cannot be no thinness), it is dissociative: There is thinness, and ignoring this is to dissociate from what is the case.

I-IV

(16) Regarding a "zero dimensional point": It is stated to have no spatial dimensions.

(17) "Zero dimensional" is a dissociation from the three dimensional, because it is stated that the three dimensional consists of an array of "zero dimensional points": The dissociation is that the claimed "zero dimensional" component of the three dimensional is conceived of as existing independently. (However, see (19) below).

(18) However, according to the dissociation "zero dimensional": When the "point" is viewed from the front view, rear view, top view, bottom view, and side views, the point would not appear. There is nothing of the universe that is of this nature, nor could there be.

(19) However, the "point" appears at each view, which demonstrates that it is three dimensional. Moreover, as was discussed in I-I about a line and plane: The point has depth, because it rose into the pictorial medium, such as a graph (that is, it rose into space), via a pictorial medium, such as a writing instrument; it has length, because it is extended horizontally on the pictorial medium, such as a graph, via a pictorial medium, such as a writing instrument; and it has height, because it is extended vertically on the pictorial medium, such as a graph, via a pictorial medium, such as a writing instrument.

(20) As such, regarding (17), the three dimensional actually consists of an array of three dimensional points.

(21) As such, the dissociation method in (17) is a failed attempt at dissociation, because the three dimensional does not consist of the "zero dimension": The "zero dimension" cannot be dissociated from the three dimensional because the amalgam (per I-II) of the three dimensional does not include the "zero dimension": The inextricable facets of the three dimensional, namely length, height, and depth, does not include the "zero dimension".

(22) Therefore: A "point without dimensions" (that is, a "zero dimensional point"), is the same as no point, or nothing. And, 'zero dimensional point' means no point, or nothing.

(23) As such, "zero dimensional" is not a dissociation, but rather a linguistic alteration, namely 'zero dimensional', which is a zeroed linguistic alteration: The meaningful language 'three dimensional' is zeroed.

(24) Linguistic alterations, because they are only linguistic alterations, cannot have conceptual meaning, and can only have the only alternative meaning, namely psychological meaning.

(25) Linguistically expressing, and mathematically expressing, that something is "zero dimensional", including attempting to provide a geometrical representation via the aforementioned pictorialization, does not mean that there is "zero dimensional", nor that there could, nor that the concept is coherent.

I-V

(26) "Extra Spatial Dimensionality" is only mathematic-linguistic additive alteration: (a) There is no conceptualization, because there are no observable "extra spatial dimensions", and (b) there is no geometry (no pictorialization), because there is no pictorialization of "extra spatial dimensions".

(27) The mathematic-linguistic additive alteration consists of using mathematic language that has meaning, namely 'three dimensional', and altering it via additive mathematic-linguistic alteration to express "extra spatial dimensions", namely 'four dimensional', 'five dimensional', etc.

(28) The expressed "four dimensional", "five dimensional", etc, are therefore only the language 'four dimensional', 'five dimensional', etc.

(29) Mathematic language that is arrived at in that way cannot have mathematic meaning, and instead has the only alternative meaning, namely psychological meaning.

(30) The linguistic alteration is engaged in to create the psychological experience of accurate scientific discernment, and accurate mathematic discernment.

(31) Aside, Randall states (Randall, 2006, p17):

"... let's think more about the meaning of the term "multidimensional space" ['multidimensional space']."

And she states (Randall, 2006, p18):

"Extra dimensions are other directions entirely. They are hard to describe, but they may be easier to understand by analogy."

She invokes *Flatland* for the analogy (Randall, 2006, p17-22; Closer To Truth, 2019), stating that, hypothetically, people who reside only in the "two dimensional" cannot experience, nor understand, the three dimensional, and that this is a basis to think that there may be "extra dimensions" that we cannot experience nor understand.

"In 1884, to explain the notion of extra dimensions, the English mathematician Edwin A. Abbott wrote a novel called Flatland." "... in Flatland three dimensions are beyond its inhabitant's comprehension. Everyone thinks it is obvious that the universe holds no more than their two perceived dimensions." "It is not until A. Square has been lifted out of Flatland into the surrounding three-dimensional world that he can truly imagine a sphere." "... while he is still confined to Flatland, he watches a three-dimensional sphere travel vertically through his two-dimensional world." "... he sees a series of disks that increase and then decrease in size, which are slices of the sphere as it passes through A. Square's plane ..."

