Proposition for a privileged frame of reference in a periodic Minkowski space-time

A. Muñoz
alfredo.o.munoz@gmail.com

April 4, 2026

Abstract

In the present article, we want to analyze a particular case of the twins paradox. As everyone knows, this paradox has been widely discussed and satisfactorily answered. So, Why is another paper necessary? We believe this problem is important because from his solution emerges the existence of a privileged reference frame within the Minkowski space-time that is consistent with the Lorentz transformations.

The Lorentz transformations are symmetric, so any solution to the paradox must show that this symmetry is broken. The purpose of this article is to deep into the implications of symmetry breaking, to explain the point of view of each observer, to point out that the characteristics of the space-time used allow to us define a reference point based on which it is possible to recognize whether an observer is in motion or at rest with respect to the space-time and to talk of an absolute time and distance, and to analyze the consequences of the existence of this privileged frame.

1 The Twin Paradox

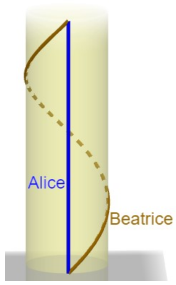
The most common formulation of this paradox was made by Paul Langevin [1] and consists of 2 twins, one of whom gets on a rocket in which he travels to a star and returns while the other brother stays on earth. From the theory of special relativity we know that time dilates for a moving observer, so on his return the traveling twin should be younger than his earthly brother, the paradox is not that one brother ages more than the other, but from the point of view of the twin going to the star: the moving object is the earth which is moving away from him, so in his reference frame it is the twin on earth in which he should age less. The contradiction is that both twins are going to say that the other is younger.

The solution to this paradox outside the context of the theory of general relativity enunciated by Einstein, is to consider that the traveling twin must successively accelerate and decelerate to go to and return from the star which would break the symmetry of the problem and explains why it is the earth twin who ages more, thus solving the problem.

2 Paradox Special Case

In the outstanding Youtube channel of Professor Javier Garcia, the subscriber Rodolfo Perez [2] presented the following variation to the paradox: In a space with a closed curvature and if a twin is already moving in the direction of the curvature, it would not be necessary for him to accelerate/decelerate or change direction to meet his brother again, it is only necessary for him to follow his trajectory so that both twins meet periodically. Since in this example there is no gravitational field or accelerated reference frames that can explain that for both twins the one who is moving (and who ages less) is his brother, so the previous explanation is no longer valid giving rise to the paradox again.

Professor Garcia himself demonstrates in a later video that this curved space can be approximated by a Minkowski metric and that the symmetry of the problem is broken, which is explained by the different topology of the reference frames [3].

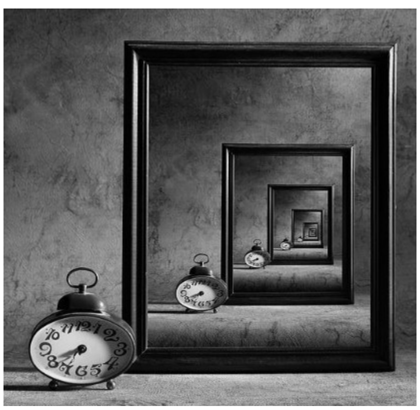


A similar case had already been solved by Professor Tevian Dray in his paper "The twin paradox revisited" [4] where he uses a cylindrical space of two dimensions (1+1) and shows that the paradox is solved, but the symmetry of the problem is broken (another solution can be found in the article "The Twin Paradox in a Closed Universe"[5] from Jeffrey R. Weeks).

For simplicity, we will use this cylindrical space which for having an extrinsic curvature is still flat and with Minkowski metric. We will call Alice the twin at rest and Beatrice the twin in motion with velocity V, we will also assume that the extension of the space-time is D and we will use the units that allow us to make c = 1.

3 Alice's frame of reference

In a space with a closed curvature, the light emitted by the observers will make a complete turn and return to its starting point, if it is not stopped it will travel infinitely completing circles and returning to the initial point with a period of D/c.



