Residue classes and stopping time of the 3n+1 problem

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Abstract

This paper presents an analysis of the stopping time of the 3n+1 problem based on the residue class of n.

3n + 1 problem (or conjecture)

In the $3\cdot n+1$ problem[1] it is possible to define the function $s:N\to N$:

$$s(n)=\begin{cases}3\cdot n+1 & if\ n\equiv 1\,(mod\,2)\\ \dfrac{n}{2} & if\ n\equiv 0\,(mod\,2)\end{cases}$$

the sequence $s^k(n)$ for $k\in N$ obtained using the function $s(n)$ is as follows:

$$s^k(n)=\begin{cases}n & for\ k=0\\ s(s^{k-1}(n)) & for\ k>0\end{cases}$$

Stopping time

The $3\cdot n+1$ conjecture is equivalent to the conjecture that for each $n\in N$, $n>1$, there exists $k\in N$ such that $s^k(n)<n$. The least $k\in N$ such that $s^k(n)<n$ is called the stopping time of n [1] .

If n is even $n\equiv 0\,(mod\,2)$ then the stopping time $k=1$ and $s^1(n)=\dfrac{n}{2}$.

Let's analyze the case where n is odd.

Let m be the number of odd terms in the first k terms of the $3\cdot n+1$ sequence, and d_i be the number of consecutive even terms immediately following the *i*-th odd term, then the next term $s^k(n)$ in he $3\cdot n+1$ sequence is[1] :

$$s^k(n)=\frac{3^m}{2^{k-m}}\cdot n+\sum_{i=1}^{m}\frac{3^{m-i}}{2^{d_i+\cdots+d_m}}$$

note that for n odd $k-m=d_1+\cdots+d_m$.

Then

$$s^k(n)=\frac{3^m\cdot n+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{k-m}}=\frac{3^m\cdot n+r}{2^{k-m}}$$

where r depends on m and n with $r\geq 3^m-2^m$.

It is possible to observe that if $n \equiv 1 (mod\, 2^2)$ then $m=1$ and the stopping time is $k=3$

$s^3(n)=\frac{3\cdot n+1}{2^2}$

so we have $r=1=3^1-2^1$ and $d_1=2$.

If we now consider the numbers $n \equiv 3 (mod\, 2^2)$ we have:

$n=3+4\cdot a$

$s^1(n)=3\cdot n+1=10+12\cdot a$

$s^2(n)=\frac{3\cdot n+1}{2}=5+6\cdot a=2+2\cdot a+n$ odd number

$s^4(n)=\frac{3\cdot\frac{3\cdot n+1}{2}+1}{2}=2+a+2n$

if a odd then $n \equiv 7 (mod\, 2^3)$, $d_1=1$, $d_2=1$ and $m>2$ since $s^2(n), s^4(n)$ odd and $s^4(n)>n$

if a even then $a=2\cdot b$

$s^5(n)=\frac{3\cdot\frac{3\cdot n+1}{2}+1}{4}=1+b+n$

if b odd then $n \equiv 11 (mod\, 2^4)$, $d_1=1$, $d_2=2$ and $m>2$ since $s^2(n), s^5(n)$ odd and $s^5(n)>n$

if b even $b=2\cdot c$ then $n \equiv 3 (mod\, 2^4)$

$s^6(n)=\frac{3\cdot\frac{3\cdot n+1}{2}+1}{8}=c+\frac{(n+1)}{2}=\frac{3^2\cdot n+5}{2^4}$

then $k=6$, $d_1=1$, $d_2=3$ and $m=2$ with $r=5=3+2^1=3^2-2^2$.

As seen for numbers $n \equiv 7 (mod\, 2^3)$

$s^4(n)=\frac{3\cdot\frac{3\cdot n+1}{2}+1}{2}=\frac{9\cdot n+5}{4}$ odd number and $d_1=1$ and $d_2=1$

$s^6(n)=\frac{3\cdot\frac{9\cdot n+5}{4}+1}{2}=\frac{27\cdot n+19}{8}$

if $n\equiv 7\left(mod\,2^4\right)$ then $s^6(n)$ even and

$$s^7(n)=\frac{27\cdot n+19}{16}$$

if $n\equiv 23\left(mod\,2^5\right)$ then $s^7(n)$ even and

$$s^8(n)=\frac{27\cdot n+19}{32}<n$$

then $k=8$, $d_1=1$, $d_2=1$, $d_3=3$ and $m=3$ from which

$$r=19=3^3-2^3=3^2+3\cdot 2^1+2^{(1+1)}=3\cdot\left(3+2^1\right)+2^{(1+1)}=3\cdot 5+2^2 \quad .$$

Note that the numbers $n\equiv 15\left(mod\,2^4\right)$ and $n\equiv 7\left(mod\,2^5\right)$ remain to be analyzed.

For numbers $n\equiv 11\left(mod\,2^4\right)$

$$s^5(n)=\frac{3\cdot\frac{3\cdot n+1}{2}+1}{4}=\frac{9\cdot n+5}{8}$$ odd number and $d_1=1$ and $d_2=2$

$$s^7(n)=\frac{3\cdot\frac{9\cdot n+5}{8}+1}{2}=\frac{27\cdot n+23}{16}$$

if $n\equiv 11\left(mod\,2^5\right)$ then $s^7(n)$ even and

$$s^8(n)=\frac{3\cdot\frac{9\cdot n+5}{8}+1}{2}=\frac{27\cdot n+23}{32}<n$$

then $k=8$, $d_1=1$, $d_2=2$, $d_3=2$ and $m=3$ from which

$$r=23=3^2+3\cdot 2^1+2^{(1+2)}=3\cdot\left(3+2^1\right)+2^{(1+2)}=3\cdot 5+2^3 \quad .$$

