

The Zitter Electron Model and the Anomalous Magnetic Moment

Oliver Consa

E-mail: consa@zitter-institute.org

Founding member of the Zitter Institute – <https://www.zitter-institute.org>

Department of Physics and Nuclear Engineering, Universitat Politècnica de Catalunya

Campus Nord, C. Jordi Girona, 1-3, 08034 Barcelona, Spain

September 29, 2026

Abstract: We explore a geometric model of the electron based on internal Zitterbewegung motion. A primary circular motion is combined with a secondary helical motion, producing an effective toroidal structure. Within this framework, we examine the associated electromagnetic fields and the corresponding distributions of energy, momentum, and electric current. These relations are then used to estimate the characteristic dimensions of the model and investigate their effect on the electron's magnetic moment. The model is consistent with several known properties of the electron and yields a magnetic correction of the expected order of magnitude. Rather than replacing the standard quantum description, it is presented as a complementary geometric approach to exploring whether intrinsic electron properties can be understood in terms of underlying internal motion.

1 Introduction

Quantum mechanics (QM) is one of the most successful theories in physics, but its interpretation has been debated since its origins. The Copenhagen interpretation, developed mainly by Bohr and Heisenberg, describes quantum systems probabilistically, with definite outcomes emerging upon measurement. Einstein and other early contributors questioned whether QM was the final description of physical reality and considered the possibility of a deeper, perhaps deterministic, underlying level. These debates remain central to the foundations of quantum mechanics. Without questioning its experimental success, it is therefore natural to ask whether some properties of individual particles might admit a simpler geometric and deterministic description. In this spirit, the present work explores whether such a model can reproduce specific observable properties of the electron.

2 Ring Electron Model

2.1 de Broglie Frequency

In 1924, de Broglie presented his famous doctoral thesis "Research on the Theory of the Quanta" [1], in which he proposed that the electron possesses wave-like properties. His work begins with a simple idea: de Broglie equated Einstein's two well-known expressions for energy, obtaining an extremely high intrinsic frequency for the electron (1.236×10^{20} Hz).

$$E = mc^2 = hf \quad (1)$$

$$f = \frac{mc^2}{h} \quad (2)$$

This relation defines a characteristic frequency and an associated timescale for the electron (8.093×10^{-21} s).

$$T = \frac{1}{f} = \frac{h}{mc^2} \quad (3)$$

2.2 Parson's Magnetron

A few years earlier, in 1915, Parson [2] had proposed a novel model of the electron as a ring-shaped structure, in which a unit charge circulates around the ring, generating a magnetic field. In this model, the electron acts not only as the unit of electric charge but also as the unit of magnetic moment, or magneton. Several notable physicists, including Webster [3] [4], Lewis [5], Grondahl [6] and Compton [7], conducted studies in support of Parson's Ring Electron Model. All of these works were compiled by Allen in 1919 in "The Case for a Ring Electron" [8] and discussed at a meeting of the Physical Society of London:

"There are many reasons why it is preferable to assume that the electron is in the form of a current circuit which can produce magnetic effects. Then the electron, in addition to exerting electrostatic forces, behaves like a small magnet. In its simplest form the magnetic electron may be looked upon as a circular anchor ring of negative electricity which rotates about its axis with a velocity which is certainly large, and is perhaps comparable with that of light."

The most significant study of the Ring Electron was conducted by Compton, who published a series of papers [9] [10] [11] demonstrating that the Compton effect was better explained using Parson's Ring Electron Model than the classical spherical model:

"The phenomena of scattering were found to be quantitatively accounted for, within the probable errors of observation, if the electron was considered to be a flexible ring of electricity with a radius of 2 pm."

2.3 Uniform Circular Motion

We model the electron as a ring in which an electric charge moves at the speed of light in uniform circular motion (UCM). The equations describing this motion are as follows:

Position:

$$\begin{cases} x(t) = R \cos wt \\ y(t) = R \sin wt \end{cases} \quad (4)$$

$$\phi = wt \quad (5)$$

Velocity:

$$\begin{cases} x'(t) = -Rw \sin wt \\ y'(t) = Rw \cos wt \end{cases} \quad (6)$$

$$v_r = Rw = c \quad (7)$$

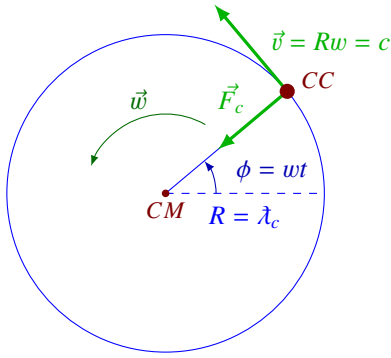


Figure 1: Uniform Circular Motion.

Taking de Broglie's frequency to be the electron's rotational frequency, the radius of the ring matches the reduced Compton wavelength (0.386 pm), and the circumference matches the Compton wavelength (2.426 pm).

$$w = 2\pi f = \frac{mc^2}{\hbar} \quad (8)$$

$$R = \frac{c}{w} = \frac{\hbar}{mc} = \lambda_c \quad (9)$$

$$\ell = 2\pi R = \frac{h}{mc} = \lambda_c \quad (10)$$

This wavelength appears experimentally in the well-known Compton effect.

$$\Delta\lambda = \lambda_c (1 - \cos \theta) \quad (11)$$

If we multiply de Broglie's frequency by the Compton wavelength, we obtain the speed of light. These quantities therefore satisfy the identity.

$$f \lambda_c = c \quad (12)$$

2.4 Center of Mass and Center of Charge

This Ring Electron Model distinguishes the experimentally point-like localization of the electron charge from the much larger Compton scale associated with its internal motion. It also shows several other notable features:

- The electron has a ring radius roughly 500 times larger than the proton radius.
- The mass of the electron is interpreted as arising from its internal energy and is distributed within the surrounding electromagnetic field.
- The Center of Mass (CM) is defined as a single point located at the geometric center of the ring.
- The Center of Charge (CC) is defined as a single point at which the electron's entire electric charge can be considered to be concentrated, analogous to the center of mass of a mass distribution.
- The CC undergoes uniform circular motion at the speed of light along a well-defined ring centered on the CM.
- The CC moves continuously around the CM without energy loss.
- The electron is considered to be at rest when the CM is at rest; in that case, the charge undergoes only rotational motion with no translational motion. The emission or absorption of energy involves acceleration of the CM.
- The electron's ring is modeled as a circular antenna whose resonant wavelength equals its circumference. Assuming propagation at the speed of light, the corresponding resonance frequency coincides with the electron's Compton frequency.
- The point-like character of the CC refers to the localization of the electric charge, while the electromagnetic field associated with the electron extends over a finite surrounding region.

2.5 Electron Magnetic Moment

The electron has a magnetic moment whose magnitude is close to one Bohr magneton (μ_B). Its magnetic properties are evidenced by phenomena such as Zeeman splitting and electron paramagnetic resonance. Within classical electromagnetism, a magnetic moment is associated with an electric current. In the present model, we therefore interpret the electron's magnetic moment as arising from the internal motion of its charge.

In the Ring Electron Model, the motion of the charge generates an electric current of 19.796 A.

$$I = ef = \frac{emc^2}{h} \quad (13)$$

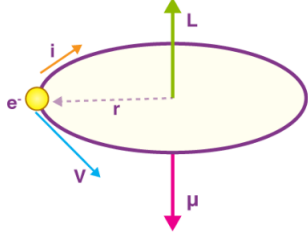


Figure 2: Magnetic Moment.