Thus, to an observer in this space, it will seem that his image is repeated every D unit back and forth as if he were in a room covered by mirrors. This repeated image provides a **reference point** on which a **privileged reference frame** can be determined, with it our observer will be able to determine whether it is in motion or at rest with respect to the space-time.

As Dray's paper explains, to prove that he is at rest the observer only needs to send two light rays in opposite directions of the curvature, if both light rays return simultaneously he will be certain that his state is at absolute rest.

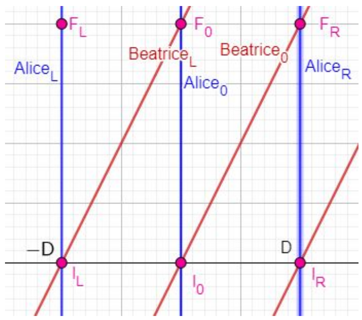
In our opinion the reference point does not create this privileged reference frame at rest, this is intrinsic to the space-time. What the curvature allows us is to be able to recognize which is the **absolute resting state**.

We have then that in Alice's reference frame, her world line will look like this:

For Alice, the space-time remains flat and without curvature, since her two-dimensional existence prevents her from observing the cylindrical shape of the space-time. The only thing that catches her attention is that she can see her "reflection" repeated both to her left and her right with a period of D units of distance.

As we can see in the graph we have $Alice_0$ at rest and her reflections on the left and on the right which for this example we will call $Alice_L$ and $Alice_R$.

Let's see how Beatrice looks like in Alice's reference frame:



It may seem complex, but this graph is trying to model the multiple images that $Alice_0$ can observe.

First, we have point I_0 which is the moment when $Alice_0$ is reunited with her sister $Beatrice_0$, their respective repetitions are going to be I_L and I_R.

Next, $Beatrice_0$ follows her worldline which leads her to meet $Alice_R$ at point F_R at her right and in turn $Beatrice_L$ from her left is who meets $Alice_0$ at F_0.

Although we know that the events are the same, the curvature of the space-time makes them look like different points of the space-time to the observer.

Translated to space-time coordinates in Alice's reference frame the points are:

$I_0 = ({I_0}^0, {I_0}^1) = (0, 0)$ $\quad$ $I_L = ({I_L}^0, {I_L}^1) = (0, -D)$ $\quad$ $I_R = ({I_R}^0, {I_R}^1) = (0, D)$

$F_0 = ({F_0}^0, {F_0}^1) = (\frac{D}{V}, 0)$ $\quad$ $F_L = ({F_L}^0, {F_L}^1) = (\frac{D}{V}, -D)$ $\quad$ $F_R = ({F_R}^0, {F_R}^1) = (\frac{D}{V}, D)$

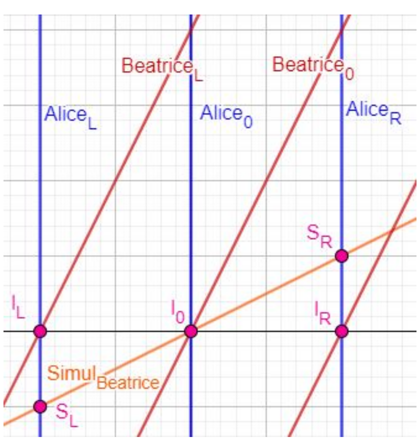
Hence, the elapsed time for $Alice_0$ from the time she is separated from her sister until they meet again is: $\Delta t = {F_0}^0 - {I_0}^0 = \frac{D}{V} - 0 = \frac{D}{V}$

And the proper time measuring for $Beatrice_0$ is: $\tau = \frac{\Delta t}{\gamma} = ({F_R}^0 - {I_0}^0)/\gamma = (\frac{D}{V} - 0)/\gamma = \frac{D}{\gamma V}$

As expected the proper time experienced by Alice turns out to be greater than that measured for Beatrice by a factor of $1/\gamma$ which at first glance may seem to give rise to a paradox. To understand that there are no contradictions between the observers it is necessary to transform these points to the reference frame experienced by Beatrice.