However, and momentarily ignoring what I have presented about "two dimensional": It is implied that she assumes that the people who reside in the "two dimensional" have "two dimensional" vision, namely that they cannot look up, nor down, and that due to this, they cannot experience, nor understand, the surrounding three dimensionality. However, there is no such kind of vision: The visual field of vision is always three dimensional, even when one's eyes remain still. Even disease that produces spatial deficit, or regional blindness, cannot replicate the absence of superior quadrant vision, and inferior quadrant vision. As such, they could see three dimensionality. Moreover: Regarding the aforementioned "... for the pages to be truly two-dimensional they would have to be infinitely thin slices with no thickness at all in the third dimension", this would entail that not only can nothing exist in the "two dimensional", but that there is no "two dimensional". And relatedly: Regarding the aforementioned sphere that A. Square visually observes, there actually would not be anything to observe because the "two dimensional" does not permit "size" (for example, "slices"). Moreover: A "two dimensional" person being "lifted" (moved upward) would not change anything for that person: The person would remain "two dimensional", and concurrently experience only the "two dimensional", regardless of where in the three dimensional the person is. Moreover: The person is always in the three dimensional; and as such, moving the person upward would not result in the person becoming three dimensional.

(32) Nima Arkani-Hamed, et al, invoke *Flatland* for the same purpose in "The Universe's Unseen Dimensions". (Arkani-Hamed, et al, 2000; p62, first paragraph; p69, final paragraph)

(33) Leonard Susskind invokes *Flatland* for the same purpose in *The Black Hole War: My Battle With Stephen Hawking To Make The World Safe For Quantum Mechanics*, at pages 234, 398, 400, 411, and 416-417. (Susskind, 2008)

(34) Paul Davies invokes *Flatland* for the same purpose in "Expanding The Universe" (Davies, 2005; p1161, first paragraph)

(35) Michio Kaku invokes *Flatland* for the same purpose in *Hyperspace: A Scientific Odyssey Through Parallel Universe, Time Warps, And The Tenth Dimension*, at pages 46-48, 55-61, 70-74, 76-77, 90-91, 96, 106, 180-181, 204, and 340-341. (Kaku, 1994)

(36) Carl Sagan invokes *Flatland* for the same purpose in *Cosmos* (Sagan, 1980) at pages 276-278; and in the television series, *Cosmos: A Personal Voyage* (Sagan et al, 1980-1981), he reconstructs an aspect of that discussion: At 0:52 to 1:07, he says, referring to what he refers to as "two dimensional" pieces of paper:

"Now, we have width and length, but no height at all. Now these little cut-outs have some little height; but let's ignore that: Let's imagine that these are absolutely flat."

Aside from what I discuss above about the issues that underlie his statement, his statement, "let's ignore that", is his request that the issues that underlie his statement be dissociated from. As is the case with Randall (15), he believes that "absolutely flat" (Randall's "infinitely thin") means no thickness; and again this is incoherent, because any degree of flatness entails some degree of thickness. He however goes farther than Randall by asking us to ignore that that is the case (namely, by asking us to ignore that there is "some little height"; (and here he uses 'height' in the sense of thickness)). His asking us to ignore this is to ask us to dissociate from something that is the case. Per (13): Mathematical and non-mathematical concepts and statements that are arrived at via dissociation cannot represent a facet of the universe, because dissociation entails ignoring what is the case.

(37) Likewise, Edward Hall (Hall, 2014) states that, for example, "an object with no thickness" (that is, "no depth"), is an "ideal", and that it, as such, should not be discounted.

(38) Likewise, Robert Thomas (Thomas, 2022), regarding the issues that I raise, states "I simply ignore them", and that "I'm ... a mathematician whose work is made simpler by ignoring" such issues.

I-V-I

(39) The prevalent theory that "extra spatial dimensionality" is compacted (the theory of compactification), such that "extra spatial dimensions" cannot be observed, nor measured, because they are too small, because they are "curled up on themselves" (for example, Greene, 1997, and Grana, 2006), (a) dissociates from spatial dimensions what they formally are (that is, what their general nature is), namely that they are the pervasive spatial context of the universe, and non-physical, and (b) associates to "extra spatial dimensions" a property that they do not have, namely that they are physical, and as such geometric, and as such of size and shape.