The circular motion of this current generates a magnetic moment with a magnitude of one Bohr magneton (μ_B).

$$\mu_m = I S = e f \pi R^2 = \frac{e \hbar}{2m} = \mu_B \quad (14)$$

The linear momentum of this circular motion is simply:

$$p = m v_r = m c \quad (15)$$

This linear momentum corresponds to an angular momentum with a value exactly equal to the reduced Planck constant.

$$\vec{L} = \vec{r} \times \vec{p} = m c R = \hbar \quad (16)$$

The ratio between magnetic moment and angular momentum, known as the gyromagnetic ratio, has the value $e/2m$. This value is consistent with the magnetic moment produced by an electric current circulating around a closed loop.

$$\mu_m = \frac{e}{2m} |\vec{L}| \quad (17)$$

The Ring Electron Model reproduces the Bohr magneton but does not by itself reproduce the electron spin-1/2.

2.6 Anomalous Magnetic Moment

The Ring Electron Model derived above gives a magnetic moment equal to one Bohr magneton. Experimentally, however, the electron magnetic moment is slightly larger. This discrepancy is known as the anomalous magnetic moment (AMM).

$$\frac{|\mu_e|}{\mu_B} = 1.00115965218046 \quad (18)$$

The conventional electron g -factor is related to AMM as

$$g_{conv} = 2 \frac{|\mu_e|}{\mu_B} \quad (19)$$

In the literature, the anomaly is commonly written as

$$a_e = \frac{g_{conv} - 2}{2} = \frac{|\mu_e|}{\mu_B} - 1 \quad (20)$$

2.7 Electron Spin and Classical Rotation

In 1926, Uhlenbeck and Goudsmit [12] proposed that the electron possesses an intrinsic self-rotation, now known as spin. This hypothesis had previously been proposed by Kronig, but Pauli rejected it. His argument was straightforward: if the electron has a size equal to or smaller than the classical electron radius, it would have to rotate at over 100 times the speed of light to account for its observed angular momentum.

$$L = m r_e v_r = \hbar \Rightarrow v_r > 100c \quad (21)$$

His classical argument does not apply to the present ring geometry, where the rotational speed is exactly the speed of light, a configuration not considered by Pauli.

Despite his initial objections, in 1927 Pauli accepted the hypothesis of spin. Nevertheless, Pauli maintained that the fourth quantum number could not be described as a geometric property of the electron, referring to it as the "classically non-describable two-valuedness". He regarded the angular momentum and magnetic moment as intrinsic properties of the electron, rather than as arising from an actual classical spinning motion. This interpretation ultimately became the standard view of electron spin.

2.8 Electromagnetic Relations

2.8.1 Resistance Quantum

The energy of the electron is very low, but the frequency of oscillation is extremely large, which defines a characteristic circulating-power scale of about 10 megawatts:

$$P = \frac{m c^2}{T} = h f^2 \quad (22)$$

The electric potential can be calculated as the electron energy per unit of electric charge, resulting in a value of approximately half a million volts:

$$V = \frac{E}{e} = \frac{h f}{e} \quad (23)$$

Multiplying the voltage of about 500 kV by the current of about 20 A, we obtain a power, again, of about 10 megawatts.

$$P = V I = h f^2 \quad (24)$$

Applying Ohm's law, we obtain a value for the impedance of the electron equal to the value of the von Klitzing constant,

$$R_K = \frac{V}{I} = \frac{h}{e^2} \quad (25)$$

This value is observable at the macroscopic level and was not discovered experimentally until 1980.

2.8.2 Magnetic Flux

We define a characteristic magnetic-flux scale associated with the voltage over one rotational period as:

$$\phi_m = VT = \frac{hf}{e} \frac{1}{f} = \frac{h}{e} \quad (26)$$

Within the present model, this relation suggests a description in terms of a quantum of electric charge and an associated magnetic flux satisfying

$$e \phi_m = h \quad (27)$$

2.8.3 Effective LC Analogy

Both the electric current and the voltage of the electron are frequency dependent. This means that the electron can be represented by an effective LC analogy, with a capacitance (C) and a self-inductance (L). We can calculate these coefficients for an electron at rest, obtaining values $L = 2.089 \times 10^{-16} \text{ H}$ and $C = 3.135 \times 10^{-25} \text{ F}$:

$$L = \frac{\phi_m}{I} = \frac{h}{e^2 f} \quad (28)$$

$$C = \frac{e}{V} = \frac{e^2}{hf} \quad (29)$$

By construction, these effective parameters satisfy the following identities:

$$\sqrt{\frac{L}{C}} = \sqrt{\frac{h/e^2 f}{e^2/hf}} = \frac{h}{e^2} = R_K \quad (30)$$

$$\frac{1}{\sqrt{LC}} = \sqrt{\frac{1}{(h/e^2 f)(e^2/hf)}} = f \quad (31)$$

Within the effective LC analogy, the electric energy is one half of the charge multiplied by the voltage, or equivalently, one half of the capacitance multiplied by the square of the voltage. Similarly, the magnetic energy is one half of the magnetic flux linkage multiplied by the current, or equivalently, one half of the inductance multiplied by the square of the current. Using the previously defined capacitance, inductance, voltage, and current, we obtain

$$E_e = \frac{1}{2} eV = \frac{1}{2} CV^2 = \frac{1}{2} hf \quad (32)$$

$$E_m = \frac{1}{2} \phi_m I = \frac{1}{2} LI^2 = \frac{1}{2} hf \quad (33)$$

Thus, within this model, the electron's total energy is equally divided between its electric and magnetic contributions:

$$E = E_e + E_m = hf \quad (34)$$

2.8.4 Vector Potential

The magnetic flux can also be calculated as the integral of the vector potential over a closed curve.

$$\phi_m = \oint \vec{A} \cdot d\vec{l} = 2\pi R |\vec{A}| = \frac{h}{e} \quad (35)$$

In this way, we obtain the value of the vector potential.

$$|\vec{A}| = \frac{mc}{e} \quad (36)$$

Both the magnetic flux and the vector potential calculated for an electron are consistent with the Aharonov–Bohm effect.

$$\Delta\varphi = \frac{e\phi_m}{\hbar} = 2\pi \quad (37)$$

3 Relativistic Helical Motion

3.1 Zitterbewegung

The Dirac equation [13] superseded the Schrödinger equation by introducing a mathematically consistent framework that incorporated both the electron's spin and the principles of special relativity.

$$i\hbar \frac{\partial \psi}{\partial t} = (-i\hbar c \vec{\alpha} \cdot \nabla + \beta mc^2) \psi \quad (38)$$

Schrödinger [14] analyzed the Dirac equation and discovered a surprising result: the equation could be decomposed into a classical motion combined with a rapid oscillatory motion, which he called Zitterbewegung.

$$\hat{x}(t) = \underbrace{\hat{x}(0) + c^2 \hat{p} H^{-1} t}_{\text{Classical}} + \underbrace{\frac{i\hbar c}{2} H^{-1} (\vec{\alpha}(0) - c \hat{p} H^{-1}) e^{-2iHt/\hbar}}_{\text{Zitter}} \quad (39)$$

Assuming $H = mc^2$ and $p = mv$, it is straightforward to see that this oscillatory motion occurs at the speed of light, with a radius on the order of the electron's Compton wavelength.

$$\hat{x}(t) = \underbrace{x_0 + vt}_{\text{Classical}} + \underbrace{\frac{i\lambda_c}{2} (\vec{\alpha}_0 - \beta) e^{-2i\omega t}}_{\text{Zitter}} \quad (40)$$

Although Schrödinger initially believed that Zitterbewegung was merely a mathematical artifact, Dirac considered it to be a real and unavoidable physical effect, as he explained in his 1933 Nobel Lecture [15]:

“It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the

velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

The oscillatory motion known as Zitterbewegung is commonly interpreted as resulting from interference between positive and negative energy states. Although this view remains dominant, a classical interpretation has continued to attract some advocates. In 1952, Kerson Huang [16] proposed an interpretation identical to that previously suggested by the Ring Electron Model:

"It is shown that the well-known Zitterbewegung may be looked upon as a circular motion about the direction of the electron spin, with a radius equal to the Compton wavelength (divided by 2π) of the electron. It is further shown that the intrinsic spin of the electron may be looked upon as the "orbital angular momentum" of this motion. The current produced by the Zitterbewegung is seen to give rise to the intrinsic magnetic moment of the electron."