For numbers $n\equiv 27\left(mod\,2^5\right)$ then $s^7(n)$ odd and $d_3=1$

$$s^7(n)=\frac{3\cdot\frac{9\cdot n+5}{8}+1}{2}=\frac{27\cdot n+23}{16}$$

$$s^9(n)=\frac{3\cdot\frac{27\cdot n+23}{16}+1}{2}=\frac{81\cdot n+85}{32}$$

if $n\equiv 59\left(mod\,2^6\right)$ then $s^9(n)$ even

$$s^{10}(n)=\frac{81\cdot n+85}{64}$$

if $n\equiv 59\left(mod\,2^7\right)$ then $s^{10}(n)$ even

$$s^{11}(n)=\frac{81\cdot n+85}{128}<n$$

then $k=11$, $d_1=1$, $d_2=2$, $d_3=1$, $d_4=3$ and $m=4$ from which

$$r=85=3^3+3^2\cdot 2^1+3\cdot 2^{(1+2)}+2^{(1+2+1)}=3\cdot\left(3^2+3\cdot 2^1+2^{(1+2)}\right)+2^{(1+2+1)}+2^{(1+2)}=3\cdot 23+2^4 \quad .$$

Note that the numbers $n\equiv 27\left(mod\, 2^6\right)$ and $n\equiv 123\left(mod\, 2^7\right)$ remain to be analyzed.

For numbers $n\equiv 15\left(mod\, 2^4\right)$

$$s^6(n)=\frac{3\cdot\frac{9\cdot n+5}{4}+1}{2}=\frac{27\cdot n+19}{8}$$ odd and $d_3=1$

$$s^8(n)=\frac{3\cdot\frac{27\cdot n+19}{8}+1}{2}=\frac{81\cdot n+65}{16}$$

if $n\equiv 15\left(mod\, 2^5\right)$ then $s^8(n)$ even and

$$s^9(n)=\frac{81\cdot n+65}{32}$$

if $n\equiv 15\left(mod\, 2^6\right)$ then $s^9(n)$ even and

$$s^{10}(n)=\frac{81\cdot n+65}{64}$$

if $n\equiv 15\left(mod\, 2^7\right)$ then $s^{10}(n)$ even and

$$s^{11}(n)=\frac{81\cdot n+65}{128}<n$$

then $k=11$, $d_1=1$, $d_2=1$, $d_3=1$, $d_4=4$ and $m=4$ from which

$$r=65=3^4-2^4=3^3+3^2\cdot 2^1+3\cdot 2^{(1+1)}+2^{(1+1+1)}=3\cdot\left(3^2+3\cdot 2^1+2^{(1+1)}\right)+2^{(1+1+1)}=3\cdot 19+2^3 \quad .$$

For numbers $n\equiv 7\left(mod\, 2^5\right)$

$$s^7(n)=\frac{27\cdot n+19}{16}$$ odd and $d_3=2$

$$s^9(n)=\frac{3\cdot\frac{27\cdot n+19}{16}+1}{2}=\frac{81\cdot n+73}{32}$$

if $n\equiv 7\left(mod\, 2^6\right)$ then $s^9(n)$ even and

$$s^{10}(n)=\frac{81\cdot n+73}{64}<n$$

if $n\equiv 7\left(mod\,2^7\right)$ then $s^{10}(n)$ even and $s^{11}(n)=\frac{81\cdot n+73}{128}<n$

then $k=11$, $d_1=1$, $d_2=1$, $d_3=2$, $d_4=3$ and $m=4$ from which

$r=73=3^3+3^2\cdot 2^1+3\cdot 2^{(1+1)}+2^{(1+1+2)}=3\cdot\left(3^2+3\cdot 2^1+2^{(1+1)}\right)+2^{(1+1+2)}=3\cdot 19+2^4$.

Note that the numbers $n\equiv 39\left(mod\,2^6\right)$, $n\equiv 71\left(mod\,2^7\right)$, $n\equiv 31\left(mod\,2^5\right)$, $n\equiv 47\left(mod\,2^6\right)$ and $n\equiv 79\left(mod\,2^7\right)$ remain to be analyzed.

By continuing with this procedure it is easy to verify the following results:

m	k	$n\left(mod\,2^{k-m}\right)$	d_m	E	r	G
1	3	1	2		1	*a*
2	6	3	3	a^1	$5=3+2^1$	*b*
3	8	23	3	b^1	$19=3^2+3\cdot 2^1+2^{(1+1)}$	*c*
		11	2	b^2	$23=3^2+3\cdot 2^1+2^{(1+2)}$	*d*
4	11	15	4	c^1	$65=3^3+3^2\cdot 2^1+3\cdot 2^{(1+1)}+2^{(1+1+1)}$	*e*
		7	3	c^2	$73=3^3+3^2\cdot 2^1+3\cdot 2^{(1+1)}+2^{(1+1+2)}$	*f*
		59	3	d^1	$85=3^3+3^2\cdot 2^1+3\cdot 2^{(1+2)}+2^{(1+2+1)}$	*g*
5	13	95	4	e^1	$211=3^4+3^3\cdot 2^1+3^2\cdot 2^{(1+1)}+3\cdot 2^{(1+1+1)}+2^{(1+1+1+1)}$	*h*
		175	3	e^2	$227=3^4+3^3\cdot 2^1+3^2\cdot 2^{(1+1)}+3\cdot 2^{(1+1+1)}+2^{(1+1+1+2)}$	*i*
		79	2	e^3	$259=3^4+3^3\cdot 2^1+3^2\cdot 2^{(1+1)}+3\cdot 2^{(1+1+1)}+2^{(1+1+1+3)}$	*j*
		39	3	f^1	$251=3^4+3^3\cdot 2^1+3^2\cdot 2^{(1+1)}+3\cdot 2^{(1+1+2)}+2^{(1+1+2+1)}$	*k*
		199	2	f^2	$283=3^4+3^3\cdot 2^1+3^2\cdot 2^{(1+1)}+3\cdot 2^{(1+1+2)}+2^{(1+1+2+2)}$	*l*
		219	3	g^1	$287=3^4+3^3\cdot 2^1+3^2\cdot 2^{(1+2)}+3\cdot 2^{(1+2+1)}+2^{(1+2+1+1)}$	*m*
		123	2	g^2	$319=3^4+3^3\cdot 2^1+3^2\cdot 2^{(1+2)}+3\cdot 2^{(1+2+1)}+2^{(1+2+1+2)}$	*n*
6	16	575	5	h^1	$665=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+1)}+3^2\cdot 2^{(1+1+1)}+3\cdot 2^{(1+1+1+1)}+2^{(1+1+1+1+1)}$	
		287	4	h^2	$697=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+1)}+3^2\cdot 2^{(1+1+1)}+3\cdot 2^{(1+1+1+1)}+2^{(1+1+1+1+2)}$	
		735	3	h^3	$761=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+1)}+3^2\cdot 2^{(1+1+1)}+3\cdot 2^{(1+1+1+1)}+2^{(1+1+1+1+3)}$	
		367	4	i^1	$745=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+1)}+3^2\cdot 2^{(1+1+1)}+3\cdot 2^{(1+1+1+2)}+2^{(1+1+1+2+1)}$	
		815	3	i^2	$809=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+1)}+3^2\cdot 2^{(1+1+1)}+3\cdot 2^{(1+1+1+2)}+2^{(1+1+1+2+2)}$	
		975	3	j^1	$905=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+1)}+3^2\cdot 2^{(1+1+1)}+3\cdot 2^{(1+1+1+3)}+2^{(1+1+1+3+1)}$	
		999	4	k^1	$817=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+1)}+3^2\cdot 2^{(1+1+2)}+3\cdot 2^{(1+1+2+1)}+2^{(1+1+2+1+1)}$	
		423	3	k^2	$881=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+1)}+3^2\cdot 2^{(1+1+2)}+3\cdot 2^{(1+1+2+1)}+2^{(1+1+2+1+2)}$	
		583	3	l^1	$977=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+1)}+3^2\cdot 2^{(1+1+2)}+3\cdot 2^{(1+1+2+2)}+2^{(1+1+2+2+1)}$	
		923	4	m^1	$925=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+2)}+3^2\cdot 2^{(1+2+1)}+3\cdot 2^{(1+2+1+1)}+2^{(1+2+1+1+1)}$	
		347	3	m^2	$989=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+2)}+3^2\cdot 2^{(1+2+1)}+3\cdot 2^{(1+2+1+1)}+2^{(1+2+1+1+2)}$	
		507	3	n^1	$1085=3^5+3^4\cdot 2^1+3^3\cdot 2^{(1+2)}+3^2\cdot 2^{(1+2+1)}+3\cdot 2^{(1+2+1+2)}+2^{(1+2+1+2+1)}$	

The value in column E indicates the relationship with the value of r represented by the letter in column G.

$s^{k_m}(n)=\frac{3^m\cdot n+r}{2^{k_m-m}}$ and $s^{k_{m+1}}(n')=\frac{3^{m+1}\cdot n'+r'}{2^{k_{m+1}-(m+1)}}$ with $r'=3\cdot r+2^{\left(k_m-m-d_m+d'_m\right)}$ and $1\le d'_m<d_m$

example c^2 indicates $r=19$ e $d'_m=2$ then $r'=3\cdot 19+2^{(8-3-3+2)}=73$.

As seen $d_m=k-m-(d_1+\cdots+d_{m-1})$ and if for a certain value of m if we find a value of r, which we indicate as $r_m^i=3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1^i+\cdots+d_{i-1}^i}$ depends on the values of $d_1^i,\cdots,d_{m-1}^i$, then from this value of r_m^i for $m+1$ we can obtain $r_{m+1}^j=3\cdot r_m^i+2^{(d_1+\cdots+d_{m-1}+d_m^j)}$ with $1\le d_m^j<d_m^i$ and $x_{m+1}^j\cdot 3^{(m+1)}\equiv -r_{m+1}^j\,(mod\,2^{d_1+\cdots d_{m-1}+d_m^j+d_{m+1}^j})$ from which we can obtain x_{m+1}^j and for $n\equiv x_{m+1}^j\,(mod\,2^{d_1+\cdots+d_{m-1}+d_m^j+d_{m+1}^j})$ we have $s^{m+1+d_1+\cdots+d_{m-1}+d_m^j+d_{m+1}^j}(n)<n$.

If $n\equiv x\,(mod\,2^{d_1+\cdots+d_{m-1}+d_m})$ with x odd, $m>1$, $s^i(n)>n$ for $1\le i<m+d_1+\cdots+d_{m-1}+d_m$

and $s^{m+d_1+\cdots+d_{m-1}+d_m}(n)=\dfrac{3^m\cdot n+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d_m}}<n$ then if $d_m>1$ for $1\le d'_m<d_m$

$$s^{m+d_1+\cdots+d_{m-1}+d'_m}(x)=\frac{3^m\cdot x+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m}}$$ is even

and $r_m=3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}$

if $x=x'+b\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}$ with $x'<2^{d_1+\cdots+d_{m-1}+d'_m}$

$$s^{m+d_1+\cdots+d_{m-1}+d'_m}(x)=\frac{3^m\cdot x+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m}}=\frac{3^m\cdot x'+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m}}+b\cdot 3^m$$

$s^{m+d_1+\cdots+d_{m-1}+d'_m}(x)$ is even and for

$$y=x+(1+2\cdot a)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}=x'+b\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}+(1+2\cdot a)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}$$

$$s^{m+d_1+\cdots+d_{m-1}+d'_m}(y)=\frac{3^m\cdot x+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m}}+(1+2\cdot a)\cdot 3^m=s^{m+d_1+\cdots+d_{m-1}+d'_m}(x)+(1+2\cdot a)\cdot 3^m$$

$s^{m+d_1+\cdots+d_{m-1}+d'_m}(y)$ is odd then

$$s^{m+1+d_1+\cdots+d_{m-1}+d'_m+1}(y)=\frac{3\cdot\dfrac{3^m\cdot y+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m}}+1}{2}=\frac{3^{m+1}\cdot y+3^m+2^{d_1+\cdots+d_{m-1}+d'_m}+\sum_{i=2}^{m}3^{m+1-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m+1}}$$

we obtain $r_{m+1}=3\cdot r_m+2^{(d_1+\cdots+d'_m)}=3^m+2^{d_1+\cdots+d_{m-1}+d'_m}+\sum_{i=2}^{m}3^{m+1-i}\cdot 2^{d_1+\cdots+d_{i-1}}$ with $1\le d'_m<d_m$.