(40) Only physical existents can be curled up on themselves, etc.

(41) Spatial dimensions, as such, cannot be "curled up on themselves" (that is, compacted).

(42) "Extra spatial dimensionality" is therefore a dissociative concept and associative concept, and as such cannot represent a facet of the universe: Regarding dissociation: Per (13): Mathematical and non-mathematical concepts and statements that are arrived at via dissociation cannot represent a facet of the universe, because dissociation entails ignoring what is the case. Regarding association: Mathematical and non-mathematical concepts and statements that are arrived at via association cannot represent a facet of the universe, because association entails ignoring what is not the case.

(43) The dissociation and association are engaged in to produce the belief (the psychological state) that novel fundamental facets of the universe have been uncovered.

(44) Aside: Randall above states: "Extra dimensions are other directions entirely", and adds that they are "... extra-dimensional passages", and adds the example, "... what he had was a device that moved in any direction, so long as it was a direction in the three-dimensions", and that it "didn't really go any way "you can think of"". (Randall, 2006, p17-18) While she demonstrates that the "extra dimensions" that she discusses are, like three dimensionality, spatial, she misbelieves that existents can move in only one of the spatial dimensions at a time. As I discuss above, existents are always three dimensional; and as such, their movements are always three dimensional, notwithstanding that they always move in a relatively greater degree of one or two of their dimensions. (It is not possible to move in a purely three dimensional way, because movement has to be up, down, right, left, in, out, and possible combinations thereof, such as up-right, up-right-out, etc. However, existents can expand and contract in a purely three dimensional way).

(45) Aside: Randall above states, regarding "extra spatial dimensions": "They are hard to describe, but they may be easier to understand by analogy." She, nor the others who are mentioned above, describe them; and stating that they are "curled up on themselves", and unobservable, is not an even minimally complete description of them; and as I discuss above, no one has observed "curled up" spatial dimensions. Moreover, using mathematic language to represent them, which is the same as using the non-mathematic language 'fifth dimension', 'sixth dimension', etc, does not mean that there are such dimensions, as I discuss above.

I-VI

(46) Notwithstanding that the following has not yet occurred in mathematics and physics, it would be consistent with the aforementioned to engage in the negative linguistic alteration of expressing "negative spatial dimensions", such as "negative one dimensional", etc.

(47) In this case, there would not be dissociation, because there would not be dissociation from the three dimensional: "Negative spatial dimensions" are not stated to comprise the three dimensional, and as such, cannot be dissociated from the three dimensional. Instead, there would only be negative linguistic alteration via the use of 'negative'.

(48) "Negative" in this context would therefore have only psychological meaning: The linguistic alteration is engaged in to create the psychological experience of accurate scientific discernment and accurate mathematic discernment.

II

INFINITY

II-I

(1) Of course, "infinity" has never been observed, nor measured. ("Infinity", when "it" is the subject of meta-analysis, should always be quoted to indicate that nothing has yet been instantiated).

(2) "Infinity", according to the form of "its" own definition, cannot be observed, nor measured: "Infinity" (including "infinite", "infinitely", "infinities", "infinitesimal", etc) partly means unobservable and unmeasurable, and as such, precludes "itself". For example:

(3) There is nothing of the universe that can be observed, nor measured, to be "infinitely divisible", "infinitely extended", of "infinite distance away" ("infinite micro-distance away", and "infinite macro-distance away"), of "infinite size" ("infinite micro-size" ("infinitesimal"), and "infinite macro-size"), "infinitely continuous" (in minimization, and maximization), of "infinite speed", etc.

(4) The concept "infinity" is, therefore, actually an antonymic linguistic alteration of the term 'finite', and as such, can not represent a facet of the universe.

(5) "Infinity" is therefore only 'infinity', and therefore has no mathematic meaning, nor scientific meaning, and instead only psychological meaning.

(6) Linguistic alterations cannot represent facets of the universe.

II-II

(7) 'Infinity', in structure, actually means not finite, which does not convey the structure of its use, namely as greater than finite. As such, what is structurally accurate is 'superfinite'.

