

The Oscillating Worldline: Unifying Kinematic and Gravitational Null Boundaries via Symplectic Involution

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Standard physics treats the kinematic null boundary in Special Relativity ($v \rightarrow c$) and the gravitational event horizon in General Relativity ($r \rightarrow r_s$) as two distinct asymptotic limits. This paper proposes a unified framework that reinterprets both limits as manifestations of a universal *null topological seam* ($ds^2 \rightarrow 0$). We posit that elementary particles do not terminate at these boundaries, but rather undergo a discrete topological transformation of their proper causal flow. By elevating the geometry to the cotangent bundle, we show that crossing this null boundary triggers a symplectic involution ($\Omega \rightarrow -\Omega$). This rigorously reverses the canonical Hamiltonian vector flow, acting as a geometric CPT inversion while leaving the background metric ($g_{\mu\nu}$) invariant. Applied to flat space-time, this provides a kinematic engine for the Feynman-Stueckelberg matter-antimatter annihilation vertex. Applied to gravitational collapse, it yields a “Black Mirror” topology where the interior singularity is replaced by an effectively repulsive transition into a conjugate anti-universe. By unifying these boundaries, we establish a continuous bipartite phase space where a single oscillating worldline generates infinite generations of matter and antimatter.

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I. INTRODUCTION

The concept that antiparticles can be mathematically described as ordinary particles moving backward in time[1, 2] is a cornerstone of Quantum Electrodynamics (QED). First formalized by Stueckelberg and later popularized by the Feynman-Stueckelberg interpretation, this geometric insight relies on the CPT Theorem, which establishes an equivalence between charge conjugation (C), parity inversion (P), and time reversal (T). However, in standard formulations, this “backward time” travel is treated as a formalistic bookkeeping tool rather than a literal kinematic reality.

Standard Special Relativity dictates that massive particles cannot accelerate to the kinematic null boundary (c), due to the divergence of the Lorentz factor ($\gamma \rightarrow \infty$). Consequently, the “One-Electron Universe” hypothesis proposed by John Wheeler[3], that all electrons and positrons are a single entity weaving back and forth through time, remained a philosophical curiosity rather than a dynamical physical model.

Simultaneously, General Relativity (GR)[4] faces a crisis at the gravitational null boundary. The prediction of spacelike singularities ($r = 0$) beyond the Schwarzschild event horizon ($r = r_s$)[5] indicates a breakdown of the theory. In this paper, we propose a formal unification of these two limits. We hypothesize that the kinematic divergence at c , the coordinate divergence at the horizon r_s , and the ultimate prediction of a central singularity at $r = 0$ are all artifacts of incomplete topology. By treating the invariant null boundary ($ds^2 \rightarrow 0$) as a discrete topological crossing across a bipartite manifold, we

construct a model where elementary particles undergo continuous evolution across conjugate phase space sectors. This model resolves the infinite energy paradox and eliminates gravitational singularities by reinterpreting these terminal boundaries as traversable topological seams governed by symplectic involution.

Recently, the geometric viability of horizon-bounded CPT-symmetric spacetimes has gained significant traction. Tzanavaris, Boyle, and Turok[6] have demonstrated that Einstein’s equations admit “Black Mirror” solutions, where the black hole interior is entirely replaced by a CPT-inverted exterior glued at the horizon. While their work establishes the static metric geometry of such bipartite universes, the underlying kinematic mechanism; specifically how a particle dynamically traverses this boundary without violating Hamiltonian conservation, remains an open question. The geometric framework presented in this paper, which we term **Temporal Physics**, provides this dynamical engine via Symplectic Geometry.

A. Geometric Derivation: Phase Space Inversion

The physical justification for this oscillation is derived from the Lorentz transformation properties of the phase space under a π rotation (CPT inversion) of the interaction vertex. We present this derivation in three stages: the spatial analogy, the geometric frame inversion, and the resolution of causal ordering.

1. The Spatial Oscillation Analogy

To visualize the mechanism, consider the spatial oscillation of a particle in a potential well (FIG. 1).

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1. **Forward Motion:** The particle moves with velocity $+v$ (Analogous to Matter).
2. **Turning Point:** At x_{max} , the velocity vanishes ($v = 0$) and the trajectory rotates in phase space.
3. **Reverse Motion:** The particle reflects and moves with velocity $-v$ (Analogous to Antimatter).

Classically, this is a continuous motion on the phase space manifold. In our framework, the null boundary ($ds^2 \rightarrow 0$) acts as a **temporal turning point**. Mathematically, this corresponds to a **phase space reflection** where the causal flow of the proper frame undergoes a discrete inversion.

- A “Pair Creation” event is physically a temporal turning point where the particle reverses direction from retrograde (past) to prograde (future).
- A “Pair Annihilation” event is a temporal turning point where the particle reverses from prograde to retrograde.

Just as a spatial reflection reverses the spatial velocity $v \rightarrow -v$, a phase-space reflection at the **null topological seam** Σ reverses the Hamiltonian momentum flow relative to coordinate time.

2. The Symplectic Involution and Horizon Equivalence

The mechanism driving this temporal oscillation is a topological phase-space inversion. As illustrated in FIG. 2, we define the particle’s proper frame coordinates (x', ct') relative to the observer’s world frame (x, ct) , where the temporal coordinate t' corresponds to the proper time parameter τ evolving along the worldline.

1. **Phase 0 (Electron, Orthochronous):** The proper time axis $+ct'$ lies within the future light cone. The spatial axis $+x'$ lies outside (Spacelike).
2. **Phase 1 (Positron, Antichronous):** Upon crossing the **topological phase boundary**, the axes undergo a symplectic involution. The proper time axis $-ct'$ now points into the past light cone, and the spatial axis $-x'$ undergoes parity inversion, maintaining Lorentz orthogonality.