In the 1980s, several researchers, such as Barut [17] [18] [19] [20] and Hestenes [21], began to take Zitterbewegung seriously as a mechanism for interpreting the Dirac equation classically. Hestenes coined the term Zitter to refer to this interpretation of Zitterbewegung as a real physical phenomenon, and proposed a "Zitterbewegung Interpretation of Quantum Mechanics" [22]. He also used the term "Zitter Electron Model" for any electron model that interprets the Zitter as an actual internal motion at the speed of light.

The Dirac position operator exhibits a factor-of-two difference from the internal motion adopted here: its oscillatory frequency is twice as large and its characteristic length scale contains an additional factor $1/2$. In the present model, we suspect that this factor-of-two structure is related to the electron spin- $1/2$.

3.2 Helical Motion

The electron is considered to be at rest when its Center of Mass (CM) is stationary. In this case, the Center of Charge (CC) undergoes rotational motion without translation. When the CM moves at a constant velocity (v), the CC follows a helical trajectory around the path of the CM.

The electron's helical motion is analogous to the observed motion of an electron in a homogeneous external magnetic field.

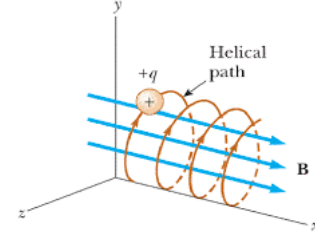


Figure 3: Electron Helical Trajectory.

The direction of the center of mass (CM) displacement is independent of the direction of the angular momentum. However, in this analysis we will restrict ourselves to the case in which the motion is collinear with the angular momentum, since the resulting trajectory is a pure helix and the calculations are greatly simplified.

This motion can be parameterized as:

$$\begin{cases} x(t) = R \cos(\omega t), \\ y(t) = R \sin(\omega t), \\ z(t) = vt. \end{cases} \quad (41)$$

The electron's helical motion can be decomposed into two orthogonal components: a rotational motion and a translational motion. The velocities of rotation and translation are not independent; they are constrained by the electron's total tangential velocity, which remains constant and equal to the speed of light.

Using the Pythagorean theorem, the relationship between these three velocities is:

$$c^2 = v_r^2 + v^2. \quad (42)$$

Then the rotational velocity of the center of charge is:

$$v_r = c \sqrt{1 - (v/c)^2} = c/\gamma. \quad (43)$$

Here, γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}. \quad (44)$$

When the electron is at rest, its rotational velocity is equal to the speed of light. As the translational velocity increases, the rotational velocity must decrease. At no time can the translational velocity exceed the speed of light.

3.3 Canonical Momentum

The decomposition of the motion into total, rotational, and translational contributions leads to the corresponding momentum relations. Multiplying the velocity relation by $(\gamma m)^2$,

$$(\gamma m)^2 c^2 = (\gamma m)^2 v_r^2 + (\gamma m)^2 v^2 \quad (45)$$

and substituting $v_r = c/\gamma$,

$$(\gamma mc)^2 = (mc)^2 + (\gamma mv)^2 \quad (46)$$

We interpret this expression as the canonical linear momentum, composed of rotational and translational components. The translational term is the usual relativistic momentum $p = \gamma mv$, while the rotational term is associated with the vector potential eA :

$$\vec{P}_c = \vec{p}_r + \vec{p}_t = e\vec{A} + \gamma m\vec{v} \quad (47)$$

The total canonical momentum and its associated total wave number are:

$$|\vec{P}_c| = \frac{E}{c} = \gamma mc, \quad k_T = \frac{P_c}{\hbar} = \frac{\gamma mc}{\hbar} \quad (48)$$

3.4 Energy and Wave Components

3.4.1 Relativistic Energy

Multiplying equation (46) by c^2 and using $p = \gamma mv$,

$$(\gamma mc^2)^2 = (mc^2)^2 + (pc)^2 \quad (49)$$

This gives the relativistic energy equation:

$$E = \gamma mc^2 = \sqrt{(mc^2)^2 + (pc)^2} \quad (50)$$

The relativistic energy is conventionally written as the sum of rest and kinetic energies,

$$E = E_0 + E_k \quad (51)$$

$$E_0 = mc^2, \quad E_k = (\gamma - 1) mc^2 \approx \frac{1}{2}mv^2 \quad (52)$$

Alternatively, multiplying the velocity relation by γm ,

$$(\gamma m)c^2 = (\gamma m)v_r^2 + (\gamma m)v^2 \quad (53)$$

and using $v_r = c/\gamma$, the same total relativistic energy can be decomposed into rotational and translational contributions:

$$\gamma mc^2 = \frac{mc^2}{\gamma} + \gamma mv^2 = E_r + E_t \quad (54)$$

Associating each energy with a frequency through $E = hf$,

$$E_T = hf_r + hf_t = hf_T = \gamma mc^2 \quad (55)$$

Therefore,

$$f_T = \frac{\gamma mc^2}{h} \quad (56)$$

and the corresponding wavelength is the relativistic Compton wavelength:

$$f_T \times \lambda_T = c, \quad \lambda_T = \frac{h}{\gamma mc} \quad (57)$$

For $v = 0$ and $\gamma = 1$, these quantities reduce to those of the Ring Electron Model.

	v	E	f	λ	p	k
Rot.	$v_r = c/\gamma$	$\frac{mc^2}{\gamma}$	$\frac{mc^2}{\gamma h}$	$\frac{h}{mc}$	eA	$\frac{1}{\lambda_c}$
Trans.	v	γmv^2	$\frac{\gamma mv^2}{h}$	$\frac{h}{\gamma mv}$	γmv	$\frac{1}{\lambda_B}$
Total	c	γmc^2	$\frac{\gamma mc^2}{h}$	$\frac{h}{\gamma mc}$	γmc	$\frac{\gamma mc}{\hbar}$

Table 1: Rotational and Translational Components.

