Multivector physics

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In 4-dim, multivectors can derive four different-dimensional Maxwell equations. One of them is original equation. From four equations, four energy densities and four forces are derived. The coordinates are transformed by the 2-basis rotor and booster. The commutation relation of angular momentum is generated by infinitesimal rotor, and the uncertainty principle, and photons in photoelectric effect are generated by infinitesimal Lorentz booster. These can be explained by the cosmic background vibration. The magnitude of the vibration is Planck's constant.

Keywords: multivector, duality, Maxwell's equations, Planck's constant

1 Introduction

The dot product of multivectors is defined , then the curl and divergence, consistent with conventional concepts, are also easily defined. Then, it is easily proven that Maxwell's equations are not experimental laws, but rather properties of multivectors and the duality of nature.
Quantization, the commutation relation of angular momentum, the uncertainty principle, and Planck's constant are also easily and accurately explained using multivectors. Therefore, it is difficult to deny.

2 Curl and divergence in multibasis

Curl and divergence are not vectors and scalar but multivectors. In addition, it can also be defined as the bases of any curvilinear coordinate system.

2.1. *Multivector and dot product*

If $v_1, v_2, \ldots, v_r$ and $w_1, \ldots, w_s$ are 1-vectors, then $(v_1 \wedge v_2)$ is a 2-vector and $(v_1 \wedge v_2 \wedge v_3)$ is a 3-vector. Multivector is anti-commutative in order.

$$(v_1 \wedge v_2) = -(v_2 \wedge v_1), \qquad (v_1 \wedge v_1) = 0$$

$$(\ldots \wedge v_{i-1} \wedge v_i \wedge \ldots) = -(\ldots \wedge v_i \wedge v_{i-1} \wedge \ldots)$$

To satisfy the anti-commutativity, if there are no two or more j, k satisfying $(v_i \cdot w_j) \neq 0,\ (w_i \cdot v_k) \neq 0$ for a given i, Then the dot product of multivectors can be defined.

$$(v_1 \wedge \ldots \wedge v_r) \cdot (w_1 \wedge \ldots \wedge w_s) = (w_1 \wedge \ldots \wedge w_s) \cdot (v_1 \wedge \ldots \wedge v_r)$$
$$= (v_2 \wedge \ldots \wedge v_r) \cdot \{v_1 \cdot (w_1 \wedge \ldots \wedge w_s)\} = (v_3 \wedge \ldots \wedge v_r) \cdot [v_2 \cdot \{v_1 \cdot (w_1 \wedge \ldots \wedge w_s)\}] = \cdots$$
$$= (w_2 \wedge \ldots \wedge w_s) \cdot \{w_1 \cdot (v_1 \wedge \ldots \wedge v_r)\} = (w_3 \wedge \ldots \wedge w_s) \cdot [w_2 \cdot \{w_1 \cdot (v_1 \wedge \ldots \wedge v_r)\}] = \cdots$$

$$w_1 \cdot (v_1 \wedge v_2 \wedge \ldots \wedge v_r) = (-1)^{i-1}(w_1 \cdot v_i)(v_1 \wedge \ldots \wedge v_{i-1} \wedge v_{i+1} \ldots \wedge v_r)$$

$$(w_1 \wedge w_2) \cdot (v_1 \wedge v_2 \wedge \ldots \wedge v_r)$$
$$= (-1)^{i+j-1}\{(w_1 \cdot v_i)(w_2 \cdot v_j) - (w_1 \cdot v_j)(w_2 \cdot v_i)\}(v_1 \wedge \ldots \wedge v_{i-1} \wedge v_{i+1} \wedge \ldots \wedge v_{j-1} \wedge v_{j+1} \wedge \ldots \wedge v_r)$$
$$= (-1)^{i+j}\{(w_2 \cdot v_i)(w_1 \cdot v_j) - (w_2 \cdot v_j)(w_1 \cdot v_i)\}(v_1 \wedge \ldots \wedge v_{i-1} \wedge v_{i+1} \wedge \ldots \wedge v_{j-1} \wedge v_{j+1} \wedge \ldots \wedge v_r)$$
$$= -(w_2 \wedge w_1) \cdot (v_1 \wedge v_2 \wedge \ldots \wedge v_r)$$

2.2. *Basis and reciprocal basis*

In n-dim curvilinear coordinates system, A point is $X(p^1, p^2, \ldots, p^n)$, which means the vector from the origin O to X. The basis u_i at X is defined as follows.

$$u_i = \frac{\partial X}{\partial p^i}$$

When $u^i \cdot u_j = \delta^i_j$, $u^1, u^2 \ldots u^n$ are reciprocal basis at X. Assuming matrices g^{ij} and g_{jk},

$$u^i = g^{ij} u_j, \qquad u^i \cdot u^k = g^{ij} u_j \cdot u^k = g^{ik}, \qquad u_i = g_{ij} u^j, \qquad u_i \cdot u_k = g_{ij} u^j \cdot u_k = g_{ik}$$

g_{jk} and g^{ij} are inverse matrices relationships.

$$u^i \cdot u_k = g^{ij} u_j \cdot u_k = g^{ij} g_{jk} = \delta^i_k$$

Therefore, find the basis at point X, find $g_{ij} = u_i \cdot u_j$, find the inverse matrix g^{ij}, and find the u^i

2.3. ***Infinitesimal volume*** $[dV]_r$ ***and its boundary*** $\partial[dV]_r$

$[dV]_r$ is an r-vector. It means something like a very small r-dimensional parallelepiped. The reference vertex is X. The r edges are $u_{\sigma_1} dp^{\sigma_1}, u_{\sigma_2} dp^{\sigma_2}, \dots, u_{\sigma_r} dp^{\sigma_r}$.

$$[dV]_r = (u_{\sigma_1} \wedge u_{\sigma_2} \wedge \dots \wedge u_{\sigma_r}) dp^{\sigma_1} dp^{\sigma_2} \dots dp^{\sigma_r}, \qquad \{\sigma_1, \sigma_2, \dots, \sigma_r\} \subset \{1,2,\dots,n\}, \qquad (\sigma_1 < \dots < \sigma_r)$$

ex) $[dV]_1 = u_2 dp^2, \; [dV]_2 = (u_1 \wedge u_2) dp^1 dp^2, \; [dV]_3 = (u_1 \wedge u_3 \wedge u_4) dp^1 dp^3 dp^4$

The 2r (r-1)-vector boundaries on the surface of $[dV]_r$ is $\partial[dV]_r$.

$$\partial[dV]_r = \frac{\pm u^i}{dp^i} \cdot [dV]_r = \begin{cases} \dfrac{+u^i}{dp^i} \cdot [dV]_r & at\ P(\dots, p^i + dp^i, \dots) \\ \dfrac{-u^i}{dp^i} \cdot [dV]_r & at\ P(\dots, p^i, \dots) \end{cases}$$

ex) $[dV]_1 = u_2 dp^2$

$$\partial[dV]_1 = \frac{\pm u^i}{dp^i} \cdot [dV]_1 = \begin{cases} \dfrac{+u^2}{dp^2} \cdot u_2 dp^2 = 1 & at\ P(\dots, p^2 + dp^2, \dots) \\ \dfrac{-u^2}{dp^2} \cdot u_2 dp^2 = -1 & at\ P(\dots, p^2, \dots) \end{cases}$$

