

Derive Schrödinger equation from $F=ma$

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Abstract

The compatibility between quantum mechanics and classical mechanics is always worth discussing. According to $F=ma$, the Schrödinger equation (SC) can be derived. Important classical physics formulas can be derived from the SC. The SC intuitively contains the mass m and the potential energy formula (this quality m can be large enough to enter the macro range and the potential energy comes from classical attraction). These three points determine that the SC is an organic combination of wave functions and classical mechanical laws. Based on this, it can be predicted that the SC, which can be used to describe macroscopic objects, can be established. It can be observed from the experiment of electron diffraction in a magnetic field that the volatility and property of moving electrons can be presented simultaneously. This experimental phenomenon, together with the mathematical analysis mentioned above, supports the conclusion that Newton's second law still applies in the microscopic world. According to the SC, it can be proven that "the energy of a moving particle described by the law of waves is equivalent to a multiple of its kinetic energy"; There is no absolute boundary between macro and micro.

Keywords: Schrödinger equation, Quantum mechanics, Classical mechanics, Organic combination, Electron diffraction experiment in magnetic field.

1. Introduction

From a mathematical perspective, the mass m in the Schrödinger equation can be so large that the described object is a macroscopic object. The Schrödinger equation directly uses the potential energy function $V = -\frac{Ze}{r}$ or $V = -\frac{GMm}{r}$ related to force in classical mechanics formulas. This indicates that the fundamental equation of quantum mechanics, the Schrödinger equation, cannot completely exclude classical mechanical formulas. However, scientists in the field of quantum mechanics have opposed quantum mechanics to classical mechanics (considering them incompatible). The reason and source of their persistence and confidence lie in directly ignoring the fact that the Schrödinger equation directly combines the differentiation of classical potential energy functions and wave functions, as well as the compatibility effect between "classical" and "quantum" brought about by the large range of mass m in the Schrödinger equation, and only emphasizing that $F=ma$ is not applicable in microscopic systems. However, we have found multiple reasons to suggest that they are compatible (It is possible to derive the Schrödinger equation that contains gravitational potential energy and can describe macroscopic objects.^{1,2,3,4,5} quantum mechanics and classical mechanics can be combined for use).

The theoretical reason is that the fundamental equations of classical mechanics can be derived from the Schrödinger equation. Conversely, starting from $F=ma$, the Schrödinger equation can be derived. This seems to undermine the confidence of orthodox quantum mechanics. Also, mathematical methods cannot determine a finite upper limit for the mass m in the Schrödinger equation. This mass m is the mass of the object described by the Schrödinger equation. When its value is large enough, it is a macroscopic object that follows the laws of classical mechanics. Therefore, the Schrödinger equation itself is a fundamental equation of wave mechanics that can describe both microscopic particles and macroscopic objects.

Through electron diffraction experiments under magnetic field interference, it can be clearly observed that the electron beam during the diffraction process can move laterally for a certain distance under the action of Lorentz force, and the diffraction pattern will deform but not disappear. We can fully explain this experimental phenomenon as follows: in the absence of quantum decoherence, the lateral acceleration of an electron is related to the force acting on it (the force acting on an object can cause acceleration. This is one of the qualitative descriptions of $F=ma$). This also proves through a simple, practical, and unambiguous experimental method that the wave and particle properties of electrons can be simultaneously maintained and presented (Complementarity is not necessary). This is also simultaneously proving that methods for describing fluctuation and methods for describing particle properties are compatible. The key is that other prophecies can also be made based on the above facts. For example, classical mechanics can be combined with quantum mechanics for use. Even in the microscopic world, 'quantum mechanics theory and methods are not in an absolute dominant position'. And we can use some computational examples to verify these predictions^{6,7,8} (especially the prediction that 'quantum mechanics and classical mechanics can be used together to describe the same object').

As mentioned above, the mathematical physical logic analysis and experimental facts jointly prove that the Schrödinger equation itself cannot limit our use of classical mechanical formulas in the microscopic field. The "limitations" mentioned here stem from the interpretation of quantum mechanics, which is determined by subjective factors. This research achievement can have a serious impact on the existing interpretation system of quantum mechanics. This kind of impact will inevitably affect the theory and methods of quantum mechanics. In fact, there have been numerous events questioning quantum mechanics, especially its interpretation.^{9,10,11} The phenomenon of supports the conclusion of this article. Although this article is relatively short, it contains a great deal of innovation and information. It is estimated to arouse great interest from scholars who are experts.