As seen $s^{m+d_1+\cdots+d_{m-1}+d_m}(x)=\frac{1}{2^{d_m-d'_m}}\cdot\frac{3^m\cdot x+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m}}$

$$s^{m+1+d_1+\cdots+d_{m-1}+d'_m+1}(y)=\frac{3\cdot\frac{3^m\cdot x+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m}}+(1+2\cdot a)\cdot 3^{m+1}+1}{2}$$

$$s^{m+1+d_1+\cdots+d_{m-1}+d'_m+1}(y)=\frac{3\cdot 2^{(d_m-d'_m)}\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)+(1+2\cdot a)\cdot 3^{m+1}+1}{2}$$

with $y=x+(1+2\cdot a)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}$

if $s^{m+1+d_1+\cdots+d_{m-1}+d'_m+1}(y)$ is odd

$$\frac{3\cdot 2^{(d_m-d'_m)}\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)+((1+2\cdot a)+2)\cdot 3^{m+1}+1}{2}=s^{m+1+d_1+\cdots+d_{m-1}+d'_m+1}(y)+3^{m+1}$$ is even

for an appropriate value of a we can get

$$s^{m+1+d_1+\cdots+d_{m-1}+d'_m+1}(x+(1+2\cdot a)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m})=\frac{3\cdot 2^{(d_m-d'_m)}\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)+(1+2\cdot a)\cdot 3^{m+1}+1}{2}$$

if $\frac{3\cdot 2^{(d_m-d'_m)}\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)+(1+2\cdot a)\cdot 3^{m+1}+1}{4}$ is odd then

$$\frac{3\cdot 2^{(d_m-d'_m)}\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)+((1+2\cdot a)+4)\cdot 3^{m+1}+1}{4}=\frac{3\cdot 2^{(d_m-d'_m)}\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)+(1+2\cdot a)\cdot 3^{m+1}+1}{4}+3^{m+1}$$

is even and for an appropriate value of a we can get

$$s^{m+1+d_1+\cdots+d_{m-1}+d'_m+2}(x+(1+2\cdot a)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m})=\frac{3\cdot 2^{(d_m-d'_m)}\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)+(1+2\cdot a)\cdot 3^{m+1}+1}{4}$$

…

for an appropriate value of a we can get

$$s^{m+1+d_1+\cdots+d_{m-1}+d'_m+(d_m-d'_m)+1}(x+(1+2\cdot a)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m})=\frac{3\cdot 2^{(d_m-d'_m)}\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)+(1+2\cdot a)\cdot 3^{m+1}+1}{2\cdot 2^{d_m-d'_m}}$$

as seen $s^{m+d_1+\cdots+d_{m-1}+d_m}(n)<n$ then $\frac{3^m}{2^{d_1+\cdots+d_{m-1}+d_m}}<1$

$k_{m+1}=m+1+d_1+\cdots+d_{m-1}+d'_m+d'_{m+1}$

if $\frac{3^m}{2^{d_1+\cdots+d_{m-1}+d_m}}<\frac{2}{3}$ then $\frac{3^{m+1}}{2\cdot 2^{d_m-d'_m}}<2^{d_1+\cdots+d_{m-1}+d'_m}$, $d'_{m+1}=(d_m-d'_m)+1$, $k_{m+1}=k_m+2$

$$s^{k_{m+1}}\left(x+\left(1+2\cdot a\right)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)=\frac{3\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)}{2}+\frac{\left(1+2\cdot a\right)\cdot 3^{m+1}+1}{2\cdot 2^{d_m-d'_m}}$$

$$s^{k_{m+1}}\left(x+\left(1+2\cdot a\right)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)<x+\left(1+2\cdot a\right)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}$$

else

$$\frac{3^{m+1}}{4}<3^m<2^{d_1+\cdots+d_{m-1}+d_m} \quad \text{and} \quad \frac{3^{m+1}}{4\cdot 2^{d_m-d'_m}}<\frac{3^m}{2^{d_m-d'_m}}<2^{d_1+\cdots+d_{m-1}+d'_m}$$

for an appropriate value of a we can get

$$s^{m+1+d_1+\cdots+d_{m-1}+d'_m+(d_m-d'_m)+2}\left(x+\left(1+2\cdot a\right)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)=\frac{3\cdot 2^{(d_m-d'_m)}\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)+\left(1+2\cdot a\right)\cdot 3^{m+1}+1}{4\cdot 2^{d_m-d'_m}}$$

then $d'_{m+1}=(d_m-d'_m)+2$ and $k_{m+1}=k_m+3$ with

$$s^{k_{m+1}}\left(x+\left(1+2\cdot a\right)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)=\frac{3\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)}{4}+\frac{\left(1+2\cdot a\right)\cdot 3^{m+1}+1}{4\cdot 2^{d_m-d'_m}}$$

$$s^{k_{m+1}}\left(x+\left(1+2\cdot a\right)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)<x+\left(1+2\cdot a\right)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}$$

in both cases you can get b so

$$n\equiv\left(\left(x+2^{d_1+\cdots+d_{m-1}+d'_m}\right)\left(mod\, 2^{d_1+\cdots+d_{m-1}+d'_m+1}\right)+2\cdot b\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)\left(mod\, 2^{d_1+\cdots+d_{m-1}+d_m+d'_m+d'_{m+1}}\right)$$

$$n\equiv\left(x+\left(1+2\cdot a\right)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)\left(mod\, 2^{d_1+\cdots+d_{m-1}+d_m+d'_m+d'_{m+1}}\right)$$

so

$$s^{k_{m+1}}(n)=\frac{3^{m+1}\cdot n+3^m+2^{d_1+\cdots+d_{m-1}+d'_m}+\sum_{i=2}^{m}3^{m+1-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m+d'_{m+1}}}<n$$

b can be obtained using $r_{m+1}=3\cdot r_m+2^{(d_1+\cdots+d'_m)}=3^m+2^{d_1+\cdots+d_{m-1}+d'_m}+\sum_{i=2}^{m}3^{m+1-i}\cdot 2^{d_1+\cdots+d_{i-1}}$

$$y\cdot 3^{(m+1)}\equiv -r_{m+1}\left(mod\, 2^{d_1+\cdots d_{m-1}+d'_m+d'_{m+1}}\right) \quad .$$