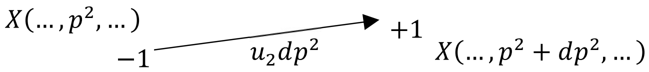

Fig.1 $\partial[dV]_1$ is ± 1. It has opposite signs to adjacent $\partial[dV]_1$

ex) $[dV]_2 = (u_1 \wedge u_2) dp^1 dp^2$

$$[dV]_2 = \frac{\pm u^i}{dp^i} \cdot [dV]_2 = \begin{cases} \dfrac{+u^1}{dp^1} \cdot (u_1 \wedge u_2) dp^1 dp^2 = +u_2 dp^2 & at\ P(\dots, p^1 + dp^1, \dots) \\ \dfrac{-u^1}{dp^1} \cdot (u_1 \wedge u_2) dp^1 dp^2 = -u_2 dp^2 & at\ P(\dots, p^1, \dots) \\ \dfrac{+u^2}{dp^2} \cdot (u_1 \wedge u_2) dp^1 dp^2 = -u_1 dp^1 & at\ P(\dots, p^2 + dp^2, \dots) \\ \dfrac{-u^2}{dp^2} \cdot (u_1 \wedge u_2) dp^1 dp^2 = +u_1 dp^1 & at\ P(\dots, p^2, \dots) \end{cases}$$

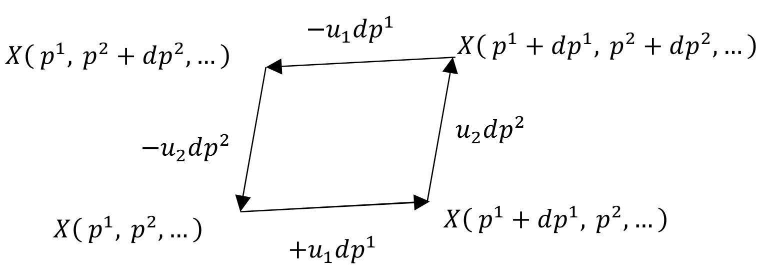

Fig.2 $\partial[dV]_2$ rotates in one direction. It rotates in the opposite direction to adjacent $\partial[dV]_2$

2.4. ***Curl***

r-vector field A in n-dim.

$$A = A_{\sigma_1 \dots \sigma_r} (u^{\sigma_1} \wedge \dots \wedge u^{\sigma_r}), \qquad (\sigma_1 < \dots < \sigma_r), \qquad \sigma_1, \dots, \sigma_r, \dots \in \{1,2, \dots n\}$$

The definition of $curl(A)$ is from the dot product of a (r-1)-vector field A and the boundary vector $\partial[dV]_r$.

$$A \cdot \partial[dV]_r = A \cdot \left(\frac{\pm u^i}{dp^i} \cdot [dV]_r\right) = \left(\frac{\pm u^i}{dp^i} \wedge A\right) \cdot [dV]_r = \left(u^i \wedge \frac{\pm A}{dp^i}\right) \cdot [dV]_r$$
$$= \left(u^i \wedge \frac{\partial A}{\partial p^i}\right) \cdot [dV]_r = curl(A) \cdot [dV]_r\,, \qquad curl(A) = \left(u^i \wedge \frac{\partial A}{\partial p^i}\right)$$

The definition itself is Stokes' theorem. This does not mean that a new vector field $curl(A)$ is created, but rather that it is a view of A from one dimension higher. A and $curl(A)$ are one entity. Also $curl(A)$ has the concept of density because $curl(A) \cdot [dV]_r$ is proportional to volume $[dV]_r$.

$$\oint A \cdot \partial[dV]_r = \oint curl(A) \cdot [dV]_r$$

In calculus, a vector field must be represented by reciprocal bases so that the bases cancel out with dot product. And differentiate only the coefficients of A to find $curl(A)$. Cartesian coordinates are actually reciprocal basis. Covariant differentiation is superfluous.

$$curl^2(A) = curl\{curl(A)\} = \left(u^j \wedge u^i \wedge \frac{\partial^2 A}{\partial p^j \partial p^i}\right) = \left(u^i \wedge u^j \wedge \frac{\partial^2 A}{\partial p^i \partial p^j}\right) = 0$$

That is, only A and $curl(A)$ are one entity.

2.5. *Comparison to conventional curl*

$A = A_r e_r + A_\theta e_\theta + A_\emptyset e_\emptyset$ in spherical coordinates.

$$u_r = \frac{\partial X}{\partial r} = e_r, \qquad u_\theta = \frac{\partial X}{\partial \theta} = r e_\theta, \qquad u_\emptyset = \frac{\partial X}{\partial \emptyset} = r \sin\theta \, e_\emptyset$$
$$u^i \cdot u_k = \delta^i_k, \qquad u^r = e_r, \qquad u^\theta = \frac{e_\theta}{r}, \qquad u^\emptyset = \frac{e_\emptyset}{r \sin\theta}$$
$$A = A_r e_r + A_\theta e_\theta + A_\emptyset e_\emptyset = A_r u^r + A_\theta r u^\theta + A_\emptyset r \sin\theta\, u^\emptyset = A_r u_r + \frac{A_\theta}{r} u_\theta + \frac{A_\emptyset}{r \sin\theta} u_\emptyset$$

$$curl(A) = \left(u^i \wedge \frac{\partial A}{\partial p^i}\right)$$

$(u^\theta \wedge u^\emptyset)$ term.

$$\left(u^\theta \wedge \frac{\partial(A_\emptyset r \sin\theta)}{\partial\theta} u^\emptyset\right) + \left(u^\emptyset \wedge \frac{\partial(A_\theta r)}{\partial\emptyset} u^\theta\right) = \left\{\frac{\partial(A_\emptyset r \sin\theta)}{\partial\theta} - \frac{\partial(A_\theta r)}{\partial\emptyset}\right\}(u^\theta \wedge u^\emptyset)$$
$$= \frac{1}{r^2 \sin\theta}\left\{\frac{\partial(A_\emptyset r \sin\theta)}{\partial\theta} - \frac{\partial(A_\theta r)}{\partial\emptyset}\right\}(e_\theta \wedge e_\emptyset)$$

Compared to the conventional results, the coefficient is the same. However, it is not the e_r term, but the $(e_\theta \wedge e_\emptyset)$ term. They are dual basis each other in 3-dim.

2.6. *Pseudoscalar and dual basis*

In n-dimension, Cartesian coordinates basis $e_1, e_2, \dots, e_n$, unit pseudoscalar I.

$$I = (e_1 \wedge \dots \wedge e_n)$$

For example

$$I = (e_1 \wedge e_2 \wedge e_3) = (e_r \wedge e_\theta \wedge e_\emptyset) = r^2 \sin\theta\,(u^r \wedge u^\theta \wedge u^\emptyset)$$

dual basis

$$e_r^d = e_r \cdot I = e_r \cdot (e_r \wedge e_\theta \wedge e_\emptyset) = (e_\theta \wedge e_\emptyset)$$

2.7. *Divergence*

$divergence(A)$ is $curl$ with respect to A^d, dual vector field of A.

$$div(A) = curl(A^d), \qquad A^d = A \cdot I$$
$$A^d = A \cdot I = \left(A_r u_r + \frac{A_\theta}{r} u_\theta + \frac{A_\emptyset}{r \sin\theta} u_\emptyset\right) \cdot r^2 \sin\theta\,(u^r \wedge u^\theta \wedge u^\emptyset)$$
$$= A_r r^2 \sin\theta\,(u^\theta \wedge u^\emptyset) - A_\theta r \sin\theta\,(u^r \wedge u^\emptyset) + A_\emptyset r (u^r \wedge u^\theta)$$
$$div(A) = curl(A^d) = \left(u^i \wedge \frac{\partial A^d}{\partial p^i}\right)$$

$$= \left\{\frac{\partial(A_r r^2 \sin\theta)}{\partial r} + \frac{\partial(A_\theta r \sin\theta)}{\partial\theta} + \frac{\partial(A_\emptyset r)}{\partial\emptyset}\right\}(u^r \wedge u^\theta \wedge u^\emptyset)$$
$$= \frac{1}{r^2 \sin\theta}\left\{\frac{\partial(A_r r^2 \sin\theta)}{\partial r} + \frac{\partial(A_\theta r \sin\theta)}{\partial\theta} + \frac{\partial(A_\emptyset r)}{\partial\emptyset}\right\}(\mathbb{e}_r \wedge \mathbb{e}_\theta \wedge \mathbb{e}_\emptyset)$$
Same coefficient, dual basis.