2. $F=ma$ Mutual Conversion with Schrödinger Equation

This operation process and result can reveal the essence and function of the Schrödinger equation. This section is a visual demonstration of the issues referred to in the title of this article, especially section 2.3.

Nowadays, it is widely believed that the classical mechanical law $F=ma$, which is important in the microscopic world or in the theoretical system of quantum mechanics, is not applicable. Another point is that everyone believes that the Schrödinger equation is written by Schrödinger based on intuition and cannot be derived (*i.e.*, cannot be derived based on axioms or fundamental principles of physics). If we use logical deduction to "derive the Schrödinger equation based on $F=ma$ " (and/or vice versa: derive $F=ma$ from the Schrödinger equation, such as classical mechanical equations), it will definitely surprise all physicists.

2.1. Derive classical mechanical formulas such as $F=ma$ from the Schrödinger equation

We first derive $F=ma$ from Schrödinger. A simple form of the Schrödinger equation is

$$\hat{H}\psi = E\psi. \quad (1)$$

When using operators to handle the wave function used in the Schrödinger equation, the following relationship can be used.

$$\langle \psi | \hat{H} | \hat{A} \psi \rangle = \langle \hat{A} \psi | \hat{H} | \psi \rangle^*. \quad (2)$$

According to Eqs. (1) and (2), it can be inferred that

$$\int \psi^* [\hat{H}, \hat{A}] \psi d\tau = E \int (\hat{A} \psi) \psi^* d\tau - E \int \psi^* \hat{A} \psi d\tau. \quad (3)$$

Among which

$$\int \psi^* [\hat{H}, \hat{A}] \psi d\tau = 0. \quad (4)$$

For \hat{A} in Eqs. (1) and (2), take $\sum_i \hat{q}_i \hat{p}_i = i\hbar \sum_i \hat{q}_i \frac{\partial}{\partial \hat{q}_i}$. According to the laws of union and distribution, we have

$$\begin{aligned} [\hat{A}, \hat{B} + \hat{C}] &= [\hat{A}, \hat{B}] + [\hat{A}, \hat{C}] \\ [\hat{A}, \hat{B}\hat{C}] &= [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}] \\ [\hat{x}, \hat{H}] &= \frac{\hbar^2}{m} \frac{\partial}{\partial x} - \frac{i\hbar}{m} \hat{p}_x. \end{aligned} \quad (5)$$

$$[\hat{p}_x, \hat{H}] = \frac{\hbar}{i} \frac{\partial V(x,y,z)}{\partial x}. \quad (6)$$

According to the relationship above, it can be concluded that

$$\begin{aligned} [\hat{H}, \sum_i \hat{q}_i \hat{p}_i] &= \sum_i [\hat{H}, \hat{q}_i \hat{p}_i] \\ &= \sum_i \hat{q}_i [\hat{H}, \hat{p}_i] + \sum_i [\hat{H}, \hat{q}_i] \hat{p}_i \\ &= i\hbar \sum_i q_i \frac{\partial V}{\partial q_i} - i\hbar \sum_i \frac{1}{m_i} \hat{p}_i^2 \\ &= i\hbar \sum_i q_i \frac{\partial V}{\partial q_i} - 2i\hbar \hat{T}. \end{aligned} \quad (7)$$

(In the Eq. (7), \hat{T} and \hat{V} are operators of kinetic energy and potential energy, respectively (\hat{V} is the same as its potential energy function).

Considering Eq. (4) again, we have

$$\langle \sum_i \psi \left| q_i \frac{\partial V}{\partial q_i} \right| \psi \rangle = 2 \langle \psi | \hat{T} | \psi \rangle. \quad (8)$$

Using the average value table of quantity B, Eq. (8) can be written as

$$\langle \sum_i q_i \frac{\partial V}{\partial q_i} \rangle = 2 \langle \hat{T} \rangle. \quad (9)$$

When we use Cartesian coordinates to represent the potential energy V , for a single electron system, we have

$$V = Ze^2 \sum \frac{1}{\sqrt{x_i^2 + y_j^2 + z_k^2}} + \sum_i \sum_{j>i} \frac{e^2}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}}. \quad (10)$$