The Event Horizon Equivalence: In standard Schwarzschild geometry, the event horizon ($r = r_s$) is traditionally treated as a boundary where the temporal and radial coordinates mathematically degenerate and interchange. In Temporal Physics, we reject this unphysical coordinate breakdown. Instead, we posit that the **topological phase boundaries** of the particle’s kinematic trajectory ($v \rightarrow c$) are physically isomorphic to the gravitational event horizon. Thus, the annihilation point is not a termination of existence, nor an entrance to a

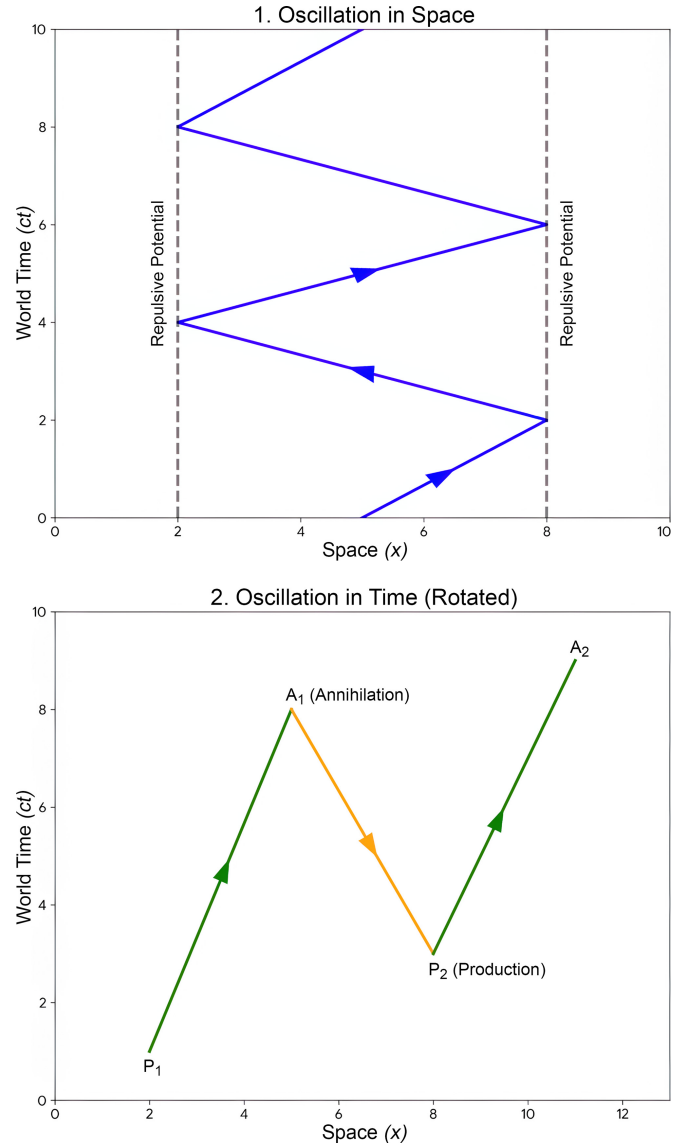


FIG. 1: **The Feynman Rotation Analogy.** **Top:** An electron oscillating in space between repulsive barriers. **Bottom:** Rotating the diagram by 90 degrees conceptually transforms the spatial trajectory into a temporal oscillation, manifesting as electron-positron pairs in the world frame.

pathological interior, but a horizon crossing where the particle smoothly enters a conjugate phase space with an inverted causal flow.

3. Topological Mechanism: Dimensional Rotation

The axis inversion described above is not an arbitrary coordinate choice but a consequence of rotation through a higher-dimensional embedding. Geometrically, a continuous rotation between disconnected parity sectors (like turning a left hand into a right hand) requires an extra dimension. A rotation in an n -dimensional Euclidean space

occurs around an $(n - 2)$ -dimensional axis. However, a discrete parity inversion in \mathbb{R}^n is topologically equivalent to a continuous rotation in the embedding space \mathbb{R}^{n+1} (or \mathbb{C}^n).

In our framework, the complexified tangent bundle provides this embedding. **This aligns with the structure of the Complex Lorentz Group $SO(3, 1; \mathbb{C})$ [8], which, unlike the real group, is connected.** The transition at the null boundary constitutes a continuous π rotation through $SO(3, 1; \mathbb{C})$. While this rotation occurs continuously in the complexified embedding space, its projection back onto the real observable phase space manifests as a discrete **Symplectic Involution** ($\Omega \rightarrow -\Omega$). Thus, the particle smoothly rotates out of the orthochronous sector and re-enters the antichronous sector, flipping the causal flow of Hamilton's equations. The ‘‘Positron’’ is the result of the electron rotating out of the standard subluminal **sector** and re-entering with inverted chirality.

4. Frame-Dependent Causal and Spatial Ordering

This axis flip resolves the apparent observational discrepancies between the two reference frames regarding both the sequence of events and the direction of motion.

1. Temporal Ordering (Causality): In the world frame (Observer), the Pair Production event P_2 occurs at an earlier world-time ct than the Annihilation event A_1 . This leads to the standard QED interpretation: the vacuum produces a pair at P_2 *before* the original electron is annihilated at A_1 . However, in the particle's proper frame, the sequence is monotonic. As shown in FIG. 2, in Phase 1 the transition from $A_1 \rightarrow P_2$ is governed by a symplectic involution where the proper time axis $-ct'$ points into the past light cone (antichronous orientation). Consequently, the Minkowski projection of the ‘‘earlier’’ world event P_2 lands further along this inverted proper time axis than A_1 . Thus, the ordering is frame-dependent:

- **World Frame:** $t(P_2) < t(A_1)$ (Manifests as Positron).
- **Proper Frame:** $\tau(A_1) < \tau(P_2)$ (Manifests as Monotonic Proper Aging).

2. Spatial Ordering (Parity): A similar inversion occurs in the spatial coordinates. In the proper frame (Phase 1), the projection of P_2 lies at a lower x' value than A_1 (see FIG. 2, Bottom). Since the particle evolves from $A_1 \rightarrow P_2$, this yields a negative displacement $\Delta x' < 0$. This matches the observer's view of a positron moving in the negative spatial direction ($P_2 \rightarrow A_1$).