2.8. *Continuity equation,* $div(A) = 0$

If A is expressed in bases and I is expressed in reciprocal bases, the following equation holds.

$$div(A) \cdot [dV]_r = curl(A^d) \cdot [dV]_r = A^d \cdot \partial[dV]_r = (A \cdot I) \cdot \partial[dV]_r = (A \wedge \partial[dV]_r) \cdot I$$

$(A \wedge \partial[dV]_r) \cdot I$ is a physical quantity that moves in and out at speed A through the boundary $\partial[dV]_r$ of $[dV]_r$. $(A \wedge \partial[dV]_r)$ is the product $\partial[dV]_r$ and only the components of A perpendicular to $\partial[dV]_r$. The parallel components cancel out in the wedge product. If $div(A) = 0$ at all points, this means that a physical quantity flows without accumulation or leakage, then A is said to be continuous.

3. Maxwell's equations

The following propositions are Maxwell's equations.

1) If vector potential A exists then $curl(A)$ exists. They are one entity.
2) Then, $curl^d(A)$ and $curl\{curl^d(A)\} = J$ also exist. It's duality.
$$curl^d(A) = curl(A) \cdot I, \qquad curl\{curl^d(A)\} = div\{curl(A)\} = Laplacian(A) = J$$
This is Gauss's law and Ampere-Maxwell's law.
3) $curl^2(A) = 0$ and $curl(J) = curl^2\{curl^d(A)\} = 0$.
These are Gauss's law for magnetism, and Faraday's law, and charge conservative law.
4) After enough time, at equilibrium, A becomes continuous. $div(A) = 0$. Lorentz condition.
Then A and J formulate the standing wave equation.

Substituting a formula into an unfamiliar proposition yields familiar Maxwell equations.

$$\mathbb{e}^t \cdot \mathbb{e}_t = 1, \qquad \mathbb{e}_t \cdot \mathbb{e}_t = -1, \qquad \mathbb{e}^t = -\mathbb{e}_t$$
$$\mathbb{e}^i \cdot \mathbb{e}_i = 1, \qquad \mathbb{e}_i \cdot \mathbb{e}_i = 1, \qquad \mathbb{e}^i = \mathbb{e}_i, \qquad (i = 1,2,3)$$
$$I = (\mathbb{e}_1 \wedge \mathbb{e}_2 \wedge \mathbb{e}_3 \wedge \mathbb{e}_t) = -(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$\partial_\mu = \frac{\partial}{\partial x^\mu}, \qquad \partial_{\mu\nu} = \frac{\partial^2}{\partial x^\mu \partial x^\nu}, \qquad (\mu, \nu = 1,2,3,t), \qquad \square = \partial_{tt} - \partial_{11} - \partial_{22} - \partial_{33}$$

1) $A = A_1\mathbb{e}^1 + A_2\mathbb{e}^2 + A_3\mathbb{e}^3 - \phi\mathbb{e}^t = A_1\mathbb{e}_1 + A_2\mathbb{e}_2 + A_3\mathbb{e}_3 + \phi\mathbb{e}_t$
$$curl(A) = \mathbb{e}^\mu \wedge \partial_\mu A = B + E$$
$$= (\partial_2 A_3 - \partial_3 A_2)(\mathbb{e}^2 \wedge \mathbb{e}^3) + (\partial_3 A_1 - \partial_1 A_3)(\mathbb{e}^3 \wedge \mathbb{e}^1) + (\partial_1 A_2 - \partial_2 A_1)(\mathbb{e}^1 \wedge \mathbb{e}^2)$$
$$-(\partial_1\phi + \partial_t A_1)(\mathbb{e}^1 \wedge \mathbb{e}^t) - (\partial_2\phi + \partial_t A_2)(\mathbb{e}^2 \wedge \mathbb{e}^t) - (\partial_3\phi + \partial_t A_3)(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$= B_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + B_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + B_3(\mathbb{e}^1 \wedge \mathbb{e}^2) + E_1(\mathbb{e}^1 \wedge \mathbb{e}^t) + E_2(\mathbb{e}^2 \wedge \mathbb{e}^t) + E_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
Conventional coefficient representation.
$$B = \nabla \times A, \qquad E = -\nabla\phi - \partial_t A$$
2) ρ: charge density, J: charge current density.
$$curl^d(A) = E_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + E_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + E_3(\mathbb{e}^1 \wedge \mathbb{e}^2) - B_1(\mathbb{e}^1 \wedge \mathbb{e}^t) - B_2(\mathbb{e}^2 \wedge \mathbb{e}^t) - B_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$J = curl(curl^d(A)) = \rho(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - J_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - J_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - J_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$= (\partial_1 E_1 + \partial_2 E_2 + \partial_3 E_3)(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - (\partial_2 B_3 - \partial_3 B_2 - \partial_t E_1)(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$-(\partial_3 B_1 - \partial_1 B_3 - \partial_t E_2)(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - (\partial_1 B_2 - \partial_2 B_1 - \partial_t E_3)(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
Conventional representation. $\nabla \cdot E = \rho$, $\nabla \times B = J + \partial_t E$
3) $curl^2(A) = 0$
$$= (\partial_1 B_1 + \partial_2 B_2 + \partial_3 B_3)(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) + (\partial_2 E_3 - \partial_3 E_2 + \partial_t B_1)(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)$$

$+(\partial_3 E_1 - \partial_1 E_3 + \partial_t B_2)(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) + (\partial_1 E_2 - \partial_2 E_1 + \partial_t B_3)(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$

Conventional representation. $\nabla \cdot B = 0,\ \nabla \times E + \partial_t B = 0$

$curl(J) = 0 = (\partial_1 J_1 + \partial_2 J_2 + \partial_3 J_3 + \partial_t \rho)I$

Conventional representation. $\nabla \cdot J + \partial_t \rho = 0$

4) $A^d = -A_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - A_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - A_1(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t) + \phi(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3)$

$div(A) = curl\{A^d\} = (\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 + \partial_t \phi)I = 0$

Conventional representation. $\nabla \cdot A + \partial_t \phi = 0$

Substituting A_i, ϕ for E_i, B_j in the 2).

$$\rho = \partial_1 E_1 + \partial_2 E_2 + \partial_3 E_3 = -\partial_{11}\phi - \partial_{1t}A_1 - \partial_{22}\phi - \partial_{2t}A_2 - \partial_{33}\phi - \partial_{3t}A_3$$
$$= -\partial_{11}\phi - \partial_{22}\phi - \partial_{33}\phi + \partial_{tt}\phi - \partial_t(\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 + \partial_t \phi)$$
$$= -\partial_{11}\phi - \partial_{22}\phi - \partial_{33}\phi + \partial_{tt}\phi = \Box\phi$$
$$J_1 = \partial_2 B_3 - \partial_3 B_2 - \partial_t E_1 = \partial_{21}A_2 - \partial_{22}A_1 - \partial_{33}A_1 + \partial_{31}A_3 + \partial_{tt}A_1 + \partial_{t1}\phi$$
$$= \partial_{tt}A_1 - \partial_{11}A_1 - \partial_{22}A_1 - \partial_{33}A_1 + \partial_1(\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 + \partial_t \phi) = \Box A_1$$
$$\rho = \Box\phi, \qquad J_i = \Box A_i$$