4 Beatrice's frame of reference

Before we do the transformations, the first thing to consider is to incorporate the $Beatrice_0$ simultaneity line into the graph:

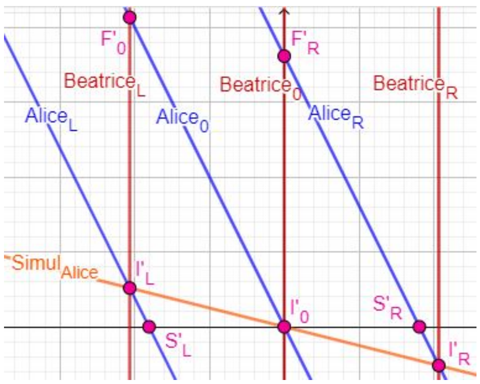


It can be seen that given the absolute velocity V carried by $Beatrice_0$ (we can say absolute because we know that the reference frame of $Alice_0$ is at absolute rest), according to its simultaneity line, I_0 is no longer simultaneous with I_R but with S_R.

This means that for $Beatrice_0$ at the instant she is reunited with her sister, if she looks at her right reflection she will observe a time in the future where both have separated and if she looks at her left reflection she will see a time in the past when they have not yet met.

It must be remembered that the restrictions of the light cones impose that the information will always arrive with delay to the observer, so in reality $Beatrice_0$ does not see the event I_R before the event I_0 happens, what happens is that when the information of the event I_R arrives to her and she tries to locate it in the space-time it will give her a time before the occurrence of I_0.

The following graph shows the same points in Beatrice's reference frame:



Here we can see that the points I'_L, I'_0 and I'_R appear displaced in time and fall on the simultaneity line of Alice as we would expect. But what is remarkable is that the same happens for points F'_L, F'_0 and F'_R that gives us the first sign that Beatrice's $\Delta t'$ is smaller than the difference she observes for $Alice_R$.

That is if we calculate ${F'}_R^0 - {I'}_0^0$ which is the interval measured by $Beatrice_0$ for the time span from the moment she is separated from her sister until she finds her again, it will give a lower value than the equivalent interval of time observed for $Alice_R$: ${F'}_R^0 - {I'}_R^0$.

To demonstrate the above we will apply the Lorentz transformations to the analyzed points:

$$I_0 = (0,0) \qquad \Longrightarrow \qquad \begin{aligned} I'_0 &= (\gamma {I_0}^0 - \gamma\beta {I_0}^1, -\gamma\beta {I_0}^0 + \gamma {I_0}^1) \\ &= (\gamma 0 - \gamma\beta 0, -\gamma\beta 0 + \gamma 0) \\ &= (0,0) \end{aligned} \tag{1}$$

$$I_R = (0,D) \qquad \Longrightarrow \qquad \begin{aligned} I'_R &= (\gamma I_R^0 - \gamma\beta I_R^1, -\gamma\beta I_R^0 + \gamma I_R^1) \\ &= (\gamma 0 - \gamma\beta D, -\gamma\beta 0 + \gamma D) \\ &= (-\gamma\beta D, \gamma D) \end{aligned} \tag{2}$$

$$F_R = (\frac{D}{V}, D) \qquad \Longrightarrow \qquad \begin{aligned} F'_R &= (\gamma {F_R}^0 - \gamma\beta {F_R}^1, -\gamma\beta {F_R}^0 + \gamma {F_R}^1) \\ &= (\gamma\frac{D}{V} - \gamma\beta D, -\gamma\beta\frac{D}{V} + \gamma D) \; if \; c = 1 \; then \; \beta = V \\ &= (\gamma\frac{D}{V} - \gamma\beta D, -\gamma\cancel{\beta}\frac{D}{\cancel{V}} + \gamma D) \\ &= (\gamma\frac{D}{V} - \gamma\beta D, -\gamma D + \gamma D) \\ &= (\gamma\frac{D}{V} - \gamma\beta D, 0) \end{aligned} \tag{3}$$

So the points are as follows:

$I'_0 = (0,0) \qquad I'_R = (-\gamma\beta D, \gamma D) \qquad F'_R = (\gamma\frac{D}{V} - \gamma\beta D, 0)$

And the proposed interval for Beatrice is:

$$\begin{aligned} {F'}_R^0 - {I'}_0^0 &= \gamma\frac{D}{V} - \gamma\beta D - 0 \\ &= \frac{D}{\gamma V}\;(\gamma^2 - \gamma^2 V\beta) \;\; if \; c = 1 \; then \; \beta = V \\ &= \frac{D}{\gamma V}\;(\frac{1}{1-V^2} - \frac{V^2}{1-V^2}) \\ &= \frac{D}{\gamma V}\;(\frac{1}{1-V^2} - \frac{V^2}{1-V^2}) \to 1 \\ &= \frac{D}{\gamma V} \end{aligned} \tag{4}$$

And for Alice is:

$$\begin{aligned} F'^{\,0}_{R} - I'^{\,0}_{R} &= \gamma \frac{D}{V} - \gamma\beta D - (-\gamma\beta D) \\ &= \gamma \frac{D}{V} - \cancelto{0}{\gamma\beta D + \gamma\beta D} \\ &= \gamma \frac{D}{V} \end{aligned} \tag{5}$$

Then the proper time that Beatrice measures for $Alice_R$ is: $\tau = \frac{\Delta t'}{\gamma} = (F'^{\,0}_{R} - I'^{\,0}_{R})/\gamma = (\gamma \frac{D}{V})/\gamma = \frac{D}{V}$

Finally, we can realize that in both reference frames the results are the same, both observers measure that the time elapsed for Alice is $\frac{D}{V}$ and for Beatrice $\frac{D}{\gamma V}$ which shows that there is no paradox.

5 Conclusions

As already demonstrated by Professors Javier Garcia, Tevian Dray, and Jeffrey R. Weeks there is no paradox and both observers agree with their measurements. The explanation is that the symmetry of the problem is broken, so even though the events are the same each observer will measure a different wordline for her twin sister. The reason is as simple as it is surprising: "There is a sister that moves and the other does not". This means that although the equations are symmetric the initial conditions of the problem are not. The solution will be symmetric if both sisters move at the same absolute speed in opposite directions.

Thus, the curvature of the space-time gives us a mechanism based on which we can recognize a privileged reference frame, this reference frame is proper to the space-time and exists previously, but it is indistinguishable from the others unless we have this reference point.

We think there might be other ways to recognize this privileged reference frame, for example:

- If space-time were discrete and an observer could see the granularity of this space-time then this observer could recognize his absolute velocity with respect to this grid at rest.
- There are some measures of the absolute velocity of the solar system relative to the CMB, If we assume that the CMB has been at rest since the Big Bang then this velocity measurement will be absolute.
- An Anti-de Sitter space-time is a special form of hyperbolic curved space, If the light could travel to the limit of space and return in a finite time then is possible to know if the observer is moving or at rest.
- A compact dimension could be periodic by definition and an observer could, in the same way as previously proposed, know whether it is in motion or not. In this respect the work of Brian Greene et al are very interesting[6][7]. They analyze a Brane moving in the compact dimension. Since their point of view is global they concluded that Lorentz symmetry is broken, however as this paper proposes, from a local point of view this is not true, and the observer still perceives the invariance of the interval.

All these methods if they were possible, might permit us to identify a privileged reference frame. If this frame at rest exists then it must respect the principles of special relativity (i.e. speed of light is constant and the laws of physics are the same in all inertial frames of reference including this frame at rest).

However, the existence of this reference frame allows us to measure absolute time and absolute speed. The proper time of the observer at rest will be the longest time for everyone in this space-time[4][5] and the time dilation should be defined as a function of the absolute speed.

It is important to note that the Lorentz transformations are still valid and the space-time remains maximally symmetric. However, an absolute time implies that simultaneity would cease to be relative since there would be a way to define which event occurs first. Since the simultaneity of the reference system at rest is definitive, the superluminal velocities no longer violate causality, and the relativity of simultaneity is only a relativistic effect just like an Einstein ring.