So, it can be concluded that $\sum_i q_i \frac{\partial V}{\partial q_i} = V$. That is,

$$2 \langle T \rangle = \langle V \rangle. \quad (11)$$

Eq. (11) is the relationship between the absolute values of kinetic energy and potential energy in a bound state system. Its simple form can be written as $2|T| = |V|$. Strangely, $2T = -|V|$ also holds true and can more accurately represent the Viry theorem: kinetic energy and potential energy have opposite signs, and kinetic energy is only half of potential energy. From the derivation process above, it can be seen that Eq. (11) is only a relationship between the magnitude of kinetic energy and potential energy. If both size and symbol relationships are considered simultaneously. The Viry theorem must be written as

$$2T = -V. \quad (12)$$

The so-called steady state that does not change with time refers to objects that have uniform linear motion or

uniform circular motion under certain conditions. As we all know, the Viry theorem holds in the steady-state bound system described by the Schrödinger equation. A free object that maintains uniform linear motion does not comply with Viry's theorem. Therefore, in theory, it can be considered that a system described by the Schrödinger equation that conforms to the Viry theorem can be an object undergoing uniform circular motion. In this case, regardless of whether the microscopic system that can be described by the Schrödinger equation is a bound state uniform circular motion system, steady state refers to the energy state and motion state of the object that do not change with time. In the existing mathematical system of quantum mechanics, using the Schrödinger equation is equivalent to assuming that the Viry theorem holds. In classical mechanics, the system in which the Viry theorem holds is a constrained system with uniform circular motion. Logically speaking, the system that acknowledges the validity of the Viry theorem must be a planetary model system undergoing uniform circular motion, and is not affected by the size of the system (*i.e.*, this is the case for both macroscopic and microscopic systems). Denying this kind of cognition appears to be very imprecise (in other words, denying this cognition seems argue irrationally and too subjective).

In quantum mechanics, people unconditionally acknowledge the Viry theorem. It does not exclude the expressions for T and V here (*i.e.*, it does not exclude that these two energy expressions can be connected using the Viry theorem). Anyway, in both classical mechanics and quantum mechanics, the Viry theorem holds, and the expressions for kinetic and potential energy are also the same. They are $T = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ and $V = -\frac{GMm}{r}$. We use the Viry theorem without any mental burden. Substituting $T = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ and $V = -\frac{GMm}{r}$ into Eq. (12), we can obtain

$$mv^2 = \frac{GMm}{r}. \quad (14)$$

Divide both sides of Eq. (14) by r and consider that $a = m\frac{v^2}{r}$ and $|F| = \frac{GMm}{r^2}$ (in Newton's Second Law, F is a resultant force, and its direction and the direction of the acceleration caused by it are determined by \vec{r}). Therefore, $m\frac{v^2}{r}$ and $\frac{GMm}{r^2}$ directions are consistent and can take the same symbol), and Eq. (14) becomes

$$ma = F. \quad (15)$$

Let $G = \sum_i p_i \cdot r_i$, so

$$\begin{aligned} \frac{dG}{dt} &= \sum_i \dot{p}_i \cdot r_i + \sum_i \dot{r}_i \cdot p_i \\ &= \sum_i m_i V_i \cdot V_i + \sum_i F_i \cdot r_i \\ &= 2T + \sum_i F_i \cdot r_i. \end{aligned} \quad (16)$$

Here, $G = \sum_i p_i \cdot r_i$, $\frac{dG}{dt} = E$, $V = \sum_i F_i \cdot r_i$, $T = -\frac{1}{2} \sum_i F_i \cdot r_i$. For the case of a single particle orbiting the nucleus, the relevant formulas above can be simplified as

$$T + V = E, \quad (17)$$

Or $2T = -V$. Comparing these two equations representing the Viry theorem, it can be concluded that

$$T = -E. \quad (18)$$

We can at least say that $F=ma$, $2T=-V$, $T+V=E$, $p=mv$, $V=Fr$, $T=(1/2)mv^2$ can all be derived from specific Schrödinger equations that contain de Broglie relations. Even if we obtain $F=-ma$ instead of Eq. (15), it still holds significant importance.