3.1. *2-vector field*

$$A = A_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + A_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + A_3(\mathbb{e}^1 \wedge \mathbb{e}^2) - \phi_1(\mathbb{e}^1 \wedge \mathbb{e}^t) - \phi_2(\mathbb{e}^2 \wedge \mathbb{e}^t) - \phi_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$curl(A) = B_\phi(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - E_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - E_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - E_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$B_\phi = \nabla \cdot A, \qquad E = \nabla \times \phi - \partial_t A$$
$$curl^d(A) = E_1\mathbb{e}^1 + E_2\mathbb{e}^2 + E_3\mathbb{e}^3 - B_\phi\mathbb{e}^t$$
$$J = \rho_1^s(\mathbb{e}^2 \wedge \mathbb{e}^3) + \rho_2^s(\mathbb{e}^3 \wedge \mathbb{e}^1) + \rho_3^s(\mathbb{e}^1 \wedge \mathbb{e}^2) + J_1^s(\mathbb{e}^1 \wedge \mathbb{e}^t) + J_2^s(\mathbb{e}^2 \wedge \mathbb{e}^t) + J_3^s(\mathbb{e}^3 \wedge \mathbb{e}^t)$$

ρ^s: spin density, J^s: spin current density.

$$\rho^s = \nabla \times E, \qquad J^s = -\nabla B_\phi - \partial_t E$$
$$\nabla \cdot E + \partial_t B_\phi = 0, \qquad \nabla \cdot \rho^s = 0, \qquad \nabla \times J^s + \partial_t \rho^s = 0$$

if $div(A) = 0$

$$\nabla \cdot \phi = 0, \qquad \nabla \times A + \partial_t \phi = 0, \qquad \rho_i^s = \Box\phi_i, \qquad J_i^s = \Box A_i$$

3.2. *3-vector field*

$$A = -A_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - A_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - A_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t) + \phi(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3)$$
$$curl(A) = B = B_\phi(-I), \qquad B_\phi = -\nabla \cdot A - \partial_t \phi$$
$$curl^d(A) = B_\phi$$
$$J = \rho_1^q\mathbb{e}^1 + \rho_2^q\mathbb{e}^2 + \rho_3^q\mathbb{e}^3 - J^q\mathbb{e}^t$$

ρ^q: quark density, J^q: quark current density(Higgs density).

$$\rho^q = \nabla B_\phi, \qquad J^q = -\partial_t B_\phi$$
$$\nabla \times \rho^q = 0, \qquad \nabla J^q + \partial_t \rho^q = 0$$

if $div(A) = 0$

$$\nabla \times A = 0, \qquad \partial_t A + \nabla\phi = 0, \qquad \rho_i^q = \Box A_i, \qquad J^q = \Box\phi$$

3.3. *scalar field*

$$A = \phi$$
$$curl(A) = E_1\mathbb{e}^1 + E_2\mathbb{e}^2 + E_3\mathbb{e}^3 - B_\phi\mathbb{e}^t, \qquad E = \nabla\phi, \qquad -B_\phi = \partial_t\phi$$
$$curl^d(A) = B_\phi(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - E_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - E_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - E_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$J = \left(\partial_t B_\phi + \partial_1 E_1 + \partial_2 E_2 + \partial_3 E_3\right)I = \rho^m(-I)$$

ρ^m: mass density.

$$\rho^m = -\partial_t B_\phi - \nabla \cdot E$$

$$\nabla \times E = 0, \qquad \nabla B_\phi + \partial_t E = 0, \qquad \rho^m = \Box\phi$$

3.4. *Particles*

$A \to curl(A) \to curl^d(A) \to J$. A is wave and J is particle, they are one entity existing in dual dimension. Duality. If $J = div\{curl(A)\} \neq 0$, then $curl(A)$ is not continuous. Particle is a phenomenon in which $curl(A)$ accumulate or leak out.

1) Quark is like a string. quark density: $\mathbb{e}^1, \mathbb{e}^2, \mathbb{e}^3$
2) Spin is like a membrane. spin density: $(\mathbb{e}^2 \wedge \mathbb{e}^3), (\mathbb{e}^3 \wedge \mathbb{e}^1), (\mathbb{e}^1 \wedge \mathbb{e}^2)$
3) Charge is like a particle. charge density: $(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3)$
4) Higgs is essential for creating mass. Higgs and mass density: $\mathbb{e}^t$, $(-\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)$
5) From a charge perspective, it's no coincidence that protons contain three quarks.
6) There is a geometric difference between a positron and an electron. $(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) = -(\mathbb{e}^2 \wedge \mathbb{e}^1 \wedge \mathbb{e}^3)$
7) Current density $(\mathbb{e}^i \wedge \mathbb{e}^j \wedge \mathbb{e}^t)$, superconductor is in a state where the spins are well aligned.

4. Energy density and force

Wedge product, which is an extension of the dot product, can be defined.

4.1. *Wedge product*

$$(u^{s_1} \wedge \dots \wedge u^{s_p}) \wedge (u_{\sigma_1} \wedge \dots \wedge u_{\sigma_r}) = (u^{s_2} \wedge \dots \wedge u^{s_p}) \wedge \{u^{s_1} \wedge (u_{\sigma_1} \wedge \dots \wedge u_{\sigma_r})\}$$
$$= (u^{s_3} \wedge \dots \wedge u^{s_p}) \wedge [u^{s_2} \wedge \{u^{s_1} \wedge (u_{\sigma_1} \wedge \dots \wedge u_{\sigma_r})\}] = \cdots$$

$$u^i \wedge (u_{\sigma_1} \wedge u_{\sigma_2} \wedge \dots \wedge u_{\sigma_r}) = \begin{cases} (u^i \wedge u_{\sigma_1} \wedge u_{\sigma_2} \wedge \dots \wedge u_{\sigma_r}), & (u^i \cdot u_{\sigma_i} = 0) \\ (-1)^{i-1}(u^i \cdot u_{\sigma_i})(u_{\sigma_1} \wedge \dots u_{\sigma_{i-1}} \wedge u_{\sigma_{i+1}} \wedge \dots \wedge u_{\sigma_r}) \end{cases}$$

$$(u^2 \wedge u^3) \wedge (u_1 \wedge u_2 \wedge u_5 \wedge u_7) = u^3 \wedge \{u^2 \wedge (u_1 \wedge u_2 \wedge u_5 \wedge u_7)\}$$
$$= u^3 \wedge \{(-1)^{2-1}(u^2 \cdot u_2)(u_1 \wedge u_5 \wedge u_7)\} = -(u^3 \wedge u_1 \wedge u_5 \wedge u_7)$$

4.2. *Energy density and force*

Energy density $eng(A)$ is pseudoscalar field, and the force F is the negative of the divergence of the energy density. The sign depends on the dimension of the vector field.

$$eng(A) = (curl(A) \wedge curl^d(A))/2$$
$$eng^d(A) = ((curl(A) \wedge curl^d(A))/2) \cdot I = \mp (curl^d(A) \cdot curl^d(A))/2$$
$$F = -div(eng(A)) = -curl(eng^d(A)) = \pm\, curl(curl^d(A) \cdot curl^d(A))/2$$
$$= \pm curl(curl^d(A)) \cdot curl^d(A) = \pm J \cdot curl^d(A)$$