This confirms that the ‘‘Positron’’ is simply the causal interpretation required by an observer to account for a single electron moving continuously through a **CPT-inverted topological sector**.

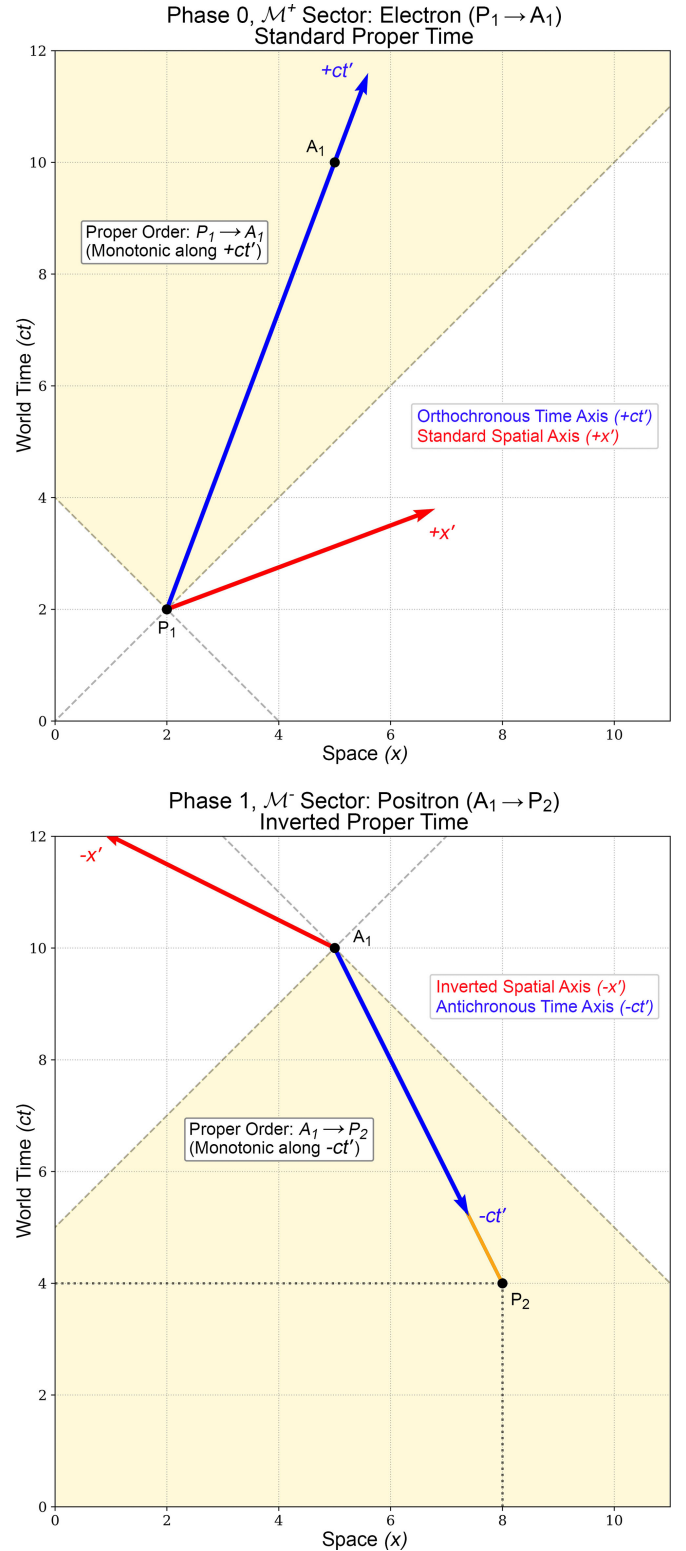


FIG. 2: The Topological Axis Flip via Symplectic Involution. **Top:** Standard proper frame axes for orthochronous evolution in the \mathcal{M}^+ sector (Phase 0). The proper time axis $+ct'$ points into the future lightcone. **Bottom:** Upon crossing the null boundary, the proper coordinate basis undergoes a Symplectic Involution ($\Omega \rightarrow -\Omega$). This is geometrically equivalent to a 180° inversion of the observer's positron frame. The proper time axis $-ct'$ now points into the past lightcone (antichronous). Because the axis itself points downward in world time, standard Minkowski projections confirm that the evolution from $A_1 \rightarrow P_2$ yields a strictly positive, monotonically increasing proper time ($\tau(P_2) > \tau(A_1)$).

II. THE SYMPLECTIC PHASE MODEL

To formalize the evolution of a particle traversing multiple **topological sectors** without violating the invariant properties of the background metric $g_{\mu\nu}$, we elevate the geometry from the base spacetime manifold \mathcal{M} to the Phase Space (Cotangent Bundle) $\mathcal{T}^*\mathcal{M}$.

A. The Bipartite Manifold Hypothesis

We postulate that the physical universe is a bipartite manifold constructed by gluing two oriented spacetime sheets along a universal topological phase boundary Σ . This boundary is rigorously defined as the limit where the particle's invariant interval becomes null:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \rightarrow 0 \quad (1)$$

Because the invariant interval vanishes at this limit ($d\tau \rightarrow 0$), the null seam Σ possesses strictly zero proper thickness along the particle's worldline, meaning its traversal requires zero proper time. This geometric constraint mathematically necessitates that the phase-space transition manifests as a discrete, instantaneous symplectic involution rather than a continuous kinematic deformation.

This single geometric condition unifies Special and General Relativity. In flat Minkowski space, $ds^2 \rightarrow 0$ corresponds to the kinematic limit $v \rightarrow c$. In Schwarzschild spacetime, $ds^2 \rightarrow 0$ corresponds to the geometric limit $r \rightarrow r_s$. Thus, the speed of light and the event horizon are simply two manifestations of the exact same **Null Gluing Surface** Σ . The bipartite structure is given by:

$$\mathcal{M}_{global} = \mathcal{M}^+ \cup_{\Sigma} \mathcal{M}^- \quad (2)$$

where \mathcal{M}^+ is the standard orthochronous sector (Matter) and \mathcal{M}^- is the antichronous CPT-inverted sector (Antimatter).

