THE FOUNDATION OF MATHEMATICS—OCCAM'S PRINCIPLE

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Abstract. This work examines the full implications of Occam's razor ["*do not multiply causes unnecessarily*"] when applied to metric existence criteria, resulting in a scientific precept and metric existence theorem of the universe—which takes the general form all natural laws must obey.

Keywords: *Occam's razor, metric, root, zero, reductive, inductive, parsimony*

1. Occam's principle

Mathematics has since its inception been without a definitive unifying theoretical principle; instead, having relied upon general heuristic standards of relative consistency. It is most notable that mathematical physics cannot be considered entirely valid without such principle established. Herein is the function of this work.

Occam's razor is one of the simplest, by definition, starting points; and when formulated in its full form becomes such definitive, necessary foundation of the theoretical sciences—the proper goal of objective mathematics. Namely, the elementary principle that the absolute cardinality (*metric measure*) of the empty set is zero; not just the ordinality.

$$x \in \{|\pm\infty|\} \mid y \neq x \Rightarrow x > (y = 0) \Rightarrow \{\pm\infty\} \to \left\{\left(\lim_{x\to\infty} x\right) \cup \left(\lim_{x\to-\infty} x\right)\right\} = \sum(\pm\infty) = y \Rightarrow \{\pm\infty\} = 0 = \emptyset \tag{1}$$

Q.E.D.

Summarized as

$$0 = \emptyset \tag{2}$$

or "zero as the root is non-multiple."

Hence, there is no element but zero, or the empty set itself—ultimately. Equivalent to the von Neumann universe hierarchy foundation of set theory, or the derivation of all of mathematics as objective as the standard Zermelo–Fraenkel set theory axioms [1]—generated as recursive iterations of the root null set.

The prime function of the principle is to transform the von Neumann hierarchy and constructible universe from heuristic assumptions and positing, into absolute proven mathematical fact and law; the principle resides in the union of the contrapositive, converse infinite limits of potential set elements, leading to not only a limit of cancellation, but a cancellation of all limits, thereby properly, actually, reductively equating zero to the root null set, by technical complementarity substitution—rather than the previous naïve set theory mere postulation thereof.

Incidentally, a proof that zero is equal to the empty set, and all is categorically, numerically reducible to such; zero being the sum of all possible elements—the proper, ultimate self-deriving natural principle, all other elements then being properly recursive iterations, as in the von Neumann hierarchy. Hence, *physically*, zero is the "*prism effect*" for all that exists.

The ultimate parsimony of all elements, and the demonstrable logical root of all measure-theoretic science as such, thus proven. Meaning "zero" is the non-multiple base of all—and all is metrically rooted, and derived, as such.

This principle does two things: (1) it establishes a root for existence criteria (the "0") according to metric unity, with resultant inductive numeric identity, and (2) constitutes an existence theorem for the metric universe as such, asserting the general character of natural uniformity of metric law according to proper tautological deduction.

In this way, the proper calculative starting point of set theory is not with assumption, but with proven, necessary, starting existence criteria. As all numbers are inductively derivable from zero, all sets are then necessarily deductively metrically constructible, completely and uniquely, from the empty set itself.

Example: $0 = \emptyset = \{\,\}, 1 = \{\{\,\}\}, 2 = \{\{\,\}, \{\{\,\}\}\}, 3 = \{\{\,\}, \{\{\,\}\}, \{\{\,\}, \{\{\,\}\}\}\}, . . .$

2. Results

The existence theorem of the constructible universe

$$\exists E = \emptyset \tag{3}$$

For E, the existential quantum, derived minimally and in a one-to-one from the root null set.

Objectified-metric set theory

Scientific reductive root axioms (*deconstructive consistency principles*)

The transitive properties of zero (and by extension, of all sets): **[ZF + (V = L)]**

2.1. Extensionality

$$\forall x \forall y [\forall z (z \in x \Leftrightarrow z \in y) \Rightarrow x = y] \tag{4}$$

Extensionality establishes that sets with identical elements are the same set.