1-vector field

$$curl(A) = B_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + B_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + B_3(\mathbb{e}^1 \wedge \mathbb{e}^2) + E_1(\mathbb{e}^1 \wedge \mathbb{e}^t) + E_2(\mathbb{e}^2 \wedge \mathbb{e}^t) + E_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$curl^d(A) = E_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + E_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + E_3(\mathbb{e}^1 \wedge \mathbb{e}^2) - B_1(\mathbb{e}^1 \wedge \mathbb{e}^t) - B_2(\mathbb{e}^2 \wedge \mathbb{e}^t) - B_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$J = \rho(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - J_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - J_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - J_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$eng(A) = ((E_i)^2 - (B_i)^2)I/2, \qquad eng^d(A) = -\,(curl^d(A) \cdot curl^d(A))/2 \neq -\,((E_i)^2 - (B_i)^2)/2$$
$$B_1(\mathbb{e}^2 \wedge \mathbb{e}^3) \wedge B_1(\mathbb{e}^2 \wedge \mathbb{e}^3)^d = -B_1(\mathbb{e}^2 \wedge \mathbb{e}^3) \wedge B_1(\mathbb{e}^1 \wedge \mathbb{e}^t) = -(B_1)^2(\mathbb{e}^3 \wedge \mathbb{e}^2 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) = -(B_i)^2 I$$
$$(B_1(\mathbb{e}^2 \wedge \mathbb{e}^3) \wedge B_1(\mathbb{e}^2 \wedge \mathbb{e}^3)^d) \cdot I = B_1(\mathbb{e}^2 \wedge \mathbb{e}^3)^d \cdot (B_1(\mathbb{e}^3 \wedge \mathbb{e}^2) \cdot I) = -B_1(\mathbb{e}^2 \wedge \mathbb{e}^3)^d \cdot B_1(\mathbb{e}^2 \wedge \mathbb{e}^3)^d$$
$$F = J \cdot curl^d(A), \qquad F_1\mathbb{e}^1 = (-\rho E_1 + J_2 B_3 - J_3 B_2)\mathbb{e}^1, \qquad F_t\mathbb{e}^t = (-J_1 E_1 - J_2 E_2 - J_3 E_3)\mathbb{e}^t$$

Conventional expression. $F = \rho E + J \times B,\ F_t = -J \cdot E$

$eng(A)$ differs from the existing one, but it is more rational. A fatal problem with the conventional energy density arises from the Lorentz booster: a moving observer changes the energy density of the rest frame. But $eng(A)$ eliminates this contradiction. Chapter 6.

2-vector field

$$eng(A) = ((E_i)^2 - (B_\phi)^2)I/2, \qquad eng^d(A) = -(curl^d(A) \cdot curl^d(A))/2$$

$$F = J \cdot curl^d(A)$$

$$F_1 e^1 = (\rho_2^s E_3 - \rho_3^s E_2 + J_1^s B_\phi) e^1, \qquad F = \rho \times E + B_\phi J$$

$$F_t e^t = (J_1^s E_1 + J_2^s E_2 + J_3^s E_3) e^t, \qquad F_t = J \cdot E$$

3-vector field

$$eng(A) = -(B_\phi)^2 I/2, \qquad eng^d(A) = -(curl^d(A) \cdot curl^d(A))/2$$

$$F = J \cdot curl^d(A), \qquad F_1 e^1 = \rho_1^q B_\phi e^1, \qquad F = \rho^q B_\phi, \qquad F_t e^t = -J^q B_\phi e^t$$

scalar field

$$eng(A) = ((E_i)^2 - (B_\phi)^2)I/2, \qquad eng^d(A) = (curl^d(A) \cdot curl^d(A))/2$$

$$F = -J \cdot curl^d(A), \qquad F_1 e^1 = -\rho^m E_1 e^1, \qquad F = -\rho^m E = -\rho^m \nabla\phi, \qquad F_t e^t = -\rho^m B_\phi e^t$$

5. Rotor and booster

Rotor and booster R transform the 1-basis of one Cartesian coordinate system into the 1-basis of another. R consists of a scalar and a plane or a velocity. The notation and meaning are as follows. e_μ is the original basis and $e_{\mu'}$ is the transformed basis.

$$e_{\mu'} = e_\mu | R = e_\mu \cdot R, \qquad e_\mu | 1 = e_\mu \cdot 1 = e_\mu$$

5.1. *Rotor*

An infinitesimal plane rotor

$$s_1 = (e_2 \wedge e_3), \qquad s_2 = (e_3 \wedge e_1), \qquad s_3 = (e_1 \wedge e_2), \qquad R_i = 1 + \Delta s_i, \qquad (\Delta \ll 1)$$

$$e_1' = e_1 | R_3 = e_1 | (1 + \Delta s_3) = e_1 | 1 + e_1 | \Delta s_3 = e_1 + e_1 \cdot \Delta s_3 = e_1 + \Delta e_2$$

$$e_2' = e_1 | R_3 = e_2 | 1 + e_2 | \Delta s_3 = e_2 - \Delta e_1, \qquad e_3' = e_1 | R_3 = e_3, \qquad e_t' = e_1 | R_3 = e_t$$

$$e_\nu | s_3 | s_3 = (e_\nu \cdot s_3) \cdot s_3$$

$$(e_1 \cdot s_3) \cdot s_3 = -e_1, \qquad (e_2 \cdot s_3) \cdot s_3 = -e_2, \qquad (e_3 \cdot s_3) \cdot s_3 = 0, \qquad (e_t \cdot s_3) \cdot s_3 = 0,$$

$$s_3 | s_3 = \begin{cases} -1 \\ 0 \end{cases}$$

Infinite series of infinitesimal plane rotors. $\Delta_1 + \Delta_2 + \Delta_3 \ldots = \theta$

$$R_3(\theta) = (1 + \Delta_1 s_3)|(1 + \Delta_2 s_3)| \ldots = \lim_{\Delta \to 0} (1 + \Delta s_3 |)^{\theta/\Delta} = e^{s_3 | \theta}$$

$$= 1 + \theta s_3 + \frac{1}{2!}\theta^2 s_3 | s_3 + \frac{1}{3!}\theta^3 s_3 | s_3 | s_3 + \cdots = \begin{cases} \cos\theta + \sin\theta\, s_3 \\ 1 + \theta s_3 \end{cases}$$

5.2. *Lorentz booster*

An infinitesimal booster

$$s_1^d = s_1 \cdot I = (e_2 \wedge e_3) \cdot I = (e_1 \wedge e_t), \qquad s_2^d = (e_2 \wedge e_t), \qquad s_3^d = (e_3 \wedge e_t), \qquad R_i^d = 1 + \Delta s_i^d$$

$$e_1' = e_1 | R_1^d = e_1 + \Delta e_t, \qquad e_t' = e_t | R_1^d = e_t + \Delta e_1, \qquad e_2' = e_2 | R_1^d = e_2, \qquad e_3' = e_3 | R_1^d = e_3$$

$$e_\nu | s_1^d | s_1^d = (e_\nu \cdot s_1^d) \cdot s_1^d$$

$$(e_1 \cdot s_1^d) \cdot s_1^d = e_1, \qquad (e_2 \cdot s_1^d) \cdot s_1^d = 0, \qquad (e_3 \cdot s_1^d) \cdot s_1^d = 0, \qquad (e_t \cdot s_1^d) \cdot s_1^d = e_t$$

$$s_1^d | s_1^d = \begin{cases} 1 \\ 0 \end{cases}$$

Infinite series of infinitesimal booster. $\Delta_1 + \Delta_2 + \Delta_3 \ldots = \theta$

$$R_1^d(\theta) = (1 + \Delta_1 s_1^d)|(1 + \Delta_2 s_1^d) \ldots = \lim_{\Delta \to 0} (1 + \Delta s_1^d |)^{\theta/\Delta} = e^{s_1^d | \theta}$$

$$= 1 + \theta s_1^d + \frac{1}{2!}\theta^2 s_1^d | s_1^d + \frac{1}{3!}\theta^3 s_1^d | s_1^d | s_1^d + \cdots = \begin{cases} \cosh\theta + \sinh\theta\, s_1^d \\ 1 + \theta s_1^d \end{cases}$$