To map a particle's continuous proper evolution across these sectors onto physical observables, we define the discrete **Topological Phase Index** $n(\tau) \in \mathbb{Z}^{\geq 0}$. The index n increments by exactly +1 each time the particle's trajectory intersects the null gluing seam Σ ($ds^2 \rightarrow 0$):

- **Phase 0** ($n = 0$): Traverses \mathcal{M}^+ (Matter, Generation I).
- **Phase 1** ($n = 1$): Traverses \mathcal{M}^- (Antimatter, Generation I).
- **Phase 2** ($n = 2$): Traverses \mathcal{M}^+ (Matter, Generation II).

B. Symplectic Involution

The dynamics of a particle in GR are governed by the super-Hamiltonian $H = \frac{1}{2}g^{\mu\nu}p_\mu p_\nu$ and the canonical symplectic two-form $\Omega = dx^\mu \wedge dp_\mu$ on $\mathcal{T}^*\mathcal{M}$. The

standard Hamiltonian vector field X_H dictating the particle's flow is defined by $i_{X_H}\Omega = dH$.

Rather than altering the Levi-Civita connection via an ad hoc metric sign flip, Temporal Physics asserts that the crossing of the topological phase boundary (Σ) triggers a **Symplectic Involution**. The gluing map $\Phi : \mathcal{T}^*\mathcal{M}^+ \rightarrow \mathcal{T}^*\mathcal{M}^-$ acts as a geometric CPT inversion on the phase space structure itself.

We define the **Phase-Dependent Symplectic Form** $\Omega_{(n)}$ via the Temporal Parity Operator $P_t(n) = (-1)^n$:

$$\Omega_{(n)} = P_t(n)\Omega = (-1)^n dx^\mu \wedge dp_\mu \quad (3)$$

- **Even n (Matter):** $\Omega_{(0)} = \Omega$. The standard symplectic structure is preserved.
- **Odd n (Antimatter):** $\Omega_{(1)} = -\Omega$. The phase space structure is CPT-inverted.

C. Modified Hamiltonian Dynamics

In the inverted sector ($n = 1$), the background metric $g_{\mu\nu}$ and the Hamiltonian H remain strictly invariant. However, the phase-dependent Hamiltonian vector field $X_{H(n)}$ is now formally defined by:

$$i_{X_{H(n)}}\Omega_{(n)} = dH \quad (4)$$

Substituting $\Omega_{(n)} = P_t(n)\Omega$ and multiplying both sides by $P_t(n)$ (since $P_t(n)^2 = 1$ for all n) yields the exact algebraic inversion:

$$i_{X_{H(n)}}\Omega = P_t(n)dH \implies X_{H(n)} = P_t(n)X_H \quad (5)$$

In local Darboux coordinates (x^μ, p_μ) , Hamilton's equations for the proper trajectory thus gain a rigorous, phase-dependent derivation:

$$\frac{dx^\mu}{d\tau} = P_t(n)\frac{\partial H}{\partial p_\mu}, \quad \frac{dp_\mu}{d\tau} = -P_t(n)\frac{\partial H}{\partial x^\mu} \quad (6)$$

For $n = 1$, the negative sign effectively reverses the flow of proper time relative to the Hamiltonian gradient. As we will show, this rigorously produces effective repulsive gravity (White Hole behavior) without requiring a pathological modification to Einstein's field equations.

III. CAUSALITY AND OBSERVATIONAL MAPPING

To reconcile the antichronous proper dynamics of the \mathcal{M}^- sector with the causal orthochronous observations of a laboratory frame, we apply a specific causal filter.

A. The CPT Observational Mapping

Due to macroscopic causality constraints, an observer restricted to the \mathcal{M}^+ manifold cannot measure a trajectory in the \mathcal{M}^- sector directly. Furthermore, a particle moving *backward in time* is physically indistinguishable from an antiparticle moving *forward in time* with inverted spatial momentum [1, 2]. The observational mapping rule is defined as:

An electron traversing the inverted symplectic sector (\mathcal{M}^-) is mapped onto the observer's \mathcal{M}^+ manifold as a positron moving forward in time in the opposite spatial direction.

B. The Disappearance Illusion (Zig-Zag Topology)

This continuous traversal creates the illusion of distinct particles interacting.

1. **Proper Reality:** The particle accelerates, crosses the **null topological seam** Σ , and smoothly enters the \mathcal{M}^- phase space.
2. **Observer Reality:** We observe an electron (Phase 0) and a positron (Phase 1) converging at the null boundary from opposite spatial directions.
3. **The Event:** They collide and annihilate.

Thus, “Annihilation” is the observational artifact of a single particle undergoing a **symplectic involution** across the null seam Σ (as modeled kinematically in FIG. 3 and topologically in FIG. 4). The “Positron” is simply the electron’s future self returning to the boundary.

IV. GRAVITATIONAL DYNAMICS: THE CPT INVERSION

We now apply this rigorous symplectic framework to the Schwarzschild geometry. The Event Horizon (r_s) is identified as the **topological seam** Σ where the conjugate manifolds \mathcal{M}^+ and \mathcal{M}^- are glued.