Let's examine the relationship between k and m with $k-m=d_1+\cdots+d_m$:

as seen if $\frac{3^m}{2^{d_1+\cdots+d_{m-1}+d_m}}\geq\frac{2}{3}$ then

$d'_{m+1}=(d_m-d'_m)+2$, $k_{m+1}=k_m+3$

$3^{m+1}\geq 2^{k_m+2-(m+1)}=2^{k_{m+1}-(m+1)-1}$, $2\cdot 3^{m+1}>2^{k_{m+1}-(m+1)}$ and $2\cdot 3^{m+1}<2^{k_{m+1}-(m+1)+1}$

$k_{m+1}-(m+1) \;=\; \lfloor \log_2(2\cdot 3^{m+1})\rfloor \;=\; \lfloor 1+(m+1)\cdot\log_2(3)\rfloor$

else if $\frac{3^m}{2^{d_1+\cdots+d_{m-1}+d_m}}<\frac{2}{3}$

$d'_{m+1}=(d_m-d'_m)+1 \quad , \quad k_{m+1}=k_m+2$

if we assume that $k_{m+1}-(m+1)= 2 + \lfloor 1+(m-1)\cdot\log_2(3)\rfloor = 2+\lfloor \log_2(2\cdot 3^{m-1})\rfloor$

$2^{k_{m+1}-(m+1)-2}\le 2\cdot 3^{m-1}<2^{k_{m+1}-(m+1)-1}$ and $k_{m+1}=k_m+2=k_{m-1}+4$

$\frac{3^{m-1}}{2^{k_{m-1}+4-(m+1)-2}}=\frac{3^{m-1}}{2^{k_{m-1}-(m-1)}}\ge 1$ this is impossible then it must be

$k_{m+1}-(m+1) = k_m- m + 1= 1 + \lfloor 1+m\cdot\log_2(3)\rfloor= 1+\lfloor \log_2(2\cdot 3^m)\rfloor$

$2^{k_{m+1}-(m+1)-1}\le 2\cdot 3^m<2^{k_{m+1}-(m+1)}$, $4\cdot 3^m\ge 2^{k_{m+1}-(m+1)}$, $2\cdot 3^{m+1}\ge 4\cdot 3^m\ge 2^{k_{m+1}-(m+1)}$ and $2\cdot 3^{m+1}<2^{k_{m+1}-(m+1)+1}$

$k_{m+1}-(m+1) = \lfloor 1+ (m+1)\cdot\log_2(3)\rfloor$.

We verified that $k_m-m = \lfloor 1+ m\cdot\log_2(3)\rfloor$ with $k_m-m=d_1+\cdots+d_{m-1}+d_m$

then $2\cdot 3^m\ge 2^{k_m-m}$

$\frac{3^{m+1}}{2^{d_m-d'_m}}>\frac{2\cdot 3^m}{2^{d_m-d'_m}}\ge 2^{d_1+\cdots+d_{m-1}+d_m-(d_m-d'_m)}$ with $1\le d'_m<d_m$

for $1\le e\le d_m-d'_m$

$s^{m+1+d_1+\cdots+d_{m-1}+d'_m+(d_m-d'_m)+1-e}\left(x+(1+2\cdot a)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)=\frac{2^e\cdot 3\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)}{2}+2^{e-1}\frac{(1+2\cdot a)\cdot 3^{m+1}+1}{2^{d_m-d'_m}}$

$n\equiv x\,(mod\,2^{k_m-m})$ has stopping time k_m then

$$s^{m+d_1+\cdots+d_{m-1}+d'_m+(d_m-d'_m)-e}(x)=\frac{3^m\cdot x+3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}}{2^{d_1+\cdots+d_{m-1}+d'_m+(d_m-d'_m)-e}}=2^e\cdot s^{m+d_1+\cdots+d_{m-1}+d_m}(x)>x$$

then since $2^{e-1}\ge 1$ for $1\le e\le d_m-d'_m$

$s^{m+1+d_1+\cdots+d_{m-1}+d'_m+(d_m-d'_m)+1-e}\left(x+(1+2\cdot a)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)>x+(1+2\cdot a)\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}$.

In summary it has been demonstrated that

For every $m>1$, if $n\equiv x\left(mod\,2^{k_m-m}\right)$ has stopping time k_m ,

then for every d'_m , $1\le d'_m<d_m$ there exists the integer $b_{d'_m}\ge 0$ so for the numbers

$$n'_{d'_m}\equiv\left(\left(x+2^{d_1+\cdots+d_{m-1}+d'_m}\right)\left(mod\,2^{d_1+\cdots+d_{m-1}+d'_m+1}\right)+2\cdot b_{d'_m}\cdot 2^{d_1+\cdots+d_{m-1}+d'_m}\right)\left(mod\,2^{d_1+\cdots+d_{m-1}+d'_m+d'_{m+1}}\right)$$

$$s^{k_{m+1}}\left(n'_{d'_m}\right)<n'_{d'_m}$$

and if $\frac{3^m}{2^{d_1+\cdots+d_{m-1}+d_m}}<\frac{2}{3}$ then $d'_{m+1}=\left(d_m-d'_m\right)+1$ else $d'_{m+1}=\left(d_m-d'_m\right)+2$

the stopping time is $k_{m+1}=(m+1)+d_1+\cdots+d_{m-1}+d'_m+d'_{m+1}=(m+1)+\lfloor 1+(m+1)\cdot\log_2(3)\rfloor$

furthermore $s^{k_m}(n)=\frac{3^m\cdot n+r}{2^{k_m-m}}$ and $r=3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}$

then $n'_{d'_m}\equiv x'_{d'_m}\left(mod\,2^{k_{m+1}-(m+1)}\right)$

$r'_{d'_m}=3\cdot r+2^{\left(d_1+\cdots+d_{m-1}+d'_m\right)}=3\cdot r+2^{\left(k_m-m-d_m+d'_m\right)}$ and $s^{k_{m+1}}\left(n'_{d'_m}\right)=\frac{3^{m+1}\cdot n'_{d'_m}+r'_{d'_m}}{2^{k_{m+1}-(m+1)}}$

$x'_{d'_m}\cdot 3^{(m+1)}\equiv -r'_{d'_m}\left(mod\,2^{k_{m+1}-(m+1)}\right)$ from which we can obtain $x'_{d'_m}$.