3.4.2 Rotational Component

The rotational energy defines the frequency

$$f_r = \frac{E_r}{h} = \frac{mc^2}{\gamma h} \quad (58)$$

Using the rotational velocity $v_r = c/\gamma$, the corresponding wavelength is

$$f_r \times \lambda_r = v_r = \frac{c}{\gamma}, \quad \lambda_r = \frac{h}{mc} = \lambda_c \quad (59)$$

Thus, the rotational wavelength remains equal to the Compton wavelength. The rotational momentum and associated wave number are

$$|\vec{p}_r| = eA = mc, \quad k_r = \frac{p_r}{\hbar} = \frac{1}{\lambda_c} \quad (60)$$

Solving for the vector potential reproduces the value obtained previously in equation (36):

$$|\vec{A}| = \frac{mc}{e} \quad (61)$$

3.4.3 Translational Component

The translational energy defines the frequency

$$f_t = \frac{E_t}{h} = \frac{\gamma mv^2}{h} \quad (62)$$

Combining this frequency with the translational velocity gives the de Broglie wavelength:

$$f_t \times \lambda_t = v, \quad \lambda_t = \frac{v}{f_t} = \frac{h}{\gamma mv} = \lambda_B \quad (63)$$

This wavelength is the characteristic scale associated with electron diffraction, as observed in the Davisson–Germer experiment. The translational momentum and its corresponding wave number are

$$|\vec{p}_t| = \gamma mv, \quad k_t = \frac{p_t}{\hbar} = \frac{1}{\lambda_B} \quad (64)$$

3.5 Helical Pitch

Although all these frequencies and wavelengths may manifest physically under certain conditions, they do not necessarily correspond to actual geometric frequencies or lengths of the electron.

If the electron moves at a constant velocity along a direction collinear with its angular momentum, its trajectory takes the form of a cylindrical helix. The geometry of the helix is defined by two constant parameters: the radius of the helix (R) and the helical pitch (H).

The frequency of this helix matches the rotational frequency calculated previously ($f_r = mc^2/\gamma h$). The radius of the helix remains constant at ($R = \hbar/mc$), while the helical pitch corresponds to the distance between two turns of the helix.

$$H = \frac{v}{f_r} = v \frac{\gamma h}{mc^2} = \gamma \beta \lambda_c. \quad (65)$$

4 Local Maxwell Field Construction

4.1 Characteristic Fields and Zitter Force

4.1.1 Fields at the Center of Mass

In the Ring Electron Model, the Biot-Savart law can be applied to calculate the magnetic field at the center of the ring, resulting in a magnetic field of 30 million tesla, comparable to magnetic fields found in neutron stars. For comparison, the magnetic field of the Earth is $50 \mu T$, and non-destructive laboratory magnetic fields can reach about $100 T$.

$$B_0 = \frac{\mu_0 I}{2R} = \alpha \frac{m^2 c^2}{e \hbar} = 3.221 \times 10^7 T \quad (66)$$

At any instant, the localized CC lies at a distance R from the CM. The corresponding instantaneous electric-field magnitude at the CM is therefore:

$$E_0 = \frac{1}{4\pi\epsilon_0} \frac{e}{R^2} = \alpha \frac{m^2 c^3}{e \hbar} = 9.656 \times 10^{15} V/m \quad (67)$$

The electric field at the center of the ring matches the value of the magnetic field multiplied by the speed of light ($E = cB$).

4.1.2 Zitter Force

The Ring Electron Model implies the existence of a centripetal acceleration and a centripetal force of $0.212 N$. This Zitter force provides the inward force required to sustain the circular motion.

$$a_c = \frac{v_r^2}{R} = \frac{c^2}{R} = \frac{mc^3}{\hbar} \quad (68)$$

$$F_c = ma = \frac{m^2 c^3}{\hbar} \quad (69)$$

Since we have consistently assumed that the electron is governed solely by electromagnetic fields, the Zitter force must be electromagnetic in origin and described by the Lorentz force. We therefore define characteristic electric and magnetic fields that independently reproduce its magnitude.

$$F_c = eE_s = ecB_s. \quad (70)$$

The corresponding characteristic field strengths are therefore:

$$E_s = \frac{|\vec{F}_c|}{e} = \frac{m^2 c^3}{e \hbar} = 1.323 \times 10^{18} V/m \quad (71)$$

$$B_s = \frac{|\vec{F}_c|}{ec} = \frac{m^2 c^2}{e \hbar} = 4.414 \times 10^9 T \quad (72)$$

These characteristic field scales coincide algebraically with the Schwinger critical fields in Quantum Electrodynamics [23]. They also match the values of E and B at the center of the ring, scaled by the fine-structure constant.

$$E_0 = \alpha E_s, \quad B_0 = \alpha B_s \quad (73)$$

4.1.3 Fields at the Center of Charge

We now construct a local electromagnetic field in the neighborhood of the Center of Charge (CC). The characteristic field amplitudes obtained in the previous section are

$$B_s = \frac{mc}{eR} \quad E_s = cB_s = \frac{mc^2}{eR} \quad (74)$$

These quantities will be interpreted as characteristic local electromagnetic field amplitudes associated with the Zitter motion. Equation (70) therefore defines two equivalent electromagnetic field scales, rather than two independent force contributions that must simultaneously produce the centripetal force. A complete dynamical description of the self-force responsible for maintaining the internal motion is not required for the local electromagnetic construction developed below and remains an open question. We will show that the characteristic amplitudes E_s and B_s can arise from conventional electromagnetic potentials satisfying Maxwell's equations locally in the neighborhood of the circular trajectory.

The Center of Charge moves along the circular trajectory

$$\rho(t) = R, \quad \phi(t) = \omega t, \quad z(t) = 0. \quad (75)$$

We introduce the rotating phase

$$\Psi(\phi, t) = \phi - \omega t. \quad (76)$$

The factor $e^{i\Psi}$ represents a field pattern rotating around the z -axis with angular frequency ω . In contrast with a derivative evaluated along the trajectory of the CC, all temporal derivatives in the following analysis are conventional partial

derivatives evaluated at fixed spatial coordinates. The complex notation is used for convenience. Since Maxwell's equations are linear, the real and imaginary parts separately define real electromagnetic fields. Physical quantities that depend on field amplitudes will be expressed in terms of E_s and B_s . The construction is intended locally in a tubular neighborhood of the trajectory $\rho = R$. A complete smooth continuation of the fields throughout all space is not assumed at this stage.

4.2 Electromagnetic Potentials

4.2.1 Vector Potential

We introduce a vector potential collinear with the tangential velocity $\vec{v} = c \hat{\phi}$.

$$\vec{A} = A_s e^{i\Psi} \hat{\phi} \quad (77)$$

Using $R = \hbar/mc$, this amplitude satisfies

$$A_s = R B_s, \quad \omega A_s = E_s \quad (78)$$

The magnetic field follows from the conventional relation

$$\vec{B} = \nabla \times \vec{A} \quad (79)$$

Since the vector potential has only an azimuthal component,

$$\vec{B} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\phi) \hat{z} = \frac{A_s}{\rho} e^{i\Psi} \hat{z} \quad (80)$$

Then,

$$\vec{B} = B_s \frac{R}{\rho} e^{i\Psi} \hat{z} \quad (81)$$

On the central circular trajectory, $\rho = R$, the field amplitude is therefore $|\vec{B}| = B_s$.