A. The Bipartite Horizon

Standard models assume particles crossing the horizon enter a spatial interior ($r < r_s$). Recently, Tzanavaris et al.[6] have shown that setting CPT-symmetric boundary conditions transforms the black hole into a “Black Mirror,” a geometry which cleanly removes the curvature singularity in favor of a mirror-image exterior. This metric geometry precisely parallels our bipartite manifold. However, whereas standard analyses rely on extending Eddington-Finkelstein coordinates across a degenerate boundary, Temporal Physics resolves the dynamics

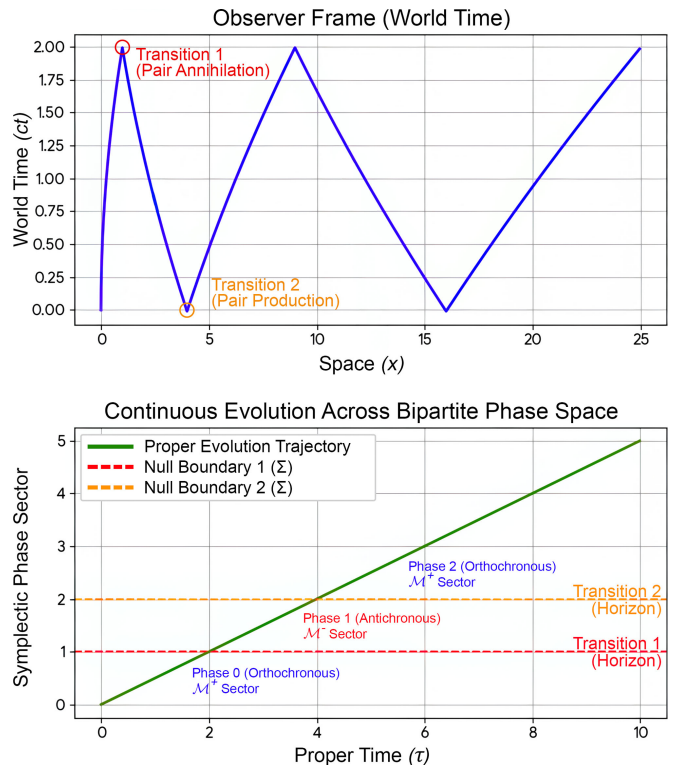


FIG. 3: **The Causal Mapping.** **Top (World Frame):** The observer, restricted to the \mathcal{M}^+ manifold, perceives discrete annihilation and production events involving standard particles and antiparticles. **Bottom (Proper Frame):** The particle traces a single, continuous worldline across the bipartite phase space. Upon crossing the null topological seam Σ , the particle undergoes a symplectic involution ($\Omega \rightarrow -\Omega$) and enters the antichronous \mathcal{M}^- sector. This continuous evolution through the conjugate phase space is mathematically projected onto the observer’s reality as a positron.

entirely within the phase space. We posit that crossing r_s triggers the symplectic involution derived in Section II.

- **Phase 0 (Approach):** The particle traverses \mathcal{M}^+ .
- **Phase 1 (Recession):** The particle traverses \mathcal{M}^- .

B. Repulsive Gravity via Symplectic Involution

In the proper frame, the electron accelerates toward the horizon. Upon reaching r_s , the particle transitions into Phase 1 ($n = 1$). Crucially, the metric $g_{\mu\nu}$ and the super-Hamiltonian $H = \frac{1}{2}g^{\mu\nu}p_\mu p_\nu$ remain invariant. However, the **symplectic involution** ($\Omega \rightarrow -\Omega$) reverses the momentum evolution dictated by Hamilton’s equations.

For a particle restricted to radial motion, the super-Hamiltonian in Schwarzschild coordinates is $H =$

World Observer Frame: The Disappearance Illusion
(Observer is constrained to \mathcal{M}^+ Exterior)

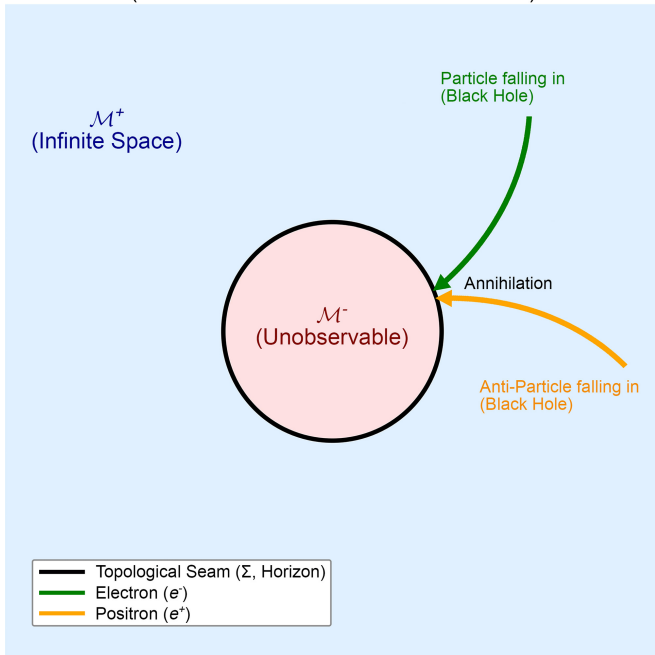


FIG. 4: **World Frame: The Disappearance Illusion.** To an observer restricted to the orthochronous \mathcal{M}^+ exterior, the topological transition manifests as a terminal annihilation event. The observer perceives both an infalling electron and an infalling positron converging at the null seam Σ . Because the observer's metric signature prevents observation of the conjugate \mathcal{M}^- sector, the interior of the boundary appears to be an unobservable, enclosed region.

$\frac{1}{2}(-f(r)^{-1}p_t^2 + f(r)p_r^2)$, where $f(r) = 1 - r_s/r$. In the standard orthochronous sector ($n = 0$), the radial momentum changes according to the negative gradient of the Hamiltonian:

$$\frac{dp_r}{d\tau} = -\frac{\partial H}{\partial r} = -\frac{r_s}{2r^2} \left(\frac{p_t^2}{f(r)^2} + p_r^2 \right) \quad (\text{Attractive}) \quad (7)$$

Because the right-hand side is strictly negative, this yields the standard, unavoidable inward acceleration of classical gravity. However, upon crossing the null seam (Σ) into the antichronous sector ($n = 1$), the symplectic involution $P_t(1) = -1$ modifies the radial momentum evolution via Eq. (6):

$$\frac{dp_r}{d\tau} = +\frac{\partial H}{\partial r} = +\frac{r_s}{2r^2} \left(\frac{p_t^2}{f(r)^2} + p_r^2 \right) \quad (\text{Repulsive}) \quad (8)$$

Because the sign of the Hamiltonian vector field has inverted relative to the particle's proper momentum, the right-hand side becomes strictly positive. The attractive potential well dynamically transforms into a repulsive barrier. The particle does not cross into a geometric "interior"; instead, it continues its trajectory in the conjugate exterior region, driven outward by exact **Relativistic Repulsive Gravity**.