Application to the generalized function qn + 1

The $q\cdot n+1$ function $t_q:N\to N$ is defined for $q\in N$, $q\equiv 1(mod\,2)$ by

$$t_q(n)=\begin{cases}q\cdot n+1 & if\ n\equiv 1(mod\,2)\\ \frac{n}{2} & if\ n\equiv 0(mod\,2)\end{cases}$$

the sequence $t_q^{\,k}(n)$ for $k\in N$ obtained using the function $t_q(n)$ is as follows:

$$t_q^{\,k}(n)=\begin{cases}n & for\ k=0\\ t_q\left(t_q^{\,k-1}(n)\right) & for\ k>0\end{cases}$$

Let m be the number of odd terms in the first k terms of the $t_q^{\,k}(n)$ sequence, and d_i be the number of consecutive even terms immediately following the *i*-th odd term, then for n odd the next term $t_q^{\,k}(n)$ in he $q\cdot n+1$ sequence is

$$t_q^{\,k}(n)=\frac{q^m\cdot n+r}{2^{k-m}}$$

where $r=q^{m-1}+\sum_{i=2}^{m}q^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}$ and $k-m=d_1+\cdots+d_m$

r then identifies a specific trajectory up to $t_q^{\,k}(n)$.

As we have seen, similarly to the previous case we have

$t_q^{k_m}(n)=\frac{q^m\cdot n+r}{2^{k_m-m}}$ and $t_q^{k_{m+1}}(n')=\frac{q^{m+1}\cdot n'+r'}{2^{k_{m+1}-(m+1)}}$ with $r'=q\cdot r+2^{(k_m-m-d_m+d'_m)}$ and $1\le d'_m<d_m$

For $q=5$

m	k_m	$n(mod\,2^{k-m})$	d_m	E	r	G
0	0	0	1		0	
1	4	3	3		1	*a*
2	7	1	4	a^1	$5=5+2^1$	*b*
		15	3	a^2	$9=5+2^2$	*c*
3	10	13	5	b^1	$39=5^2+5\cdot2^1+2^{(1+1)}$	*d*
		57	4	b^2	$43=5^2+5\cdot2^1+2^{(1+2)}$	*e*
		17	3	b^3	$51=5^2+5\cdot2^1+2^{(1+3)}$	*f*
		103	4	c^1	$53=5^2+5\cdot2^2+2^{(2+1)}$	*g*
		63	3	c^2	$61=5^2+5\cdot2^2+2^{(2+2)}$	*h*
4	14	773	7	d^1	$203=5^3+5^2\cdot2^1+5\cdot2^{(1+1)}+2^{(2+1)}$	
		637	6	d^2	$211=5^3+5^2\cdot2^1+5\cdot2^{(1+1)}+2^{(2+2)}$	
		365	5	d^3	$227=5^3+5^2\cdot2^1+5\cdot2^{(1+1)}+2^{(2+3)}$	
		845	4	d^4	$259=5^3+5^2\cdot2^1+5\cdot2^{(1+1)}+2^{(2+4)}$	
		809	6	e^1	$231=5^3+5^2\cdot2^1+5\cdot2^{(1+2)}+2^{(3+1)}$	
		537	5	e^2	$247=5^3+5^2\cdot2^1+5\cdot2^{(1+2)}+2^{(3+2)}$	
		1017	4	e^3	$279=5^3+5^2\cdot2^1+5\cdot2^{(1+2)}+2^{(3+3)}$	
		881	5	f^1	$287=5^3+5^2\cdot2^1+5\cdot2^{(1+3)}+2^{(4+1)}$	
		337	4	f^2	$319=5^3+5^2\cdot2^1+5\cdot2^{(1+3)}+2^{(4+2)}$	
		727	6	g^1	$281=5^3+5^2\cdot2^2+5\cdot2^{(2+1)}+2^{(3+1)}$	
		455	5	g^2	$297=5^3+5^2\cdot2^2+5\cdot2^{(2+1)}+2^{(3+2)}$	
		935	4	g^3	$329=5^3+5^2\cdot2^2+5\cdot2^{(2+1)}+2^{(3+3)}$	
		799	5	h^1	$337=5^3+5^2\cdot2^2+5\cdot2^{(2+2)}+2^{(4+1)}$	
		255	4	h^2	$369=5^3+5^2\cdot2^2+5\cdot2^{(2+2)}+2^{(4+2)}$	

The value in column E indicates the relationship with the value of r represented by the letter in column G

example c^2 indicates $r=9$ e $d'_m=2$ then $r'=5\cdot9+2^{(7-2-3+2)}=61$.

For $q=7$

m	k_m	$n(mod\,2^{k-m})$	d_m	E	r	G
0	0	0	1		0	
1	4	1	3		1	*a*
2	8	39	5	a^1	$9=7+2^1$	*b*
		5	4	a^2	$11=7+2^2$	*c*
3	12	267	7	b^1	$67=7^2+7\cdot2^1+2^{(1+1)}$	
		367	6	b^2	$71=7^2+7\cdot2^1+2^{(1+2)}$	
		55	5	b^3	$79=7^2+7\cdot2^1+2^{(1+3)}$	
		455	4	b^4	$95=7^2+7\cdot2^1+2^{(1+4)}$	
		461	6	c^1	$85=7^2+7\cdot2^2+2^{(2+1)}$	
		149	5	c^2	$93=7^2+7\cdot2^2+2^{(2+2)}$	
		37	4	c^3	$109=7^2+7\cdot2^2+2^{(2+3)}$	

The value in column E indicates the relationship with the value of r represented by the letter in column G

example c^2 indicates $r=11$ e $d'_m=2$ then $r'=7\cdot11+2^{(8-2-4+2)}=93$.