4.2.2 Scalar Potential

We introduce the scalar potential

$$V = E_s \rho e^{i\Psi} \quad (82)$$

Its gradient is

$$\nabla V = E_s e^{i\Psi} \hat{\rho} + i E_s e^{i\Psi} \hat{\phi} \quad (83)$$

The partial time derivative of the vector potential is

$$\frac{\partial \vec{A}}{\partial t} = -i \omega A_s e^{i\Psi} \hat{\phi} = -i E_s e^{i\Psi} \hat{\phi} \quad (84)$$

The electric field is now obtained from the conventional electromagnetic potential relation

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla V \quad (85)$$

Substituting the previous expressions, the two azimuthal terms cancel exactly,

$$\vec{E} = i E_s e^{i\Psi} \hat{\phi} - E_s e^{i\Psi} \hat{\rho} - i E_s e^{i\Psi} \hat{\phi} \quad (86)$$

so that

$$\vec{E} = -E_s e^{i\Psi} \hat{\rho} \quad (87)$$

The electric field is therefore purely radial. On the central trajectory $|\vec{E}| = E_s$, and consequently

$$\frac{|\vec{E}|}{|\vec{B}|} = \frac{E_s}{B_s} = c \quad (88)$$

The amplitudes of the potentials on the central trajectory are

$$|\vec{A}|_{\rho=R} = \frac{mc}{e}, \quad |V|_{\rho=R} = E_s R = \frac{mc^2}{e} \quad (89)$$

We recover the value of the vector potential previously obtained from the magnetic flux in equation (36) and independently from the linear momentum in equation (61).

4.2.3 Lorenz Gauge Condition

We calculate the divergence of the vector potential and the temporal derivative of the scalar potential (voltage).

$$\nabla \cdot \vec{A} = \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} = i \frac{A_s}{\rho} e^{i\Psi} \quad (90)$$

$$\frac{1}{c^2} \frac{\partial V}{\partial t} = -i \frac{\omega E_s \rho}{c^2} e^{i\Psi} \quad (91)$$

Therefore, on the central trajectory $\rho = R$, the potentials satisfy the Lorenz gauge condition, although not throughout the surrounding local region in this particular gauge.

$$\nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} = 0 \quad (92)$$

The present potentials therefore do not satisfy the Lorenz gauge throughout the full local domain. This is a property of the chosen potentials rather than of the electromagnetic fields themselves; the fields below are determined directly from the gauge-invariant relations between V , A , E and B .

4.3 Poynting Vector

The electric field is radial and the magnetic field is axial. The electromagnetic energy flux is therefore tangential. Using the characteristic field amplitudes,

$$|\vec{S}| = \frac{1}{\mu_0} |\vec{E}| |\vec{B}| \quad (93)$$

From the local fields,

$$|\vec{S}(\rho)| = \frac{E_s B_s R}{\mu_0 \rho}. \quad (94)$$

On the central trajectory,

$$|\vec{S}|_{\rho=R} = \frac{E_s B_s}{\mu_0} \quad (95)$$

The direction is tangential to the circular trajectory. With the orientation adopted here,

$$\vec{S}_s = \frac{E_s B_s}{\mu_0} \hat{\phi} \quad (96)$$

Reversing the orientation of the internal circulation reverses the sign of $\hat{\phi}$ but leaves all scalar quantities unchanged.

Using $E_s = cB_s$, the characteristic electromagnetic energy density on the central trajectory is

$$u_s = \frac{1}{2} \epsilon_0 E_s^2 + \frac{1}{2\mu_0} B_s^2 = \epsilon_0 E_s^2 \quad (97)$$

Using the fine-structure constant and $R = \hbar/mc$, this can be written as

$$u_s = \frac{mc^2}{4\pi\alpha R^3} \quad (98)$$

Similarly, the characteristic energy flux is

$$|\vec{S}_s| = \frac{mc^3}{4\pi\alpha R^3}. \quad (99)$$

4.4 Local Electromagnetic Relations

We now verify the local electromagnetic relations generated by the potentials introduced above.

4.4.1 Gauss's Law for Magnetism

The simplest of Maxwell's equations to verify is Gauss's law for magnetism.

$$\nabla \cdot \vec{B} = 0 \quad (100)$$

We verify that the law is satisfied since the magnetic field has only a z-component, and its derivative with respect to z is zero.

$$\nabla \cdot \vec{B} = \frac{\partial B_z}{\partial z} = 0 \quad (101)$$

4.4.2 Faraday's Law

Since the electric field has only a radial component, its curl has only a z component,

$$\nabla \times \vec{E} = -\frac{1}{\rho} \frac{\partial E_\rho}{\partial \phi} \hat{z} = i \frac{E_s}{\rho} e^{i\Psi} \hat{z} \quad (102)$$

On the other hand,

$$-\frac{\partial \vec{B}}{\partial t} = i\omega B_s \frac{R}{\rho} e^{i\Psi} \hat{z} = i \frac{E_s}{\rho} e^{i\Psi} \hat{z} \quad (103)$$

Hence,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (104)$$

Faraday's law is therefore satisfied throughout the local domain, not only on the trajectory of the Center of Charge.

4.4.3 Gauss's Law for Electricity

Gauss's law for electricity is

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_0} \quad (105)$$

The divergence of the electric field is

$$\nabla \cdot \vec{E} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho E_\rho) = -\frac{E_s}{\rho} e^{i\Psi} \quad (106)$$

The corresponding effective local charge density is therefore

$$\rho_e(\rho, \phi, t) = -\epsilon_0 \frac{E_s}{\rho} e^{i\Psi} \quad (107)$$

On the central circular trajectory,

$$|\rho_e|_{\rho=R} = \epsilon_0 \frac{E_s}{R} = \epsilon_0 \frac{mc^2}{eR^2} \quad (108)$$

4.4.4 Ampère–Maxwell Law

The Ampère–Maxwell equation is

$$\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J} \quad (109)$$

The curl of the magnetic field is

$$\nabla \times \vec{B} = i \frac{A_s}{\rho^2} e^{i\Psi} \hat{\rho} + \frac{A_s}{\rho^2} e^{i\Psi} \hat{\phi} \quad (110)$$

The temporal derivative of the electric field is

$$\frac{\partial \vec{E}}{\partial t} = i\omega E_s e^{i\Psi} \hat{\rho} = i \frac{c^2 A_s}{R^2} e^{i\Psi} \hat{\rho} \quad (111)$$

the Ampère–Maxwell equation gives

$$\vec{J} = \frac{A_s}{\mu_0} e^{i\Psi} \left[i \left(\frac{1}{\rho^2} - \frac{1}{R^2} \right) \hat{\rho} + \frac{1}{\rho^2} \hat{\phi} \right] \quad (112)$$

The local current therefore has radial and tangential components away from the central trajectory. However, the radial component vanishes exactly at the Center of Charge trajectory ($\rho = R$),

$$\vec{J}(R, \phi, t) = \frac{A_s}{\mu_0 R^2} e^{i\Psi} \hat{\phi} = \frac{mc}{\mu_0 e R^2} e^{i\Psi} \hat{\phi} \quad (113)$$

Combining this expression with the effective charge density derived above,

$$\rho_e(R, \phi, t) = -\epsilon_0 \frac{E_s}{R} e^{i\Psi} \quad (114)$$

and using

$$\frac{1}{\mu_0 \epsilon_0} = c^2 \quad (115)$$

we obtain

$$\vec{J}(R, \phi, t) = -\rho_e(R, \phi, t)c \hat{\phi} \quad (116)$$

Thus, on the central trajectory, the effective current-density amplitude is equal to the effective charge-density amplitude multiplied by the speed of light.

Furthermore, because the electric field is radial and the current density is purely tangential on the central trajectory,

$$\vec{J} \cdot \vec{E} \Big|_{\rho=R} = 0 \quad (117)$$

There is therefore no local Ohmic energy dissipation on the central circular trajectory.