For example the events I'_0 and I'_R are the same event but $Beatrice_0$ perceives they occur at distinct times, it is so because she perceives that the speed of light is constant and not perceive the proper motion. Because of the constant of light speed $Beatrice_0$ says that I'_R occurs before I'_0 but this is only a perception problem.

In order to understand time dilation, it is interesting to reformulate the relativistic interval equation:

$$\tau^2 c^2 = \Delta t^2 c^2 - \Delta x^2$$

$$\Delta t^2 c^2 = \tau^2 c^2 + \Delta x^2 \tag{6}$$

$$\Delta t^2 = \tau^2 + \frac{\Delta x^2}{c^2}$$

Where we could now define Δt as the time between events measured by the observer at absolute rest, we will rename T_u (Universal Time) and depends of the absolute distance and absolute speed of the moving observer. τ is the proper time of the moving observer and depends on the absolute distance and his absolute speed. And $\frac{\Delta x}{c}$ is the time it takes a massless particle moving at lightspeed to travel the distance x, we will rename T_m (Time massless particle) and depends of the absolute distance. So the equation 6 is now:

$$T_u^2(x, \dot{x}) = \tau^2(x, \dot{x}) + T_m^2(x) \tag{7}$$

As we see, the universal time measured by the observer at rest splits between the proper time and the time its take to the particle to cover the distance x at speedlight. We might speculate that time dilation is a gap in the time product of motion. Perhaps it is because the perception of time is the product of a discrete interaction that delays the particle and gives it mass (i.e. Brout–Englert–Higgs interaction in which the massless particles: photons, gravitons do not participate, so these particles do not perceive time at all) so the observer only perceives the time in which occur the interaction, which is shorter as the observer's absolute speed increases just as the wavelength of its particles is shorter. Because of that the absolute motion will add these gaps between interactions decreasing the perceived time by the observer, even though the infinitesimal size of this gap prevents his detection.

In summary the observer only perceives the time during Higgs interaction the rest of the time the particle moves at the speed of light and does not perceive time at all.

Finally another interesting consequence of Lorentz transformations is that the extension of the universe is larger in a γ factor ("γD" in equation 2) and for an observer in an accelerated reference frame this would translate into seeing an expanding space[8]. It is possible that the equivalence principle could explain in part the accelerated expansion of our universe also as a relativistic effect.

We believe that this privileged reference frame probably exists in our present universe, although we lack a mechanism to recognize it yet.

References

[1] Galina Weinstein. *Einstein's Clocks and Langevin's Twins.* 2012. DOI: https://doi.org/10.48550/arXiv.1205.0922. URL: https://arxiv.org/.

[2] Javier Garcia. *Ejercicio de RG propuesto por el suscriptor Rodolfo Pérez.* 2022. URL: https://www.youtube.com/watch?v=SRLN-nM4lQE&t=339s.

[3] Javier Garcia. *Paradoja de los Gemelos en el universo estático de Einstein ('Rodolfo Pérez Paradox').* 2022. URL: https://www.youtube.com/watch?v=vsjg2ybVcoM.

[4] Tevian Dray. "The twin paradox revisited." In: *American Journal of Physics* 58 (1990), pp. 822–825.

[5] Jeffrey R. Weeks. "The Twin Paradox in a Closed Universe." In: *Mathematical Association of America* 108.7 (2001), pp. 585–590.

[6] Brian Greene et al. *Superluminal Propagation on a Moving Braneworld.* 2022. DOI: https://doi.org/10.48550/arXiv.2206.13590. URL: https://arxiv.org/.

[7] Brian Greene et al. *Back to the Future: Causality on a Moving Braneworld.* 2023. DOI: https://doi.org/10.48550/arXiv.2208.09014. URL: https://arxiv.org/.

[8] P. C. Peters. "Periodic boundary conditions in special relativity". In: *American Journal of Physics* 51 (1983), pp. 791–795.