5.3. *Multibasis transformations*

The multibasis transformations is as follows. Pseudoscalar I is not changed by the rotor and booster.

$$(\mathbb{e}'_\mu \wedge \mathbb{e}'_\nu) = (\mathbb{e}_\mu \wedge \mathbb{e}_\nu)|R = ((\mathbb{e}_\mu|R) \wedge (\mathbb{e}_\nu|R))$$
$$(\mathbb{e}'_\mu \wedge \mathbb{e}'_\nu \wedge \mathbb{e}'_\lambda)|R = (\mathbb{e}_\mu \wedge \mathbb{e}_\nu \wedge \mathbb{e}_\lambda)|R = ((\mathbb{e}_\mu|R) \wedge (\mathbb{e}_\nu|R) \wedge (\mathbb{e}_\lambda|R))$$
$$I' = I|R = I$$
$$(\mathbb{e}'_\mu \cdot \mathbb{e}'_\nu) = (\mathbb{e}_\mu \cdot \mathbb{e}_\nu)|R = ((\mathbb{e}_\mu|R) \cdot (\mathbb{e}_\nu|R))$$

For examples in $R_1^d(\theta)$

$$\mathbb{e}^{1'} = cosh\,\theta\,\mathbb{e}^1 - sinh\,\theta\,\mathbb{e}^t, \qquad \mathbb{e}^{2'} = \mathbb{e}^2, \qquad \mathbb{e}^{3'} = \mathbb{e}^3, \qquad \mathbb{e}^{t'} = cosh\,\theta\,\mathbb{e}^t - sinh\,\theta\,\mathbb{e}^1$$
$$(\mathbb{e}^{1'} \wedge \mathbb{e}^{2'}) = ((cosh\,\theta\,\mathbb{e}^1 - sinh\,\theta\,\mathbb{e}^t) \wedge \mathbb{e}^2) = cosh\,\theta\,(\mathbb{e}^1 \wedge \mathbb{e}^2) + sinh\,\theta\,(\mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$(\mathbb{e}^{1'} \wedge \mathbb{e}^{t'}) = (\mathbb{e}^1 \wedge \mathbb{e}^t)$$
$$I' = -(\mathbb{e}^{1'} \wedge \mathbb{e}^{2'} \wedge \mathbb{e}^{3'} \wedge \mathbb{e}^{t'}) = -(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) = I$$

6. Vector field

A vector field A is a one entity. However, its representation varies depending on the coordinate system. Its coefficients also differ. Yet, in reality, only the coefficients can be felt and measured. If A is a vector field then $curl(A)$, $curl^d(A)$, J, $eng(A)$, F, I are also vector field.

$$X(p^1, \dots, p^n) = X(q^1, \dots, q^n), \qquad u_i = \frac{\partial X}{\partial p^i}, \qquad v_j = \frac{\partial X}{\partial q^j}$$
$$u_i = \alpha_i^j v_j = (u_i \cdot v^j)v_j, \qquad u_i \cdot v^k = (a_i^j v_j) \cdot v^k = \alpha_i^k$$
$$u_i = \frac{\partial X}{\partial p^i} = \frac{\partial q^j}{\partial p^i}\frac{\partial X}{\partial q^j} = \frac{\partial q^j}{\partial p^i} v_j = (u_i \cdot v^j)v_j, \qquad (u_i \cdot v^j) = \frac{\partial q^j}{\partial p^i}, \qquad (v_j \cdot u^i) = \frac{\partial p^i}{\partial q^j}$$
$$u^i = (u^i \cdot v_j)v^j = \frac{\partial p^i}{\partial q^j} v^j$$

$A(p) = A(q)$ then

$$curl(A(p)) = \left(u^i \wedge \frac{\partial A(p)}{\partial p^i}\right) = \left(\frac{\partial p^i}{\partial q^j} v^j \wedge \frac{\partial A(p)}{\partial p^i}\right) = \left(v^j \wedge \frac{\partial p^i}{\partial q^j}\frac{\partial A(q)}{\partial p^i}\right) = \left(v^j \wedge \frac{\partial A(q)}{\partial q^j}\right) = curl(A(q))$$

I is obviously the same in all spaces. $|m_{ij}|$ is determinant of matrix m_{ij}

$$u_i \cdot v^j = (u_i \cdot w^k)w_k \cdot w^l(w_l \cdot v^j) = (u_i \cdot w^k)(w_k \cdot v^j)$$
$$v_i \cdot v^j = \delta_i^j = (v_i \cdot \mathbb{e}^k)(\mathbb{e}_k \cdot v^j), \qquad |v_i \cdot \mathbb{e}^j||\mathbb{e}_i \cdot v^j| = 1$$
$$(u_1 \wedge \dots \wedge u_n) = ((u_1 \cdot \mathbb{e}^{\sigma_1})\mathbb{e}_{\sigma_1} \wedge \dots \wedge (u_n \cdot \mathbb{e}^{\sigma_n})\mathbb{e}_{\sigma_n}) = |(u_i \cdot \mathbb{e}^j)|(\mathbb{e}_1 \wedge \dots \wedge \mathbb{e}_n) = |(u_i \cdot \mathbb{e}^j)|I$$
$$(u_1 \wedge \dots \wedge u_n) = |u_i \cdot v^j|(v_1 \wedge \dots \wedge v_n) = |(u_i \cdot \mathbb{e}^j)||(\mathbb{e}_i \cdot v^j)|(v_1 \wedge \dots \wedge v_n)$$
$$I(p) = (u_1 \wedge \dots \wedge u_n)/|(u_i \cdot \mathbb{e}^j)| = (v_1 \wedge \dots \wedge v_n)/|(v_i \cdot \mathbb{e}^j)| = I(q)$$

$$curl^d(A(p)) = curl(A(p)) \cdot I = curl(A(q)) \cdot I = curl^d(A(q))$$
$$J(p) = curl(curl^d(A(p))) = curl(curl^d(A(q))) = J(q)$$
$$eng(A(p)) = (curl(A) \wedge curl^d(A))/2 = eng(A(q))$$
$$F(p) = J(p) \cdot curl^d(A(p)) = J(q) \cdot curl^d(A(q)) = F(q)$$

For example, electromagnetic field in Lorenz boost $R_1^d(\theta)$

$$curl(A)$$
$$= B_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + B_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + B_3(\mathbb{e}^1 \wedge \mathbb{e}^2) + E_1(\mathbb{e}^1 \wedge \mathbb{e}^t) + E_2(\mathbb{e}^2 \wedge \mathbb{e}^t) + E_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$= B'_1(\mathbb{e}^{2'} \wedge \mathbb{e}^{3'}) + B'_1(\mathbb{e}^{3'} \wedge \mathbb{e}^{1'}) + B'_1(\mathbb{e}^{1'} \wedge \mathbb{e}^{2'}) + E'_1(\mathbb{e}^{1'} \wedge \mathbb{e}^{t'}) + E'_1(\mathbb{e}^{2'} \wedge \mathbb{e}^{t'}) + E'_1(\mathbb{e}^{3'} \wedge \mathbb{e}^{t'})$$

Obtain the coefficient relationship by substituting $(\mathbb{e}^{\mu'} \wedge \mathbb{e}^{\nu'})$ with $(\mathbb{e}^\mu \wedge \mathbb{e}^\nu)$.