4.4.5 Charge Continuity Equation

The effective source densities obtained from Gauss's law and the Ampère–Maxwell equation must satisfy the charge continuity equation,

$$\nabla \cdot \vec{J} + \frac{\partial \rho_e}{\partial t} = 0 \quad (118)$$

The divergence of the current density obtained above is,

$$\nabla \cdot \vec{J} = -i \frac{A_s}{\mu_0 \rho R^2} e^{i\Psi} \quad (119)$$

The temporal derivative of the effective charge density is

$$\frac{\partial \rho_e}{\partial t} = i \epsilon_0 \frac{\omega E_s}{\rho} e^{i\Psi} \quad (120)$$

Since

$$\epsilon_0 \omega E_s = \frac{A_s}{\mu_0 R^2} \quad (121)$$

we obtain

$$\frac{\partial \rho_e}{\partial t} = i \frac{A_s}{\mu_0 \rho R^2} e^{i\Psi} \quad (122)$$

Therefore,

$$\nabla \cdot \vec{J} + \frac{\partial \rho_e}{\partial t} = 0 \quad (123)$$

The effective charge and current densities therefore satisfy the conventional charge continuity equation throughout the local electromagnetic domain. The present construction provides a local spacetime-dependent electromagnetic field satisfying Maxwell's equations with ordinary partial derivatives. On the central trajectory it reproduces the characteristic quantities

$$|\vec{E}| = E_s, \quad |\vec{B}| = B_s \quad (124)$$

$$|\rho_e| = \epsilon_0 \frac{mc^2}{eR^2}, \quad |\vec{J}| = \frac{mc}{\mu_0 e R^2} \quad (125)$$

together with

$$u_s = \frac{mc^2}{4\pi\alpha R^3}, \quad |\vec{S}_s| = \frac{mc^3}{4\pi\alpha R^3} \quad (126)$$

These provide the characteristic local electromagnetic scales used in the toroidal extension developed in the following section. A complete global electromagnetic solution, including smooth spatial localization, boundary conditions, self-force balance, and the stability of the internal motion, is not established by this local construction and remains a subject for further investigation.

5 Helical Solenoid Electron Model

5.1 Secondary Poloidal Motion

The Ring Electron Model, in its present form, does not derive the electron's anomalous magnetic moment (AMM) or account for its spin-1/2 value. To address these limitations, we introduce an additional postulate: a second rotational motion with a radius much smaller than the Compton wavelength, producing a secondary helical trajectory.

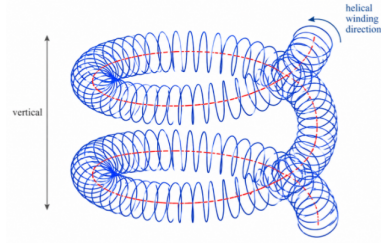


Figure 4: Helical Solenoid Electron Model.

For an electron at rest, this additional motion transforms the geometry from a simple ring into a toroidal solenoid. For a moving electron, the helical trajectory becomes a helical solenoid. We refer to this extended model as the Helical Solenoid Electron Model [24].

5.2 Toroidal Geometry

A toroidal solenoid is characterized by a major radius R , a minor radius r , and a winding ratio N . Compared with the ring geometry, it therefore introduces two additional parameters: r and N . The position of the charge on the toroidal surface can be parameterized using two angular coordinates:

$$\begin{cases} x(t) = (R + r \cos \psi) \cos \theta \\ y(t) = (R + r \cos \psi) \sin \theta \\ z(t) = r \sin(\psi) \end{cases} \quad (127)$$

Here, $\theta(t)$ denotes the toroidal angular position and $\psi(t)$ the poloidal angular position. The winding ratio N specifies the number of poloidal revolutions per toroidal revolution and is defined by

$$\psi(t) = N\theta(t) \quad (128)$$

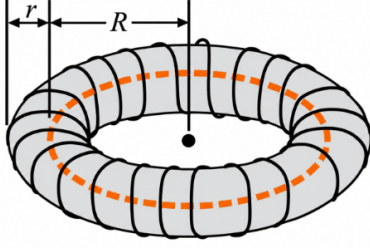


Figure 5: Toroidal solenoid for integer N .

Substituting this relation into the toroidal parameterization gives

$$\begin{cases} x(t) = (R + r \cos N\theta) \cos \theta \\ y(t) = (R + r \cos N\theta) \sin \theta \\ z(t) = r \sin(N\theta) \end{cases} \quad (129)$$

Since N is constant, differentiation with respect to time gives

$$N = \frac{\dot{\psi}(t)}{\dot{\theta}(t)} \quad (130)$$

Thus, although the instantaneous toroidal and poloidal angular velocities may vary along the trajectory, they vary proportionally and their ratio remains constant. Consequently, N remains well defined independently of the variation of the instantaneous angular velocities.

If N were an integer, the trajectory closes after one complete revolution around the major axis of the torus, with N revolutions around the minor cross-section. More generally, if N were rational, $N = n/m$, the trajectory is periodic and closes after m toroidal revolutions and n poloidal revolutions.

We postulate the winding ratio

$$N = \frac{1}{\sqrt{2}}, \quad \text{so that} \quad N^2 = \frac{1}{2} \quad (131)$$

As N is irrational, the trajectory never closes and is dense on the toroidal surface, approaching every point arbitrarily closely.

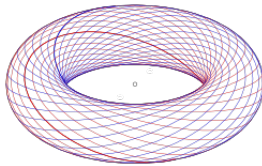


Figure 6: Toroidal solenoid for irrational N .

5.3 Uniform Toroidal Charge Distribution

Since N is irrational, the Center of Charge (CC) follows a non-periodic trajectory that is dense on the toroidal surface.

For the electromagnetic estimates developed below, we approximate the long-time charge distribution by a uniform toroidal distribution. We further represent this distribution by an equivalent uniform volumetric charge density that preserves the total electric charge.

The torus is defined by its major radius $R = \lambda_c$ and minor radius r .

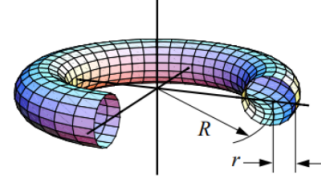


Figure 7: Toroidal Electron Geometry.

The volume of the torus is calculated as the area of the minor circle multiplied by the circumference with radius R :

$$A = \pi r^2, \quad V = 2\pi R \pi r^2 \quad (132)$$

5.4 The Minor Radius

Using the volume of the torus together with the effective electromagnetic quantities derived in Section 4, the minor radius r can be evaluated using five related electromagnetic quantities: (i) charge density, (ii) current density, (iii) energy density, (iv) energy flux density, and (v) momentum density. Since these quantities are mutually related, these calculations should not be regarded as five independent determinations of r , but rather as internal consistency checks of the model.