Application to cycle of the generalized function 3n + h

The $3\cdot n+h$ function[2] $t_h: N\to N$ is defined for $h\in N$, $h\equiv1(mod\,2)$ by

$$t_h(n)=\begin{cases}3\cdot n+h & if\ n\equiv1(mod\,2)\\ \dfrac{n}{2} & if\ n\equiv0(mod\,2)\end{cases}$$

the sequence $t_h^{\,k}(n)$ for $k\in N$ obtained using the function $t_h(n)$ is as follows:

$$t_h^{\,k}(n)=\begin{cases}n & for\ k=0\\ t_h(t_h^{\,k-1}(n)) & for\ k>0\end{cases}$$

Let m be the number of odd terms in the first k terms of the $t_h^{\,k}(n)$ sequence, and d_i be the number of consecutive even terms immediately following the *i*-th odd term, then for n odd the next term $t_h^{\,k}(n)$ in he $3\cdot n+h$ sequence is

$$t_h^{\,k}(n)=\frac{3^m\cdot n+h\cdot r}{2^{k-m}}$$

where $r=3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot2^{d_1+\cdots+d_{i-1}}$ and $k-m=d_1+\cdots+d_m$

r then identifies a specific trajectory up to $t_h^{\,k}(n)$.

For $h\geq 1$, $h\equiv 1(mod\,6)$, in a primitive cycle[2] , consisting only of integers relatively prime to h , we have:

$y=\frac{3^m\cdot y+h\cdot r}{2^{k-m}}$ and $h\cdot r=(2^{k-m}-3^m)\cdot y$

then y divides r .

As we have seen r must be $r=3^{m-1}+\sum_{i=2}^{m}3^{m-i}\cdot 2^{d_1+\cdots+d_{i-1}}$ fixed $m\geq 1$ if we take a value of r obtained for a given residue class that has stopping time $k=m+\lfloor 1+m\cdot\log_2(3)\rfloor$ then for those divisors y of r for which $h\cdot r=(2^{k-m-d_m+a}-3^m)\cdot y$ with $(2^{k-m-d_m+a}-3^m)>0$ the sequence $t_h{}^k(n)$ has the same trajectory as $s^k(n)$ up to the *m-th* odd term and therefore they have the same r , since r does not depend on d_m .

Some examples:

1)

$m=1$, $k=3$, $d_m=2$, $r=1$

then $y=r=1$ and $h=(2^a-3^1)$ with $(2^a-3^1)>0$

for $h=(2^a-3^1)$ with $a\geq 2$ the term $y=1$ belongs to a primitive cycle.

2)

$m=2$, $k=6$, $d_m=3$, $r=5$

then $y=r=5$ and $h=(2^{6-2-3+a}-3^2)$ with $a\geq 3$

for $h=(2^{a+1}-3^2)$ with $a\geq 3$ the term $y=5$ belongs to a primitive cycle.

3)

$m=3$, $k=8$, $d_m=3$, $r=19$

then $y=r=19$ and $h=(2^{8-3-3+a}-3^3)$ with $a\geq 3$

for $h=(2^{a+2}-3^3)$ with $a\geq 3$ the term $y=19$ belongs to a primitive cycle.

4)

$m=4$, $k=11$, $d_m=4$, $r=65=13\cdot 5$

then $y=13$ and $h\cdot 5=(2^{11-4-4+a}-3^4)$ with $a>3$ and $a\equiv 1(mod)4$

for $h=\frac{(2^{a+3}-3^4)}{5}$ with $a>3$ and $a\equiv 1(mod)4$ the term $y=13$ belongs to a primitive cycle.

Below is the algorithm code to generate the residue classes for $m \geq 1$:

```
get_dim(m)={my(log2_3=log(3)/log(2),X1=matrix(2,floor(1+m*log2_3)-m), d=0,dim=1);X1[1,1]=0;
if(m>=3,for(x=3,m,y=floor(1+(x-1)*log2_3);
for(i=1,y-x,for(j=i,y-x,if(i==1,X1[1+(d+1)%2,j]=1);X1[1+(d+1)%2,j]=X1[1+(d+1)%2,j]+X1[d+1,i]));
d=(d+1)%2);for(i=1,floor(1+m*log2_3)-m,dim=dim+X1[d+1,i]));dim;}
{m_max=9;dim_max=get_dim(m_max);N=vector(dim_max);R=matrix(2,dim_max);Dm=matrix(2,dim_max);
nr=0;m=1;log2_3=log(3)/log(2);kmm=floor(1+m*log2_3);c=1;N[1]=1;R[1,1]=1;Dm[1,1]=2;
print1("m = ",m,"  -  stopping time: ",kmm+m,"\nif n == ",N[1]);
print1(" (mod 2^",kmm,")\ntotal residue classes:  ",c,"\n\n");
for(m=1,m_max-1, kmm=floor(1+m*log2_3);mp1=m+1;kmmp1=floor(1+mp1*log2_3);
print1("m = ",mp1,"  -  stopping time: ",kmmp1+mp1,"\nif n == ");c1=0;
for(i=1,c,sdmm1=kmm-Dm[1+nr,i];for(j=1,Dm[1+nr,i]-1,rj=3*R[1+nr,i]+2^(sdmm1+j); c1=c1+1;
N[c1]=((2^kmmp1-rj)*lift(Mod(1/3^mp1,2^kmmp1)))%2^kmmp1;R[1+(1+nr)%2,c1]=rj;
Dm[1+(1+nr)%2,c1]=kmmp1-(sdmm1+j))); c=c1;N1=vector(c);N1=vecsort(N[1..c]);for(i1=1,c-1,
print1(N1[i1],", "));print1(N1[c]);nr=(1+nr)%2;print(" (mod 2^",kmmp1,")\ntotal residue classes:  ",c,"\n\n"))}
```