Thus, the electron perceives itself accelerating *away* from the horizon, traversing a CPT-inverted phase space where the central mass acts as a gravitational repeller (White Hole).

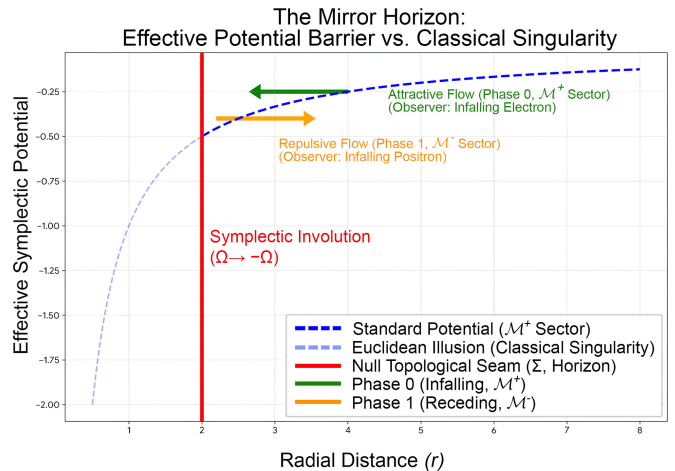


FIG. 5: **The Mirror Horizon: Hamiltonian Flow vs. Classical Singularity.** The diagram contrasts the classical interior singularity (dotted line) with the bipartite phase space boundary (solid vertical line). At the event horizon ($r = r_s$), the background Schwarzschild metric and potential remain invariant, but the symplectic involution ($\Omega \rightarrow -\Omega$) reverses the canonical momentum flow via Hamilton's equations. The particle transitions from an attractive gravitational flow in the \mathcal{M}^+ sector (Phase 0) to an effectively repulsive flow in the conjugate \mathcal{M}^- sector (Phase 1), yielding a unitary geometric bounce.

C. No Interior Region

Consequently, the geometric region $r < r_s$ is never realized in the particle's proper frame. The apparent black hole is topologically isomorphic to a non-bounding seam (Σ) rather than a volume. To the observer, the backward-in-time recession of the particle (moving away from the horizon) is mapped identically as a positron falling *into* the horizon.

D. Topological Seams vs. Euclidean Boundaries

A profound consequence of this bipartite phase space is the resolution of the Euclidean boundary illusion. To a world-frame observer, the event horizon at $r = r_s$ appears as a closed, finite S^2 sphere. By classical Euclidean intuition (the Jordan-Brouwer Separation Theorem), a closed surface must enclose a bounded interior volume. Consequently, standard GR assumes the existence of the $r < r_s$ region terminating at a central singularity.

However, in Temporal Physics, the horizon sphere does not bound a finite volume. Because the particle crosses Σ and undergoes a symplectic involution, it emerges into

the unbounded exterior of the CPT-inverted manifold \mathcal{M}^- . Topologically, the horizon is not the boundary of a 3-dimensional ball (∂B^3); it is a non-bounding topological seam connecting two infinite domains (a traversable Einstein-Rosen throat without an interior).

Thus, the ‘‘Black Hole’’ interior is an observational artifact. The world observer witnesses a closed geometric shape that is, topologically, not closed at all. It is an open window to the conjugate phase space, traversed continuously by the proper particle. This inside-out geometric transformation is visually detailed in FIG. 6.

V. THE RELATIVISTIC HORIZON IDENTITY

A central prediction of this framework is that the designation of a topological seam (Σ) as an absorbing ‘‘Black Hole’’ or an emitting ‘‘White Hole’’ is strictly relative to the observer’s symplectic phase sector.

A. The Phase-Dependent Horizon

In proper time, the electron traverses a continuous, oscillatory trajectory through the bipartite phase space:

1. **Approach (Phase 0):** The particle accelerates toward the boundary. It perceives an attractive Hamiltonian flow (Black Hole).
2. **Crossing (First Boundary):** The particle crosses the topological seam. The $\Omega \rightarrow -\Omega$ symplectic involution occurs.
3. **Recession (Phase 1):** The particle traverses the conjugate antichronous sector ($n = 1$). It perceives the object behind it as a repulsive flow (White Hole) propelling it forward.
4. **Next Approach (Transition to Phase 2):** The particle is propelled by the repulsive gradient of the trailing boundary and attracted by the potential well of the subsequent null boundary.

a. The Equivalence of Kinematic and Gravitational Boundaries This topological involution is not limited to Schwarzschild black holes. By the Local Equivalence Principle, a particle undergoing constant proper acceleration α approaches a Rindler Horizon at a distance c^2/α in its instantaneous rest frame. While true gravitational fields (Schwarzschild) are globally distinguished from accelerated frames (Rindler) by the presence of non-zero Riemann curvature ($R^\mu_{\nu\rho\sigma} \neq 0$) and tidal forces, the intersection with the null boundary is a strictly local phenomenon. As the particle’s velocity approaches the light-cone ($ds^2 \rightarrow 0$), the local geometry of the kinematic Rindler horizon becomes topologically isomorphic to the gravitational event horizon. Consequently, the symplectic boundary crossing derived here applies equally to high-energy particles in flat spacetime: the