Generally speaking, the inverse process of mathematical derivation also holds true. The inverse process of the above derivation is the process of deriving the Schrödinger equation from Newton's law $F=ma$. Since $F=ma$ is a classical mechanical formula, classical mechanical methods can be used throughout the entire process of deriving the Schrödinger equation from it. This makes the derivation process much simpler. However, in this guidance process, the relationship of $mv^2 = hv$ is required. We have to deduce this relationship first. The process by which Schrödinger wrote the Schrödinger equation is not called a derivation process. The reason is that he cannot explain why wave functions are used. In fact, there is no need to explain this point mathematically. We just need to be grateful that we can get the correct result after using the wave function.

2.2. The relationship between the wave energy of a moving particle and the particle energy

The wave function of a one-dimensional moving particle in a straight line can be written as

$$\psi(x,t) = \psi_0 e^{-\frac{i}{\hbar}(Et - px)}. \quad (19)$$

In the Schrödinger equation, there is a term $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi$. We use slightly different methods to calculate its value.

Considering Eq. (19), the calculation result of this term is $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi = \frac{p^2}{2m} \psi = (\frac{1}{2}mv^2)\psi$. When $p=mv$, $\frac{p^2}{2m} = \frac{1}{2}mv^2 = T$.

Write the wave function in the form of $\psi(x,t) = A e^{-i2\pi(vt - x/\lambda)}$. Considering that $\lambda = h/p$ and $E = hv$, calculate the term

$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi$. We can get it again $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi = \frac{(hv)^2}{2mv^2} \psi$. So, $\frac{(hv)^2}{2mv^2}$ is also equal to T . Eq. (19) and $\psi(x,t) = A e^{-i2\pi(vt - x/\lambda)}$ are two

different expressions of the same equation. So, $\frac{1}{2}mv^2 = \frac{(hv)^2}{2mv^2}$. By organizing this relationship equation (with only positive square roots), we can obtain

$$mv^2 = hv. \quad (20)$$

By substituting the de Broglie wave relationship $\lambda = h/mv$ into Eq. (20), we can organize it to obtain

$$v = \lambda v. \quad (21)$$

The left side of Eq. (20) represents energy expressed using classical particle laws (it has twice the kinetic energy), while the right side represents energy expressed using wave laws. These two energy expressions are aimed at the same moving particle. Eq. (21) is the relationship between the phase velocity and frequency of de Broglie waves.

If Eq. (19) is the wave equation of the de Broglie wave of a moving particle, then the relationship of $c = \lambda v$ cannot be used in it or in its differential results. If Eq. (19) is not considered to be the wave equation of the de Broglie wave, then the de Broglie wave described by the Schrödinger equation is not consistent with the wave (noumenon) of the wave function ψ in the Schrödinger equation, and using the Schrödinger equation cannot calculate the correct energy value of moving particles. From this point of view (from a physics perspective), these two waves must be consistent. The mathematical derivation results also support this point. It is frustrating that in the past, people believed that waves in wave functions were quasi electromagnetic waves, while de Broglie waves were matter waves.

Eq. (20) is a very important relationship. The left side represents the particle-property energy of the described object, while the right side represents the wave energy of the described particle. Both originate from the same differential

in the Schrödinger equation. This indicates that we can simultaneously or separately represent the energy of a moving particle in two different ways (wave law and particle law), and they are quantitatively equivalent. This seems to indicate that the waves of moving particles are the result of forming teams according to the rules under the influence of external conditions of the particles. In other words, a moving particle exhibits two forms of energy without a time difference. Eq. (20) can also indicate that the classical kinetic energy of a bound particle is half of the wave energy of its de Broglie waves. Although the volatile and particle-properties of moving particles can be manifested simultaneously, the energy of moving particles (i.e. the energy of de Broglie waves) is not the sum of particle energy and wave energy. This fact can be explained as follows: the wave characteristics of moving particles are the result of their particle formation (i.e., the statistical results of multi particle behavior). Simply put, individual moving particles are particles, while groups have the characteristics of waves. It can be said that they are both particles and waves, not complementary. The experimental results shown in Figure 2 can intuitively prove this point. Substituting Eq. (21) into the phase velocity formula $v=E/p$ yields $E=hv \neq \sqrt{m_0^2 c^4 + c^2 p^2}$. The phase velocity formula for de Broglie waves is

$$v = hv/p. \quad (22)$$

Eq. (22) can intuitively represent that the energy of the de Broglie wave of a particle moving at velocity v is not the total energy of the particle (i.e., the sum of kinetic energy and "energy equivalent to rest mass"). Artificially described by wave patterns, its energy can be expressed in the form of hv , while artificially described by particle patterns, its kinetic energy is $(1/2)mv^2$. Artificially selecting two different forms of homologous energy does not necessarily have to be equal, let alone additive. The above analysis indicates that the de Broglie waves of moving elementary particles are not wave groups but monochromatic waves, and the phase velocity is equal to the particle's motion velocity. Static particles do not possess wave particle duality but only particle properties.