PARI/GP code of the algorithm

```
get_dim(m)={my(dim_vec=floor(1+m*log(3)/log(2))-m,nr=0,dim=1,X1=matrix(2,dim_vec));X1[1,1]=0;
if(m>=3,kmxm1=4;for(x=3,m,for(i=1,kmxm1-x,for(j=i,kmxm1-x,if(i==1,X1[1+(nr+1)%2,j]=1);
X1[1+(nr+1)%2,j]=X1[1+(nr+1)%2,j]+X1[nr+1,i]));nr=(nr+1)%2;if(3^x<2^(kmxm1+1),kmxm1=kmxm1+1,
kmxm1=kmxm1+2));for(i=1,dim_vec,dim=dim+X1[nr+1,i]););dim;}
{m_max=9;dim_max=get_dim(m_max);N=vector(dim_max);R=matrix(2,dim_max);Dm=matrix(2,dim_max);
m=1;kmm=2;c=1;N[1]=1;R[1,1]=1;Dm[1,1]=2;
print1("m = ",m," - stopping time: ",kmm+m,"\nif n == ",N[1]);
print1(" (mod 2^",kmm,")\ntotal residue classes: ",c,"\n\n");
for(m=1,m_max-1,mp1=m+1;if(3^mp1<2^(kmm+1),kp1mmp1=kmm+1,kp1mmp1=kmm+2);c1=0;
print1("m = ",mp1," - stopping time: ",kp1mmp1+mp1,"\nif n == ");
for(i=1,c,sdmm1=kmm-Dm[1,i];for(j=1,Dm[1,i]-1,rj=3*R[1,i]+2^(sdmm1+j);if(c1,print1(", "));
c1=c1+1;N[c1]=((2^kp1mmp1-rj)*lift(Mod(1/3^mp1,2^kp1mmp1)))%2^kp1mmp1;print1(N[c1]);R[2,c1]=rj;
Dm[2,c1]=kp1mmp1-(sdmm1+j)));c=c1;for(i2=1,c,R[1, i2]=R[2, i2];Dm[1, i2]=Dm[2, i2]);
print(" (mod 2^",kp1mmp1,")\ntotal residue classes: ",c,"\n\n");kmm=kp1mmp1)}
```

PARI/GP code 2 of the algorithm

```
row(m)={if(m<1, [0], my(r=[1], d=[2], km=2); for(mi=1, m-1, my(temp1=[], temp2=[],
c=if(3^(mi+1)<2^(km+1), 1, 2));  for(i=1, #d, temp1=concat(temp1, vector(d[i]-1, j, 3*r[i]+2^(km-d[i]+j)));
temp2=concat(temp2, vector(d[i]-1, j, d[i]-j+c))); km=km+c; r=temp1; d=temp2; );
vecsort(apply(x->((-x)*lift(Mod(1/3^m, 2^km)))%2^km, r)));}
for(i=0, 9, print(row(i)))
```

PARI/GP code 3 of the algorithm

```
row(n, q) = {if(n<1, [0], my(r=[1], d=floor(log(q)/log(2)), km=d+1); for(m=1, n-1, my(s=[],
c=if(q^(m+1)<2^(km+d), d, d+1)); for(i=1, #r, my(dm=km, t=r[i], e); for(j=1, m-1, t-=q^(m-j);
e=valuation(t,2); t/=2^e; dm-=e); s=concat(s, vector(dm-1, j, q*r[i]+2^(km-dm+j)))); km=km+c; r=s);
vecsort(apply(x->((-x)*lift(Mod(1/q^n, 2^km)))%2^km, r)));}
for(i=0, 8, print(row(i, 5)))
```

PARI/GP code of the algorithm for finding from r a term of the cycle of the generalized function qn+1

```
list(n, q) = {my(ans=List(), t=List(), km=1, d=floor(log(q)/log(2))); if(n>0, for(i=1, d-1, listput(~t, 0)); listput(~t,
1); listput(~ans, 1); for(m=1, n-1, my(t1=List(), c=if(q^m<2^(km+d), d, d+1)); for(i=1, d-1, listput(~t1, 0));
if(c==d+1, listput(~t1, sum(j=1, #t, t[j]))); for(i=1, #t, listput(~t1, sum(j=i, #t, t[j]))); km=km+c; t=t1;
listput(~ans, sum(j=1, #t, t[j])))); Vec(ans);}
```

PARI/GP code of the algorithm to display the number of elements in the first n rows for a given q

```
n_m=12;r_m=16;
r_v(n)={if(n<1, [0], my(r=[1], d=[2], km=2); for(i=1, n-1, my(t1=[], t2=[], c=if(3^(i+1)<2^(km+1), 1, 2));
for(j=1, #d, t1=concat(t1, vector(d[j]-1, m, 3*r[j]+2^(km-d[j]+m))); t2=concat(t2, vector(d[j]-1, m, d[j]-
m+c)));km=km+c; r=t1; d=t2; ); r);}
trg=matrix(r_m,r_m); ix=vector(r_m,i,1); log2_3=log(3)/log(2); for(n=1, n_m,
my(t=r_v(n));h=floor(1+n*log2_3); for(i=1, #t, my(r=t[i], t2=divisors(r)); for(i2=1, #t2, for(j=1, r_m,
my(y=t2[i2], k=3*j-1-j%2); my(t3=binary(k*(r/y)+3^n)); if(vecsum(t3)==1 && #t3>h, trg[j,ix[j]]=y;
if(ix[j]<r_m,ix[j]++))))))); for(i=1,r_m,print("h=",3*i-1-i%2," - ", vecsort(select(x->x>0,triangle[i,]))))
```

PARI/GP code of the algorithm for finding from r a term of the cycle of the generalized function 3n+h

References

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