5.4.1 Charge Density

We can compute the charge density by dividing the total charge by the volume of the torus:

$$\rho_e = \frac{e}{V} = \frac{e}{2\pi R \pi r^2} \quad (133)$$

By equating this result to the charge density calculated in (125), we can calculate the minor radius of the torus.

$$|\rho_e| = \frac{e}{2\pi R \pi r^2} = \epsilon_0 \frac{mc^2}{eR^2} \quad (134)$$

The resulting radius is 26.32 fm, approximately nine times the classical electron radius

$$r = R \sqrt{\frac{2\alpha}{\pi}} \quad (135)$$

5.4.2 Current Density

Similarly, we can calculate the current density by dividing the total electric current by the area of the toroid's circular cross-section.

$$|\vec{J}| = \frac{I}{A} = \frac{ef}{\pi r^2} = \frac{ec}{2\pi R \pi r^2} \quad (136)$$

By equating this result to the current density calculated in (113), we can calculate the minor radius of the torus.

$$|\vec{J}| = \frac{ec}{2\pi R \pi r^2} = \frac{mc}{\mu_0 e R^2} \quad (137)$$

As expected from the relation between charge density and current density obtained previously, this calculation yields the same value of r .

$$r = R \sqrt{\frac{2\alpha}{\pi}} \quad (138)$$

5.4.3 Energy Density

Using the standard electromagnetic energy-density expression, we obtain:

$$u = \frac{1}{2} \epsilon_0 |\vec{E}|^2 + \frac{1}{2\mu_0} |\vec{B}|^2 \quad (139)$$

$$u = \frac{mc^2}{4\pi\alpha R^3} \quad (140)$$

We can also calculate the energy density by dividing the electron's total energy by the volume of the torus.

$$u = \frac{E}{V} = \frac{mc^2}{2\pi R \pi r^2} \quad (141)$$

By equating these two values, we can calculate the minor radius of the torus.

$$u = \frac{mc^2}{4\pi\alpha R^3} = \frac{mc^2}{2\pi R \pi r^2} \quad (142)$$

The resulting value of r is consistent with that obtained from the charge and current densities.

$$r = R \sqrt{\frac{2\alpha}{\pi}} \quad (143)$$

5.4.4 Energy Flux Density

We can calculate the power per unit area by dividing the electron's energy by the rotation period and by the circular cross-sectional area of the torus.

$$\frac{E}{T \pi r^2} = \frac{mc^2}{\pi r^2} \frac{1}{T} = \frac{mc^2}{\pi r^2} \frac{mc^2}{h} = \frac{m^2 c^4}{h \pi r^2} \quad (144)$$

The Poynting vector represents the power flux of an electromagnetic field, that is, the power per unit area. By equating the magnitude of the Poynting vector obtained in (126) with this energy flux density, the minor radius of the torus can be determined.

$$|\vec{S}| = \frac{mc^3}{4\pi\alpha R^3} = \frac{m^2 c^4}{h \pi r^2} \quad (145)$$

Since the Poynting vector is constructed from the same electromagnetic fields used to calculate the energy density, the resulting value of r provides an additional consistency check and agrees with the previous calculations.

$$r = R \sqrt{\frac{2\alpha}{\pi}} \quad (146)$$

5.4.5 Momentum Density

Finally, we can calculate the linear momentum density from the Poynting vector using the formula:

$$p_{vol} = \frac{|\vec{S}|}{c^2} = \frac{mc}{4\pi\alpha R^3} \quad (147)$$

On the other hand, the characteristic circulating momentum-density magnitude is obtained by dividing the local momentum magnitude mc by the torus volume:

$$p_{vol} = \frac{|\vec{p}|}{V} = \frac{mc}{2\pi R \pi r^2} \quad (148)$$

By equating both expressions, we obtain the same value. This agreement provides another algebraic consistency check.

$$\frac{mc}{4\pi\alpha R^3} = \frac{mc}{2\pi R \pi r^2} \quad (149)$$

$$r = R \sqrt{\frac{2\alpha}{\pi}} \quad (150)$$

6 Geometric Estimate of the AMM

6.1 Toroidal Geometric Factor

The electric current has two components, a toroidal component (blue) and a poloidal component (red).

The toroidal component produces an axial magnetic moment, while the poloidal component generates a secondary magnetic moment tangential to the ring.

$$\vec{\mu}_e = \mu_\phi \hat{\phi} + \mu_m \hat{z} \quad (151)$$

The secondary magnetic moment, tangential to the ring, vanishes in the long-time average by symmetry and therefore produces no net time-averaged external dipole moment.

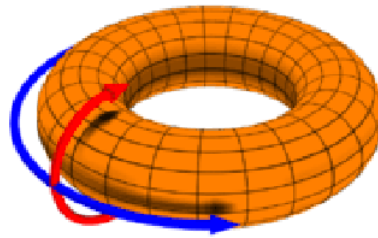


Figure 8: Toroidal and Poloidal currents.

The axial magnetic moment can be evaluated geometrically over many toroidal revolutions [25]. Since $ds = v dt$, the result is independent of the instantaneous angular velocity with which the trajectory is traversed. For M toroidal revolutions, the mean axial contribution per revolution is

$$\langle \mu_m \rangle = \frac{I}{2} \lim_{M \rightarrow \infty} \frac{1}{M} \int_0^{2\pi M} (R + r \cos N\theta)^2 d\theta \quad (152)$$

Expanding the integrand,

$$(R + r \cos N\theta)^2 = R^2 + 2Rr \cos N\theta + r^2 \cos^2 N\theta \quad (153)$$

In the long-time limit, for $N \neq 0$,

$$\langle \cos N\theta \rangle = 0, \quad \langle \cos^2 N\theta \rangle = \frac{1}{2}. \quad (154)$$

Therefore,

$$\langle \mu_m \rangle = \frac{I}{2} [2\pi R^2 + \pi r^2], \quad (155)$$

since $I = ef$,

$$\langle \mu_m \rangle = ef \pi R^2 \left[1 + \frac{1}{2} \left(\frac{r}{R} \right)^2 \right] \quad (156)$$

The result coincides with that of a simple current loop, but includes an additional time-averaged contribution proportional to the square of the ratio between the minor and major radii of the torus. We refer to this geometry-dependent correction as the "toroidal geometric factor", which quantifies the anomalous contribution arising from the solenoidal geometry.

$$g_T = \left[1 + \frac{1}{2} \left(\frac{r}{R} \right)^2 \right] \quad (157)$$

For simplicity, in the following we denote the averaged axial magnetic moment simply by μ_m .

$$\mu_m = ef \pi R^2 g_T \quad (158)$$

6.2 Helical Geometric Factor

Differentiating the position vector with respect to time yields the velocity components:

$$\begin{cases} x'(t) = -(R + r \cos N\theta) \dot{\theta} \sin(\theta) - rN\dot{\theta} \sin(N\theta) \cos(\theta) \\ y'(t) = (R + r \cos N\theta) \dot{\theta} \cos(\theta) - rN\dot{\theta} \sin(N\theta) \sin(\theta) \\ z'(t) = rN\dot{\theta} \cos(N\theta) \end{cases} \quad (159)$$

We postulate that the tangential speed is always equal to the speed of light, so

$$|\vec{v}(t)|^2 = c^2 = \left(\frac{d\theta}{dt} \right)^2 [(R + r \cos N\theta)^2 + (rN)^2] \quad (160)$$

If the geometrical parameters R , r , and N are kept constant and the total tangential velocity remains constant and equal to the speed of light, the instantaneous angular velocity must vary along the toroidal trajectory. We define that instantaneous angular velocity as

$$w_{\text{inst}}(\theta) = \frac{d\theta}{dt} = \frac{c}{\sqrt{(R + r \cos N\theta)^2 + (rN)^2}} \quad (161)$$

Then,

$$dt = \frac{1}{c} \sqrt{(R + r \cos N\theta)^2 + (rN)^2} d\theta \quad (162)$$

Although the instantaneous angular velocity varies continuously, the long-time mean angular velocity remains well defined and constant.

$$w = \frac{c/R}{\frac{1}{2\pi} \int_0^{2\pi} \sqrt{\left(1 + \frac{r}{R} \cos\phi\right)^2 + \left(\frac{rN}{R}\right)^2} d\phi} \quad (163)$$