As mentioned above, a simple deduction analysis in this section yielded five conclusions: (1) The energy of moving particles represented by wave laws is equivalent to the energy of particles represented by classical mechanics laws (the two energy expressions of moving particles — wave energy expression and particle energy expression — are equivalent); (2) The fluctuation characteristics of moving particles are the manifestation of particle queuing and the statistical result of the photosensitive performance of individual particles; (3) The energy expression form determines that the wave energy of the de Broglie wave of a moving particle is twice its kinetic energy of particle-property; (4) The essence of the wave function in the Schrödinger equation describing a particle is its de Broglie wave (the two are consistent); (5) The de Broglie wave of a moving particle (especially a fundamental particle) is a monochromatic wave rather than a wave group, and its phase velocity is the velocity of the particle.

2.3. Derive the Schrödinger equation based on $F=ma$ and classical mechanics methods

The purpose and effect of this action is to reveal the essence and function of the Schrödinger equation. It is the most intuitive work of bring out the theme.

Nowadays, it is widely believed that one of the most important classical laws of mechanics, $F=ma$, does not apply in the microscopic world or in the theoretical system of quantum mechanics. Another point is that everyone believes that the Schrödinger equation is written by Schrödinger based on intuition and cannot be derived (not based on axioms or fundamental principles of physics). If we derived the Schrödinger equation based on $F=ma$, it would surely surprise all physicists.

Newton's second law states that the force required to accelerate an object is equal to the product of acceleration and mass.

$$F=ma. \quad (23)$$

In classical mechanics, Eq. (17) is a consequence of Viry's theorem for a system of uniform circular motion. In classical mechanics, a definite uniform circular system satisfies both the virial law and $F=mv^2/r=ma$. This allows us to use $T=\frac{p^2}{2m}$, Eqs. (17) and (23) together.

The formula for kinetic energy in classical mechanics is $T=\frac{p^2}{2m}$. For an object in uniform circular motion, Eq. (23) can be written as $F=mv^2/r$. Multiplying both sides of $F=mv^2/r$ by $(1/2)r$, we can obtain $(1/2)Fr=(1/2)mv^2=T$. We can also write the potential energy as $V=Fr$. Substituting $(1/2)mv^2=T$ and $V=Fr$ into Eq. (17), we can obtain $(1/2)mv^2+Fr=E$. We feel that using the code V to represent potential energy is more intuitive and can represent different potential energies. Therefore, we write the classical mechanical formula $(1/2)mv^2+Fr=E$ as

$$\frac{p^2}{2m}+V=E. \quad (24)$$

If all terms of Eq. (24) are multiplied by ψ , we can obtain $\frac{1}{2}hv\psi+V\psi=E\psi$, or

$$\frac{p^2}{2m}\psi+V\psi=E\psi. \quad (25)$$

The process from Eq. (24) to Eq. (25) is the process of combining classical mechanics laws with wave functions.

Take the form of (19) and let $f(\hbar,m)\frac{\partial^2}{\partial x^2}\psi=\frac{p^2}{2m}\psi=\frac{mv^2}{2}\psi=\frac{1}{2}hv\psi$, we can obtain $f(\hbar,m)=-\frac{\hbar^2}{2m}$. Substituting $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi=\frac{p^2}{2m}\psi$ into Eq. (25), we can obtain

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+V\psi=E\psi. \quad (26)$$

Here, the operator of potential energy V and the operator of energy E can be taken in the same form as their expressions. Therefore, Eq. (26) is the one-dimensional stationary Schrödinger equation. At this point, we have achieved the Schrödinger equation, one of the fundamental equations of quantum mechanics, starting from $F=ma$ in classical mechanics and taking the planetary model as an example.

Even if the starting point of quantum mechanics is not the Schrödinger equation but the expression of unitary evolution of states in Hilbert space, it cannot be denied that the above derivation and argumentation are valid. Because using the Schrödinger equation is the most important fundamental method in quantum mechanics, and is also an important component of quantum mechanics theory (mathematical formal system).