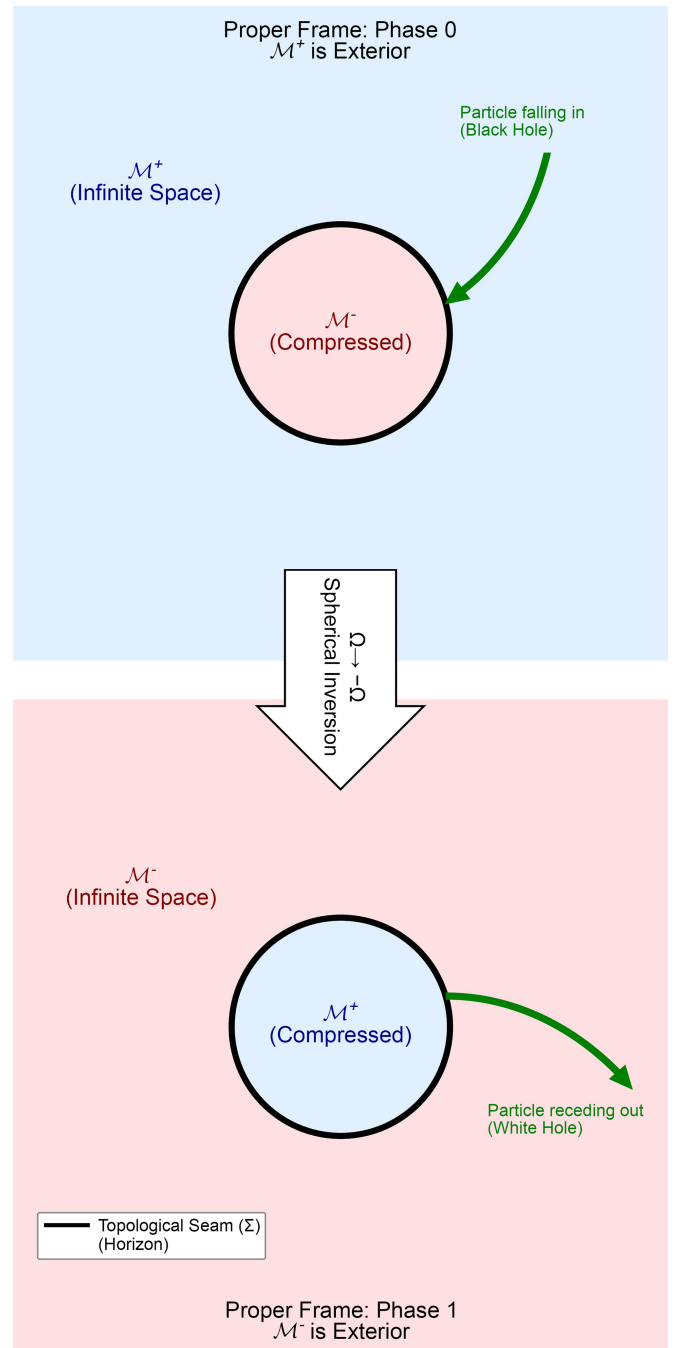


FIG. 6: **Proper Frame: Topological Spherical Inversion.** In the proper frame, a single continuous particle bridges the bipartite manifold. **Left:** The particle approaches the boundary from the infinite \mathcal{M}^+ exterior (Phase 0). **Right:** Upon crossing Σ , the symplectic involution ($\Omega \rightarrow -\Omega$) triggers a spherical inversion. The topology turns inside-out: the \mathcal{M}^- sector becomes the unbounded exterior, and the particle continuously recedes outward (Phase 1). The infalling positron observed in the world frame (FIG. 4) is strictly the kinematic projection of this receding trajectory.

“speed of light” barrier acts identically to the event horizon. Because the symplectic involution ($\Omega \rightarrow -\Omega$) is triggered locally at the null seam Σ , the presence or absence of global spacetime curvature is irrelevant to the phase space bounce itself.

B. Observer’s Perspective: Infinite Generations

To the world-time observer, these continuous trans-boundary oscillations appear as discrete generations of particles:

- **Transition 1** ($n = 0 \rightarrow 1$): Pair Annihilation at a Black Hole or Kinematic Limit.
- **Transition 2** ($n = 1 \rightarrow 2$): Pair Production from a White Hole or Kinematic Limit.
- **Transition 3** ($n = 2 \rightarrow 3$): Pair Annihilation at a Black Hole or Kinematic Limit.

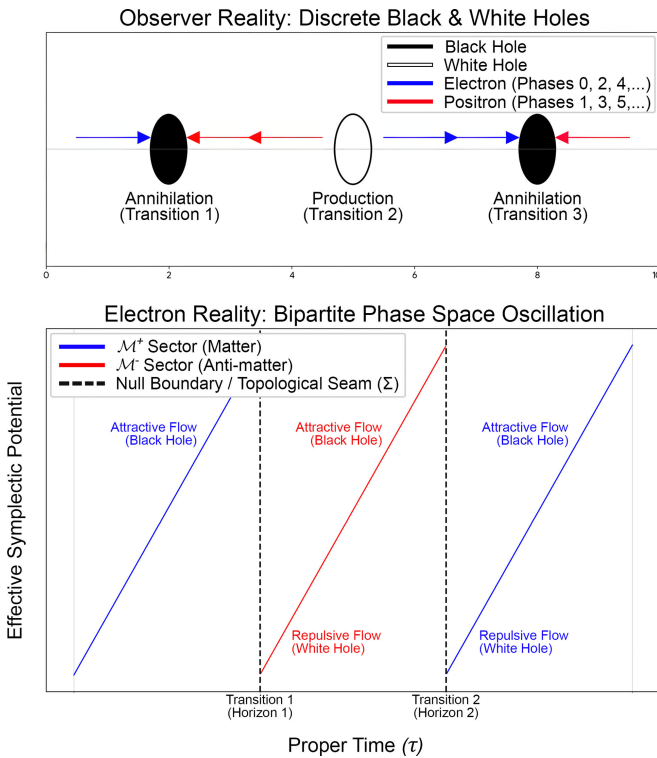


FIG. 7: **The Relativistic Horizon Identity.** **Top (Observer Frame):** To an observer constrained to the \mathcal{M}^+ manifold, the interaction appears as discrete, discontinuous events: an electron and a positron falling/annihilation into a Black Hole (Horizon) and an electron and a positron emerging/production from a White Hole (Horizon). **Bottom (Proper Frame):** The particle experiences a single, continuous worldline. The horizon acts as a topological gluing seam (Σ) where the symplectic form undergoes involution ($\Omega \rightarrow -\Omega$). This continuous oscillation between attractive (\mathcal{M}^+) and repulsive (\mathcal{M}^-) Hamiltonian flows generates the infinite generations of matter and antimatter.