We define the "helical geometric factor" as the ratio between the speed of light and the rotational speed in this toroidal geometry.

$$g_H = \frac{c}{v_r} \quad (164)$$

Since $v_r = Rw$, we obtain

$$g_H = \frac{c}{Rw} = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{\left(1 + \frac{r}{R} \cos\phi\right)^2 + \left(\frac{rN}{R}\right)^2} d\phi \quad (165)$$

The rotational velocity depends on this helical geometric factor and is slightly lower than the speed of light:

$$v_r = c/g_H \quad (166)$$

The frequency in the toroidal geometry is also slightly lower than on the ring geometry,

$$f = \frac{v_r}{2\pi R} = \frac{mc^2}{g_H h} \quad (167)$$

Combining the frequency obtained in Eq. (167) with the toroidal magnetic moment defined in Eq. (158), we obtain an expression that depends on both the helical and toroidal geometric factors.

$$\mu_m = ef \pi R^2 g_T = \mu_B \frac{g_T}{g_H} \quad (168)$$

6.3 Normalized Magnetic Moment

In Quantum Electrodynamics (QED), the electron's anomalous magnetic moment arises from radiative corrections. In our model, we explore whether a similar correction can emerge from the electron's internal geometry. To evaluate the normalized magnetic moment, we use the minor-to-major radius ratio derived in Section 5 and the postulated winding ratio:

$$\frac{r}{R} = \sqrt{\frac{2\alpha}{\pi}}, \quad N^2 = \frac{1}{2} \quad (169)$$

Substituting these parameters into Eqs. (157) and (165) gives the following geometric factors:

$$g_T = 1 + \frac{1}{2} \left(\frac{r}{R} \right)^2 = 1 + \frac{\alpha}{\pi} \approx 1.0023228195 \quad (170)$$

$$g_H = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{\left(1 + \frac{r}{R} \cos\phi\right)^2 + \frac{1}{2} \left(\frac{r}{R}\right)^2} d\phi \quad (171)$$

$$g_H \approx 1.0011634338 \quad (172)$$

The resulting magnetic moment is slightly greater than one Bohr magneton.

$$\frac{|\mu_e|}{\mu_B} = \frac{g_T}{g_H} \approx 1.0011580384 \quad (173)$$

Source / Model	$ \mu_e /\mu_B$
CODATA 2022	1.0011596522
This model: g_T/g_H	1.0011580384

Table 2: Theoretical and experimental values.

An interesting relationship emerges between the winding ratio N and the leading-order QED correction to the electron's magnetic moment, known as the Schwinger term. To investigate this relationship, we expand the geometric factors g_T and g_H for a small minor-to-major radius ratio $r/R \ll 1$. Expanding the integral expression for the helical factor to second order gives

$$g_H = 1 + \frac{N^2}{2} \left(\frac{r}{R}\right)^2 + O\left(\frac{r^4}{R^4}\right) \quad (174)$$

Consequently, the normalized magnetic moment becomes

$$\frac{|\mu_e|}{\mu_B} = \frac{g_T}{g_H} = 1 + (1 - N^2) \frac{\alpha}{\pi} + O(\alpha^2) \quad (175)$$

Choosing $N = 1/\sqrt{2}$, as postulated in Section 5.2, makes this leading geometric term coincide with the first-order QED correction calculated by Schwinger,

$$a_e^{(1)} = \frac{\alpha}{2\pi} \quad (176)$$

Our approach offers a transparent geometric interpretation and a comparatively simple derivation, suggesting that alternative theoretical perspectives on the electron's magnetic moment may still be physically informative.

6.4 Limitations and Open Questions

Although the model provides a geometric estimate of the electron's anomalous magnetic moment, several questions remain open. The electromagnetic construction of Section 4 satisfies Maxwell's equations locally, but a complete global solution including spatial localization, self-interaction, force balance, stability, and the absence of radiative losses has not yet been established. The connection between the effective local

source densities and the coarse-grained toroidal distribution also remains a modeling assumption. In addition, the value $N^2 = 1/2$ is selected because it reproduces the leading-order Schwinger correction. Its physical origin, and its possible relation to the electron's spin-1/2 structure, remain unknown.

References

- [1] L. de Broglie. Recherches sur la théorie des quanta. *Annales de Physique*, 3:22–128, 1925.
- [2] A.L. Parson. A magneton theory of the structure of the atom. *Smithsonian Miscellaneous Collections*, 65:2–80, 1915.
- [3] D.L. Webster. The theory of electromagnetic mass of the parson magneton. *Phys. Rev.*, 9(6):484–499, 1917.
- [4] D.L. Webster. The scattering of alpha rays as evidence on the parson magneton hypothesis. *J. Am. Chem. Soc.*, 40:375–379, 1918.
- [5] G.N. Lewis. The atom and the molecule. *J. Am. Chem. Soc.*, 38:762–785, 1916.
- [6] L.O. Grondahl. Experimental evidence for the parson magneton. *Phys. Rev.*, 10:586–588, 1917.
- [7] A.H. Compton. The size and shape of the electron i. *Washington Academy of Sciences*, 8(1):1–11, 1918.
- [8] H.S. Allen. The case for a ring electron. *Proceedings of the Physical Society*, 31:49–68, 1919.
- [9] A.H. Compton. The size and shape of the electron ii. *Phys. Rev.*, 14:20–43, 1919.
- [10] A.H. Compton. The size and shape of the electron iii. *Phys. Rev.*, 14:247–259, 1919.
- [11] A.H. Compton. The magnetic electron. *Journal of the Franklin Institute*, 192:145–155, 1921.
- [12] G.E. Uhlenbeck and S. Goudsmit. Spinning electrons and the structure of spectra. *Nature*, 117:264–265, 1926.
- [13] P.A.M. Dirac. The quantum theory of the electron. *Proceedings of the Royal Society A*, 117:610–624, 1928.
- [14] E. Schrödinger. On the free motion in relativistic quantum mechanics. *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, pages 418–428, 1930.
- [15] P.A.M. Dirac. Theory of electrons and positrons (nobel lecture). *Les Prix Nobel*, 1933.
- [16] K. Huang. On the zitterbewegung of the dirac electron. *Am. J. Phys.*, 20:479–484, 1952.
- [17] A.O. Barut and A.J. Bracken. Zitterbewegung and the internal geometry of the electron. *Phys. Rev. D*, 23(10):2454–2463, 1981.
- [18] A.O. Barut and N. Zanghi. Classical model of the dirac electron. *Phys. Rev. Lett.*, 52:2009–2012, 1984.
- [19] A.O. Barut and M. Pavsic. Quantisation of the classical relativistic zitterbewegung in the schrodinger picture. *Classical and Quantum Gravity*, 4(4):L131, 1987.
- [20] A.O. Barut. Excited states of zitterbewegung. *Physics Letters B*, 237(3):436–439, 1989.
- [21] D. Hestenes. Quantum mechanics from self-interaction. *Found Phys.*, 15(1):63–87, 1985.
- [22] D. Hestenes. The zitterbewegung interpretation of quantum mechanics. *Found. Phys.*, 20(10):1213–1232, 1990.
- [23] J. Schwinger. On gauge invariance and vacuum polarization. *Phys. Rev.*, 82:664–679, 1951.
- [24] O. Consa. Helical solenoid model of the electron. *Progress in Physics*, 14:80–89, 2018.
- [25] K. Marinov, A.D. Boardman, and V.A. Fedotov. Toroidal metamaterial. *New Journal of Physics*, 9:324–335, 2007.