3. The Schrödinger equation has a composition and structure that directly accepts classical mechanics

The above introduces the function of mining the hidden acceptable classical mechanical formulas in the Schrödinger equation. This section points out that the composition and structure of the Schrödinger equation determine that it can directly accept some classical mechanical formulas. We can also intuitively see from Eq. (26) that it contains the potential energy function V (which can be $V=-Ze^2/r$ or $V=-GMm/r$) and the mass m of the described object. Potential energy is not an original existence, it originates from classical forces or field interactions. Since the Schrödinger equation cannot eliminate the expression of classical potential energy, it does not completely reject the concepts and laws of classical force and potential energy. The Schrödinger equation itself cannot deny that the range of values for the mass m can be large enough to allow the object it describes to be a well-defined macroscopic object. The macroscopic system follows the laws of classical mechanics (theories that describe macroscopic objects cannot exclude macroscopic laws, that is, quantum mechanics cannot be completely opposed to classical mechanics). In reality, people believe that the fundamental

equations of quantum mechanics cannot describe macroscopic systems (or in other words, it is not suitable to discuss classical mechanical laws in quantum mechanics). This is a subjective choice guided by the old notion that "quantum" and "classical" are incompatible. Limiting the mass m in the Schrödinger equation to the microscopic mass range is also a subjective choice (guided by the subjective consciousness of quantum and classical incompatibility). Limiting the mass m in the Schrödinger equation to the microscopic range is also a subjective choice. This section discusses the reasons why the Schrödinger equation itself does not exclude classical mechanical formulas, which can be seen in a more intuitive way. Corresponding to the hidden "similar reasons" excavated earlier.

4. Electron diffraction experiment in magnetic field

This experiment can also be called an electron diffraction experiment under magnetic field interference. Place a high-intensity permanent magnet near one end of the fluorescent screen of the working electron diffractometer and observe the effect of the magnetic field on the electron diffraction pattern. The observed phenomenon is that the electron beam that has passed through the slit has moved a certain distance in the direction of the Lorentz force (the diffraction pattern can move as a whole), but the diffraction pattern has not disappeared (indicating that the quantum decoherence process has not occurred. See Fig. 1 and Video 1 for details). This experimental phenomenon proves that in the absence of quantum decoherence, Lorentz force causes transverse motion of the electron beam ("force acting on an object can cause acceleration in the direction of force", which is one of the qualitative expressions of Newton's second law $F=ma$). This experimental phenomenon supports the conclusion that ' $F=ma$ still applies in the microscopic world'. From this experimental phenomenon, it can be directly observed that the particle (particle property) and wave (volatile) characteristics of moving electrons can be presented simultaneously. In other words, electron diffraction experiments under magnetic field interference show that the particle and wave characteristics of electrons during the diffraction process are presented simultaneously (similar to the simultaneous presentation of water and water waves). The two expressions of the energy of moving particles can be converted into each other. That is to say, for a moving particle, the energy expressed by the wave law is equivalent to the energy expressed by the particle characteristics. We cannot add up the values obtained from the two expression methods. This situation is very similar to quality and energy.

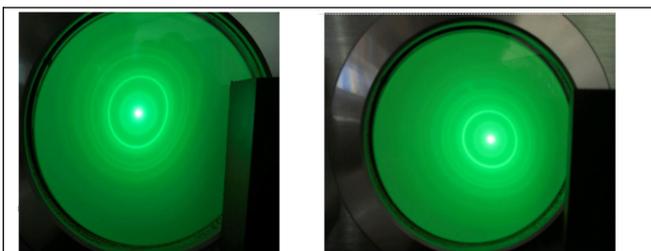


Fig. 1. The result of using magnetic field to interfere with electron diffraction. The electron beam moves laterally along the direction of force, causing diffraction fringes to deform and move as a whole, but not disappear. During the diffraction process, electron rays can exhibit both particle characteristics and wave law simultaneously.

The experimental phenomenon shown in **Fig. 2** was completed by previous researchers. References. The experiment shown in **Fig. 1** was completed by the author of this article. Interested readers can complete the repeated experiment within a few minutes.