$$B_1 = B'_1, \qquad B_2 = cosh\,\theta\,B'_2 - sinh\,\theta\,E'_3, \qquad B_3 = cosh\,\theta\,B'_3 + sinh\,\theta\,E'_2$$

$$E_1 = E_1', \qquad E_2 = sinh\,\theta\, B_3' + cosh\,\theta\, E_2', \qquad E_3 = -\,sinh\,\theta\, B_2' + cosh\,\theta\, E_3'$$

$$eng(A) = ((E_i)^2 - (B_i)^2)I/2 = ((E_i')^2 - (B_i')^2)I/2 = eng'(A)$$

The energy density, I field is the same for a moving observer and a rest frame observer.

$$J = -\rho(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - J_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - J_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - J_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$= -\rho'(\mathbb{e}^{1'} \wedge \mathbb{e}^{2'} \wedge \mathbb{e}^{3'}) - J_1'(\mathbb{e}^{2'} \wedge \mathbb{e}^{3'} \wedge \mathbb{e}^{t'}) - J_2'(\mathbb{e}^{3'} \wedge \mathbb{e}^{1'} \wedge \mathbb{e}^{t'}) - J_3'(\mathbb{e}^{1'} \wedge \mathbb{e}^{2'} \wedge \mathbb{e}^{t'})$$
$$\rho = cosh\,\theta\, \rho' - sinh\,\theta\, J_1', \qquad J_1 = cosh\,\theta\, J_1' - sinh\,\theta\, J_t', \qquad J_2 = J_2', \qquad J_3 = J_3'$$

$$F = F_1\mathbb{e}^1 + F_2\mathbb{e}^2 + F_3\mathbb{e}^3 + F_t\mathbb{e}^t = F_1'\mathbb{e}^{1'} + F_2'\mathbb{e}^{2'} + F_3'\mathbb{e}^{3'} + F_t'\mathbb{e}^{t'}$$
$$F_t = cosh\,\theta\, F_t' - sinh\,\theta\, F_1', \qquad F_1 = cosh\,\theta\, F_1' - sinh\,\theta\, F_t', \qquad F_2 = F_2', \qquad F_3 = F_3'$$

Check in another way

$$F_1 = (-J_tE_1 + J_2B_3 - J_3B_2)$$
$$= -(cosh\,\theta\, \rho' - sinh\,\theta\, J_1')E_1' + J_2'(cosh\,\theta\, B_3' + sinh\,\theta\, E_2') - J_3'(cosh\,\theta\, B_2' - sinh\,\theta\, E_3')$$
$$= cosh\,\theta\,(-\rho' E_{1'} + J_2'B_3' - J_3'B_2') - sinh\,\theta\,(-J_1'E_1' - J_2'E_2' - J_3'E_3')$$
$$= cosh\,\theta\, F_t' - sinh\,\theta\, F_1'$$

7. Quantization

R^* is the Hermitian rotor with respect to R.

$$\mathbb{e}_\mu' = \mathbb{e}_\mu | R, \qquad \mathbb{e}_\mu = \mathbb{e}_\mu' | R^*$$
$$\mathbb{e}_\mu = \mathbb{e}_\mu \big| (R|R^*), \qquad \mathbb{e}_\mu' = \mathbb{e}_\mu' \big| (R^*|R), \qquad (R|R^*) = (R^*|R) = 1$$
$$R_i^*(\theta) = R_i(-\theta), \qquad R_i^{d*}(\theta) = R_i^*(-\theta)$$

$$\mathbb{e}_\mu \cdot \mathbb{e}_{\nu'} = \mathbb{e}_\mu \cdot (\mathbb{e}_\nu | R) = (\mathbb{e}_\mu \cdot (\mathbb{e}_\nu | R))|R^* = (\mathbb{e}_\mu | R^*) \cdot (\mathbb{e}_\nu | (R|R^*)) = (\mathbb{e}_\mu | R^*) \cdot \mathbb{e}_\nu$$

7.1. *Commutator*

Two rotors, R_A, R_B. two path, $(R_A|R_B), (R_B|R_A)$. And the commutator, the difference of the two paths. The symmetry breaking of the two paths creates quantization.

$$R_A(\theta), R_B(\theta) \in \{(cos\,\theta + sin\,\theta\, \mathbb{s}_i), (1 + \theta \mathbb{s}_j), (cosh\,\theta + sinh\,\theta\, \mathbb{s}_k^d), (\,1 + \theta \mathbb{s}_l^d)\}$$
$$[R_A(\theta_1), R_B(\theta_2)] = (R_A(\theta_1)|R_B(\theta_2)) - (R_B(\theta_2)|R_A(\theta_1))$$
$$= [R_A^*(\theta_1), R_B^*(\theta_2)] = (R_A(-\theta_1)|R_B(-\theta_2)) - (R_B(-\theta_2)|R_A(-\theta_1))$$

The following equation holds. ε_{ab} is Levi-Civita symbol

$$\mathbb{e}_\mu \cdot (\mathbb{e}_\nu | (R_A|R_B)) = (\mathbb{e}_\mu | R_B^*) \cdot (\mathbb{e}_\nu | (R_A)) = (\mathbb{e}_\mu | (R_B^*|R_A^*)) \cdot \mathbb{e}_\nu$$
$$\mathbb{e}_\mu \cdot (\mathbb{e}_\nu | [R_A, R_B]) = (\mathbb{e}_\mu | [R_B^*, R_A^*]) \cdot \mathbb{e}_\nu = -(\mathbb{e}_\mu | [R_A, R_B]) \cdot \mathbb{e}_\nu$$

$$(\mathbb{e}_\mu \wedge \mathbb{e}_\nu) \cdot (\mathbb{e}_{\sigma_1}' \wedge \mathbb{e}_{\sigma_2}') = (\mathbb{e}_\mu \wedge \mathbb{e}_\nu) \cdot ((\mathbb{e}_{\sigma_1} \wedge \mathbb{e}_{\sigma_2}) | (R_A|R_B))$$
$$= (\mathbb{e}_\mu \wedge \mathbb{e}_\nu) \cdot \{((\mathbb{e}_{\sigma_1} | (R_A|R_B)) \wedge (\mathbb{e}_{\sigma_2} | (R_A|R_B)))\} = \varepsilon_{ab}\{\mathbb{e}_\mu \cdot (\mathbb{e}_{\sigma_a} | [R_A, R_B])\}\{\mathbb{e}_\nu \cdot (\mathbb{e}_{\sigma_b} | [R_A, R_B])\}$$
$$= \varepsilon_{ab}\{(\mathbb{e}_\mu | [R_A, R_B]) \cdot \mathbb{e}_{\sigma_a}\}\{(\mathbb{e}_\nu | [R_A, R_B]) \cdot \mathbb{e}_{\sigma_b}\} = ((\mathbb{e}_\mu | [R_A, R_B]) \wedge (\mathbb{e}_\nu | [R_A, R_B])) \cdot (\mathbb{e}_{\sigma_1} \wedge \mathbb{e}_{\sigma_2})$$
$$= ((\mathbb{e}_\mu \wedge \mathbb{e}_\nu) | (R_A|R_B)) \cdot (\mathbb{e}_{\sigma_1} \wedge \mathbb{e}_{\sigma_2})$$

By substituting various cases, the following equation is obtained. To summarize.