(8) Per (I-6), 'superfinite' is actually an additive linguistic alteration of 'finite', and as such, does not represent a facet of the universe.

(9) There is a vast extent of mathematic language and non-mathematic language that can be structurally altered in the aforementioned ways; and that they can be altered as such does not entail that they must therefore represent facets of the universe.

III

ZERO

(1) Whereas there can be zero particular existents, phenomena, etc, there cannot be "zero itself": "Zero itself" has the same meaning as "nothing"; and for the reasons that are presented in IV below about "nothing", there is no "zero itself".

(2) Therefore, "zero" is actually a dissociation, namely a dissociation from zero particular existents, phenomena, etc, such that it is believed that there is "zero itself". That is: Let $0x$ represent zero particular existents; and the 0 is dissociated from x .

(3) Per (I 13): Mathematical and non-mathematical concepts and statements that are arrived at via dissociation cannot represent a facet of the universe, because the dissociation entails dissociating inextricably combined facets of the universe from one another. That is, dissociation entails ignoring what is the case.

IV

NOTHING

(1) "Nothing" would be a region of the absence of universe, which is impossible:

(2) Anything that exists is universe; and there are no regions of the universe that are "non-universe".

(3) "Nothing" would be a region of "non-universe".

(4) Moreover, "non-universe" cannot be conceived of, and instead "it" is only 'non-universe', namely an antonymic linguistic alteration of 'universe', which as such is conceptually meaningless, and which instead has the only alternative meaning, namely psychological meaning.

V

PURE GEOMETRY

- (1) The concept "Amplituhedron" (Arkani-Hamed, et al, 2013), namely an object of "pure positive geometry", is arrived at via dissociation of the geometry of an object from the object.
- (2) However, geometry is not an existent. Conceiving of it as such is analogous to conceiving of the various properties of matter as existing in the universe without the matter. For example, temperature itself, weight itself, mass itself, color itself, spin itself, etc.
- (3) Notwithstanding: Per (*I 13*): Mathematical and non-mathematical concepts and statements that are arrived at via conceptual dissociation cannot represent a facet of the universe, because such dissociation entails dissociating inextricably combined facets of the universe from one another.
- (4) What is further conceived is that the "pure geometry" is "positive geometry", namely that it is the causal source of the universe. That is, that the universe arose from the geometry that was imparted to it by Amplituhedra.
- (5) However, since there is no geometry that is apart from geometric things, geometry cannot be a causal source.
- (6) The dissociation is engaged in to produce the belief (the psychological state) that a novel fundamental facet of the universe has been uncovered.

VI

CURVED SPACE

- (1) Space (that is, spatial field) is not the kind of facet of the universe that can be curved: It cannot have internal geometry, except for three dimensional extension. Only its boundary (macro boundary) could be curved.
- (2) Objects can be curved, but only at their macro boundaries.
- (3) The spatial field interiors of objects cannot be curved: The interiors are extended without curvature to the objects' macro boundaries.
- (4) One or more substances that comprise the spatial field interiors of objects cannot be curved: They can only be condensed to different degrees, and in different locations. And in this case, when something of a particular nature, and particular speed, travels though that spatial field, it may have one or more curved paths of travel.
- (5) The prevalent demonstration of "curved space" via placing a weighted object on an elastic surface, thereby causing the surface to become concave, entails only the curving of the two macro boundaries of the elastic surface, namely the inner macro boundary, and outer macro boundary. Moreover: The interior of the elastic surface, however minimal, is not curved: It is extended three dimensionally, and as such without shape, to the above inner macro boundary and outer macro boundary.
- (6) The concept "curved space" is therefore actually an associative concept: associated to space is a property that it does not have.
- (7) Moreover: The concept "curved space" is therefore actually only a combinatorial linguistic alteration, namely 'curved' is combined with 'space', and as such, does not represent a facet of the universe.

(8) "Curved space" is therefore only 'curved space', and therefore has no mathematical meaning, nor physical meaning, and instead only psychological meaning: The associative conceptualization, and combinatorial linguistic alteration, are engaged in to produce the belief (the psychological state) that a novel fundamental facet of the universe has been uncovered.

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