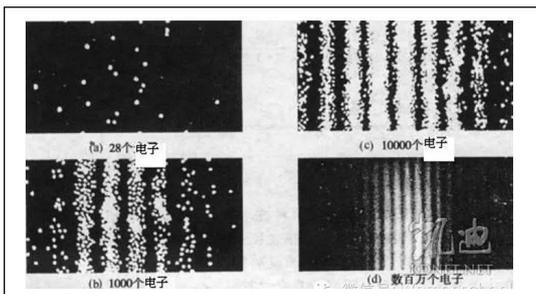


Fig.2. The diffraction pattern formed by allowing electrons to pass through a slit one by one. The relationship between diffraction pattern and electron number indicates that the diffraction pattern is synthetic. The experimental results also indicate that both particle and wave characteristics can be presented simultaneously.

References

- [1] Tu, R. Establishing the Schrödinger Equation for Macroscopic Objects and Changing Human Scientific Concepts. *Adv Theo Comp Phy*, 7(4), 01-03 (2024). <https://dx.doi.org/10.33140/ATCP.07.04.18>
- [2] Tu, R. Research Progress on the Schrödinger Equation of Gravitational Potential Energy. *Adv Theo Comp Phy*, 8(1), 01-07 (2025). <https://dx.doi.org/10.33140/ATCP.08.01.01>
- [3] Tu, R. (2025). Research Progress on the Schrödinger Equation of Gravitational Potential Energy. *Adv Theo Comp Phy*, 8(1), 01-07. <https://dx.doi.org/10.33140/ATCP.08.02.01>
- [4] Tu, R. Schrödinger-Tu Equation: A Bridge Between Classical Mechanics and Quantum Mechanics. *Int J Quantum Technol*, 1(1), 01-09 (2025). <https://www.primeopenaccess.com/scholarly-articles/schrodingertu-equation-a-bridge-between-classical-mechanics-and-quantum-mechanics.pdf>
- [5] Runsheng Tu. Research Progress on the Schrödinger Equation that Can Describe the Earth's Revolution and its Applications, *London Journal of Research in Science: Natural & Formal*. 25(1) , 17-28 (2025). https://journalspress.com/LJRS_Volume25/Research-Progress-on-the-Schrodinger-Equation-that-Can-Describe-the-Earths-Revolution-and-its-Applications.pdf
- [6] Tu, R. A Review of Research Achievements and Their Applications on the Essence of Electron Spin. *Adv Theo Comp Phy*, 7(4), 01-19 (2024). <https://dx.doi.org/10.33140/ATCP.07.04.10>
- [7] Tu, R. Some Successful Applications for Local-Realism Quantum Mechanics: Nature of Covalent-Bond Revealed and Quantitative Analysis of Mechanical Equilibrium for Several Molecules. *Journal of Modern Physics*. 5(6), 309-318 (2014). DOI: [10.4236/jmp.2014.56041](https://doi.org/10.4236/jmp.2014.56041)
- [8] Runsheng Tu. The principle and application of experimental method for measuring the interaction energy between electrons in atom. *International Journal of Scientific Reports*. 2(8), 187–200 (2016). DOI: [10.18203/issn.2454-2156.intjsciirep20162808](https://doi.org/10.18203/issn.2454-2156.intjsciirep20162808)
- [9] Sean Carroll. Why even physicists still don't understand quantum theory 100 years on. *Nature*. 638 , 31-34 (2025). doi: <https://doi.org/10.1038/d41586-025-00296-9>
- [10] Lee Billings. Quantum Physics Is on the Wrong Track, Says Breakthrough Prize Winner Gerard 't Hooft. Breakthrough Prize Winner Gerard 't Hooft Says Quantum Mechanics Is 'Nonsense'. *Scientific American*. 34(2), 104 (2025). <https://www.scientificamerican.com/article/breakthrough-prize-winner-gerard-t-hooft-says-quantum-mechanics-is-nonsense>

nse/, doi: 10.1038/scientificamerican062025-3ndxLHOdGJV2u6twkhWFzi

<https://www.scientificamerican.com/article/breakthrough-prize-winner-gerard-t-hoof-t-says-quantum-mechanics-is-nonsense/>

[11] Gibney, E. Physicists disagree wildly on what quantum mechanics says about reality, Nature survey shows. *Nature* 643, 1175-1179 (2025). <https://doi.org/10.1038/d41586-025-02342-y>