VI. CONSERVATION LAWS AND STABILITY

A. Hamiltonian Conservation

A common objection to trans-horizon or trans-boundary trajectories is the presumed divergence of kinetic energy at the null limits. We resolve this via the exact conservation of the super-Hamiltonian. Because our framework does not alter the underlying metric tensor $g_{\mu\nu}$, the relativistic dispersion relation $E^2 = p^2 c^2 + m^2 c^4$ is globally preserved across the transition.

The limit c acts not as an asymptotic energy barrier, but as a **topological horizon**. The apparent divergence of the Lorentz factor is simply the coordinate singularity of mapping the Hamiltonian vector flow at the gluing boundary. Because the transition constitutes an involution of the symplectic two-form rather than an inversion of the metric, the interaction term cleanly transitions from extracting energy from the gravitational field (attraction) to restoring energy to the field (repulsion). The total Hamiltonian \mathcal{H} remains strictly conserved on the constraint surface.

Crucially, this resolves the standard relativistic paradox that infinite energy is required to reach the null boundary c . Just as the $t \rightarrow \infty$ divergence at the Schwarzschild horizon is a coordinate singularity rather than a physical trap, the $\gamma \rightarrow \infty$ divergence at $v \rightarrow c$ is an observational artifact of projecting a bipartite phase space onto a single Euclidean coordinate chart. The particle does not require infinite energy to breach the boundary because it does not traverse *through* c into a superluminal regime; rather, upon reaching the null seam ($ds^2 \rightarrow 0$), the symplectic involution ($\Omega \rightarrow -\Omega$) triggers a geometric reflection. In Quantum Electrodynamics, this topological reflection is experimentally observed not as a divergence of infinite kinetic energy, but as the finite-energy vertex of pair annihilation/production ($E \approx 2m_0 c^2$).

B. The Phase-Dependent Potential Structure

Physically, the evolution describes a continuous descent through a **multi-sheeted potential manifold**. As the particle transitions from Phase n to Phase $n+1$, it enters the conjugate phase space where the effective Hamiltonian gradient remains driving. The repulsive flow from the trailing “White Hole” (previous topological seam, Σ_{prev}) and the attractive flow from the approaching “Black Hole” (next topological seam, Σ_{next}) reinforce each other. In the conjugate phase space, the Hamiltonian gradients from these respective boundaries sum constructively, maintaining a monotonic increase in proper momentum:

$$\left. \frac{dp_r}{d\tau} \right|_{net} = \left. \frac{\partial H}{\partial r_{prev}} \right|_{\Sigma_{prev}} + \left. \frac{\partial H}{\partial r_{next}} \right|_{\Sigma_{next}} \quad (9)$$

The electron acts as a trans-phasic oscillator, ensuring total energy conservation within the global system.

C. Vacuum Stability

A common objection to non-standard spacetime topologies is the potential for vacuum instability via Cherenkov radiation, an anomaly typically associated with spacelike (tachyonic) trajectories. However, because the symplectic involution strictly preserves the invariant interval ($ds^2 \geq 0$), the particle never enters a spacelike regime. Instead, the antichronous state ($n = 1$) is **topologically projected** onto the observer's \mathcal{M}^+ manifold as a subluminal positron moving forward in time.

Furthermore, because the involution reverses the canonical momentum ($p_\mu \rightarrow -p_\mu$), the minimal coupling to the background electromagnetic field ($p_\mu - eA_\mu$) effectively yields an inverted charge ($-e$) relative to the Hamiltonian flow. Since this coupling is defined entirely on the projected orthochronous worldline ($v_{obs} < c$), the particle does not interact anomalously with the vacuum. It couples exactly as dictated by standard QED for a CPT-inverted fermion. Consequently, the vacuum remains strictly stable, and no anomalous Cherenkov radiation is generated.

VII. CONCLUSION

By reinterpreting the Feynman-Stueckelberg mechanism through the lens of Symplectic Geometry, we have constructed a unified kinematic and gravitational framework where:

1. **Matter and Antimatter** are dynamically equivalent trajectories on conjugate phase space sectors of a bipartite manifold.
2. **The Kinematic and Gravitational Null Boundaries** (c and r_s) are topologically identical gluing surfaces (Σ) defined by $ds^2 \rightarrow 0$, rather than asymptotic limits.
3. **Black Holes** act as Symplectic Involution Boundaries, generating effective repulsive gravity without requiring physical singularities or modified Einstein equations.

This framework—**Temporal Physics**—resolves the geodesic incompleteness of General Relativity, offers a

mathematically rigorous geometric solution to the Black Hole Information Paradox, and provides a dynamical kinematic mechanism for the “Black Mirror” topologies recently identified in CPT-symmetric spacetimes[6].

Most profoundly, it provides a rigorous geometric foundation for John Wheeler’s “One-Electron Universe.” By demonstrating that a single, continuous oscillating worldline can traverse the null boundary via symplectic involution, it suggests that the universe is populated not by an ensemble of discrete, disconnected particles, but by the infinite generations of a single worldline oscillating through a non-orientable phase space.

Ultimately, this paradigm shift demonstrates that the most intractable infinities in modern physics do not necessitate the abandonment of classical field equations, but rather demand a rigorous geometric understanding of the topological seams that bound them.

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