$$\mathbb{e}_1 \cdot [\mathbb{s}_1, \mathbb{s}_2] = (\mathbb{e}_1 \cdot \mathbb{s}_1) \cdot \mathbb{s}_2 - (\mathbb{e}_1 \cdot \mathbb{s}_2) \cdot \mathbb{s}_1 = -\mathbb{e}_2 = \mathbb{e}_1 | (-\mathbb{s}_3)$$
$$\mathbb{e}_2 \cdot [\mathbb{s}_1, \mathbb{s}_2] = \mathbb{e}_1 = \mathbb{e}_2 | (-\mathbb{s}_3), \qquad \mathbb{e}_3 \cdot [\mathbb{s}_1, \mathbb{s}_2] = 0 = \mathbb{e}_3 | (-\mathbb{s}_3), \qquad \mathbb{e}_t \cdot [\mathbb{s}_1, \mathbb{s}_2] = 0 = \mathbb{e}_t | (-\mathbb{s}_3)$$
$$[\mathbb{s}_1, \mathbb{s}_2] = -\mathbb{s}_3, \qquad \mathbb{e}_\mu \cdot (\mathbb{e}_\nu | [\mathbb{s}_1, \mathbb{s}_2]) = \mathbb{e}_\mu \cdot \big(\mathbb{e}_\nu | (-\mathbb{s}_3)\big) = -(\mathbb{e}_\mu | [\mathbb{s}_1, \mathbb{s}_2]) \cdot \mathbb{e}_\nu = (\mathbb{e}_\mu | \mathbb{s}_3) \cdot \mathbb{e}_\nu$$
$$[\mathbb{s}_i, \mathbb{s}_j] = -\varepsilon_{ijk}\mathbb{s}_k, \qquad [\mathbb{s}_i^d, \mathbb{s}_j^d] = \varepsilon_{ijk}\mathbb{s}_k, \qquad [\mathbb{s}_i, \mathbb{s}_j^d] = [\mathbb{s}_i^d, \mathbb{s}_j] = -\varepsilon_{ijk}\mathbb{s}_k^d$$

7.2. *ladder*

The well-known ladder is obtained by adding and subtracting the two equations above.

$[\mathbb{s}_k, \mathbb{s}_j^d] = -\varepsilon_{kji}\mathbb{s}_i^d = \varepsilon_{ijk}\mathbb{s}_i^d, \qquad [\mathbb{s}_i^d, \mathbb{s}_j^d] = \varepsilon_{ijk}\mathbb{s}_k$

$[(\mathbb{s}_k \pm \mathbb{s}_i^d), \mathbb{s}_j^d] = \pm\varepsilon_{ijk}(\mathbb{s}_k \pm \mathbb{s}_i^d), \qquad [\mathbb{s}_{j\pm}, \mathbb{s}_j^d] = \pm\varepsilon_{ijk}\mathbb{s}_{j\pm}, \qquad \mathbb{s}_{j\pm} = \mathbb{s}_k \pm \mathbb{s}_i^d$

$\mathbb{s}_{j\pm}|\mathbb{s}_j^d = \mathbb{s}_j^d|\mathbb{s}_{j\pm} \pm \mathbb{s}_{j\pm}$

For example, if 1-basis field V^0 exists, then $V^{+1}, V^{+2}, V^{+3}, \ldots$ satisfying the ladder may also exist.

$V^0 = \mathbb{e}^2 - \mathbb{e}^t, \qquad \mathbb{s}_{2+}|\mathbb{s}_2^d = \mathbb{s}_2^d|\mathbb{s}_{2+} + \mathbb{s}_{2+}, \qquad \mathbb{s}_{2+} = \mathbb{s}_3 + \mathbb{s}_1^d$

$V^0|\mathbb{s}_2^d = V^0 \cdot (\mathbb{e}_2 \wedge \mathbb{e}_t) = (\mathbb{e}^2 - \mathbb{e}^t) \cdot (\mathbb{e}_2 \wedge \mathbb{e}_t) = \mathbb{e}_t + \mathbb{e}_2 = \mathbb{e}^2 - \mathbb{e}^t = V^0$

$V^0|((\mathbb{s}_2^d|\mathbb{s}_{2+}) + \mathbb{s}_{2+}) = ((V^0|\mathbb{s}_2^d) + V^0)|\mathbb{s}_{2+} = 2V^0|\mathbb{s}_{2+} = 2V^{+1}, \qquad V^0|\mathbb{s}_{2+} = V^{+1}$

$V^0|(\mathbb{s}_{2+}|\mathbb{s}_2^d) = (V^0|\mathbb{s}_{2+})|\mathbb{s}_2^d = V^{+1}|\mathbb{s}_2^d = 2V^{+1}$

$V^{+1}|((\mathbb{s}_2^d|\mathbb{s}_{2+}) + \mathbb{s}_{2+}) = ((V^{+1}|\mathbb{s}_2^d) + V^{+1})|\mathbb{s}_{2+} = 3V^{+1}|\mathbb{s}_{2+} = 3V^{+2}, \qquad V^{+1}|\mathbb{s}_{2+} = V^{+2}$

$V^{+1}|(\mathbb{s}_{2+}\mathbb{s}_2^d) = (V^{+1}|\mathbb{s}_{2+})|\mathbb{s}_2^d = V^{+2}|\mathbb{s}_2^d = 3V^{+2}$

...

$V^{+m}|\mathbb{s}_2^d = (m+1)V^{+m}, \qquad V^{+m} = V(|\mathbb{s}_{2+})^{m-1}$

8. Planck's constant

There is no rest frame! The universe is always vibrating. The current magnitude of the cosmic background vibration, namely Δ, is Planck's constant $\hbar$.

First, $(1 + \Delta\mathbb{s}_i)$ vibration creates commutation relations of angular momentum operators. $\hbar$ has the dimension of angular momentum, in 7.2.

$R_A = 1 + \hbar\mathbb{s}_i, \qquad R_B = 1 + \hbar\mathbb{s}_j$

$[\hbar\mathbb{s}_i, \hbar\mathbb{s}_j] = -\varepsilon_{ijk}\hbar(\hbar\mathbb{s}_k), \qquad [L_i, L_j] = -\varepsilon_{ijk}\hbar L_k, \qquad L_i = \hbar\mathbb{s}_i$

Second, $(1 + \Delta\mathbb{s}_i^d)$ vibration creates uncertainty principle. The uncertainty of time is interpreted as the uncertainty of momentum. It can also be applied to physical quantities corresponding to 2-basis or 3-basis.

$X = x^1\mathbb{e}_1 + t\mathbb{e}_t = x^{1\prime}\mathbb{e}_1' + t'\mathbb{e}_t' = x^{1\prime}(\mathbb{e}_1 + \Delta\mathbb{e}_t) + t'(\mathbb{e}_t + \Delta\mathbb{e}_1) = (x^{1\prime} + \Delta t')\mathbb{e}_1 + (t' + \Delta x^{1\prime})\mathbb{e}_t$

$x^1 = x^{1\prime} + \Delta t', \qquad t = t' + \Delta x^{1\prime}$

$$\left(\frac{x^1}{x^{1\prime}}\right)\left(\frac{t}{t'}\right) = \left(1 + \Delta\frac{t'}{x^{1\prime}}\right)\left(1 + \Delta\frac{x^{1\prime}}{t'}\right) \geq 1 + 2\Delta = 1 + 2\hbar$$

Third, photon is a concept created out of ignorance of the existence of beat waves.

$x^1 - t = (x^{1\prime} - t')(1 - \Delta), \qquad e^{i(x^1 - t)} = e^{i(x^{1\prime} - t')}e^{-i\Delta(x^{1\prime} - t')}$

$$\psi = \frac{e^{i(x^1 - t)}}{e^{i(x^{1\prime} - t')}} = e^{-i\Delta(x^{1\prime} - t')} = e^{-i\hbar(x^{1\prime} - t')}$$

The frequency ν is due to wave $e^{i(x^{1\prime} - t')}$, and the energy $h\nu = \hbar\omega$ is due to the beat wave $e^{-i\hbar(x^{1\prime} - t')}$. This interpretation is more consistent with the concept of measuring Planck's constant in the photoelectric effect. Schrödinger's wave is also a beat wave. $\hbar$ is not a constant coefficient of the equation.