2.2. Regularity

$$\forall x \left[x \neq \emptyset \Rightarrow \exists y \left(y \in x \land \forall z \big(z \in x \Rightarrow \neg (z \in y) \big) \right) \right] \tag{5}$$

Regularity establishes the distinction between non-empty set element and the set containing it; avoiding ill-defined contradictions, like "Russell's paradox".

2.3. Separation

$$\forall u_1 \ldots \forall u_k \big[\forall w \exists v \forall r \big(r \in v \Leftrightarrow r \in w \land \psi(r, u_1, \ldots, u_k) \big) \big] \tag{6}$$

Separation establishes that set elements may be related to different sets' elements by a mapping formula.

2.4. Pairing

$$\forall x \forall y \exists z \forall w (w \in z \Leftrightarrow w = x \lor w = y) \tag{7}$$

Pairing establishes that for any two sets there is exactly one set containing both of them.

2.5. Union

$$\forall x \exists y \forall z [z \in y \Leftrightarrow \exists w (w \in x \land z \in w)] \tag{8}$$

Union establishes that a set of subsets has a general set containing all the elements of the subsets.

2.6. Replacement

$$\forall u_1 \ldots \forall u_k \left[\forall x \exists!\, y \phi(x, y, u_1, \ldots, u_k) \Rightarrow \forall w \exists v \forall r \left(r \in v \Leftrightarrow \exists s \big(s \in w \land \phi(s, r, u_1, \ldots, u_k) \big) \right) \right] \tag{9}$$

Replacement establishes that mappings of set elements will always transform them to elements of an existing set.

2.7. Power set

$$\forall x \exists y \forall z[z \in y \Leftrightarrow \forall w(w \in z \Rightarrow w \in x)] \tag{10}$$

Power set establishes that a set of all subsets exists for all the designated elements of a given set.

2.8. Infinity

$$\exists x\big[\emptyset \in x \wedge \forall y\big(y \in x \Rightarrow \bigcup\{y, \{y\}\} \in x\big)\big] \tag{11}$$

Infinity establishes arbitrary ordinal iteration, or unlimited counting.

2.9. Constructibility

$$V = L \tag{12}$$
$$\left\{L_0 \coloneqq \emptyset, L_{\alpha+1} \coloneqq \mathrm{Def}(L_\alpha), L_\lambda \coloneqq \bigcup_{\alpha<\lambda} L_\alpha \,, L \coloneqq \bigcup_{\alpha\in\mathrm{Ord}} L_\alpha\right\}$$

Constructibility establishes that all sets are constructible according to recursive metric iteration.

As these axioms are minimally sufficient with maximal economy of principle to derive the von Neumann universe and to generalize the reductive rules of Occam's principle, it is determined that their very existence constitutes a technical *uniqueness theorem* as foundation for all subsequently objectively true mathematics, following minimally from the *existence theorem* of the metric universe. These resultantly self-generating, recursive principles conjoin to mathematical physics, through Occam's principle, to properly establish the physical universe as such—namely, *metrizable*; with mathematics as science properly objectively calibrated.

3. Conclusions

The *axiom of constructibility* establishes the final uniqueness of the cumulative hierarchy generating ZF axioms in necessitating unique metric ordinal set and number derivation without contradiction, accurately reflecting the objective condition of the metric universe—also proving the *axiom of global choice* and the *generalized continuum hypothesis* [1]—hence, the proper function of axioms are as *metric laws*, of which there number 9 in total—with the implicit root *zeroth* of the null set itself as generator, in Occam's principle and the *existence theorem of the constructible universe*, the *zero*. The final criterion of ultimate scientific objective consistency; derived in a tautologically objectively-calibrated fashion, from sheer deduction to total induction—demonstrating the final necessity of metric law.

REFERENCES

[1] Quine, W. V., Set Theory and Its Logic, Belknap Press, 1971.