Virgin and Under-Investigated Areas of Research in Non-Newtonian Fluid Mechanics

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Abstract: This paper is mainly about the areas of non-Newtonian fluid mechanics which are not investigated or not appropriately and sufficiently investigated. In fact, this should also include emerging areas of research in the field of non-Newtonian fluid mechanics due to new scientific and technological developments and advancements. The purpose of the paper is to highlight and draw the attention to these areas so that researchers (especially the young researchers and new-comers to research such as PhD students) invest their resources and efforts in these areas instead of investing in other areas which are previously investigated and hence they are of less priority from this aspect. Apart from the obvious benefit of "leveling up" in research, the attention to these rather neglected and non-explored areas of research can be beneficial at the scientific and individual levels since it can lead to breakthroughs and new discoveries in these areas of research by inspecting and assessing their potentials and impacts at the theoretical and practical levels and probing their beneficial applications. We will also provide a brief discussion about the possibility of introducing novel tools and methods in these areas of research (and in non-Newtonian fluid mechanics research in general) as well as highlighting some of the existing limitations of the past and current research in the field of non-Newtonian fluid mechanics (noting that this discussion should help in achieving the ultimate objective of this investigation).

Keywords: Fluid mechanics, fluid dynamics, non-Newtonian fluids, time-independent fluids, time-dependent fluids, memory fluids, complex fluids, shear-thinning, shear-thickening, yield stress, viscoelasticity, thixotropy, rheology.

Contents

1 Introduction

Non-Newtonian fluid mechanics (and related subjects in mechanics and rheology) is a huge branch of science. It has been under intensive investigation and research from many aspects and perspectives especially in the last decades (starting approximately from the 1950s although the interest in non-Newtonian flow phenomena goes back well beyond this date as can be inferred for instance from Maxwell viscoelastic fluid model). The subject has been investigated from physical, mathematical and computational angles and by different methodologies. It was also a subject to many experimental as well as theoretical investigations (see for instance [1, 2]). This should come as no surprise because non-Newtonian flow is a common physical phenomena in nature (and actually it is the norm rather than the exception) and has many technological and industrial applications (e.g. in oil extraction, medicine, engineering, food processing, and so on).

However, thorough inspection to the previous and current research in this field should reveal the fact that there are many gaps and neglected areas in these investigations where some areas are ignored or insufficiently investigated while other areas enjoyed much more attention and investments in resources and efforts. This is partly understandable because (for instance) some areas are easier to investigate or have exceptional practical benefits (e.g. to the petroleum industry) and hence they were the focus of many investigations while other areas are more difficult to investigate or have less practical benefits and hence they were ignored or under-investigated. We should also consider in this context newly emerging areas of research in this field due to new developments in science, medicine and technology.

Anyway, this fact should be admitted and hence addressed by the research community to level up the situation by directing the attention to the ignored and under-investigated areas and drawing more resources and efforts to these areas. This is not only a necessity to science to have a better and more thorough understanding of the non-Newtonian flow phenomena but it should also have many scientific and personal benefits where new areas of research could lead to breakthroughs and novelties that to be celebrated by science and scientists.

As indicated already in the abstract, the main purpose of this paper is to draw the attention to these rather neglected or insufficiently investigated areas of non-Newtonian fluid mechanics to help identifying these areas first, and to encourage the researchers in this field to spend more resources and efforts in these areas by directing some or all of their resources and efforts to these areas instead of accumulating their attention and resources on the traditional areas of this subject. In fact, we also want this paper to be a call and motive to the funding bodies and academic and research institutes (especially the leading ones) to give these areas more attention in their funding and projects (such as PhD scholarships and programs and post-doctoral research projects).

As indicated already, there are many serious challenges in the research of these virgin and under-investigated areas and hence this could be a deterrent (or at least makes them less attractive) to many researchers to enter these areas (especially those who come to research temporarily and accidentally and for other objectives such as getting formal academic qualifications). However, this can be a motive to other types of researchers (and actually the talented and exceptional researchers) who enjoy the challenges and difficulties and are keen to find areas of research that can have lasting effects and impacts at the scientific and personal levels by making breakthroughs and new discoveries.

In short, although there are many challenges, deterrents and disadvantages in conducting research in these virgin and under-investigated areas, there are also many benefits, incentives and advantages in conducting research in these areas (such as dealing with and managing less complexities in many cases where investigations of novel subjects require little more than addressing and dealing with fundamentals and generalities, possibility of making excited discoveries and breakthrough which are less likely in old areas of research and hence getting precedence and honors, and so on).

Our plan in this paper (following this introduction) is to present and discuss some research areas that are not explored or they require further attention and investigation (see § 2). We then (for the purpose of completeness) discuss briefly some of the potential tools that should be considered in these investigations (and actually in the investigations of non-Newtonian fluid mechanics in general) especially with the current revolution in computing and artificial intelligence (see § 3). We then (also for the purpose of completeness) discuss briefly some of the limitations of the past and current research in non-Newtonian fluid mechanics (see § 4). The paper will be concluded by listing a number of achievements and conclusions that we obtain from the present paper.

2 Virgin and Under-Investigated Areas of Research

We present in the following subsections some of the neglected or emerging or under-explored topics that should benefit from deeper and more thorough investigation and hence they should be given more attention in the future research in the field of non-Newtonian fluid mechanics. However, before we go through this investigation it is important to note that the following items of discussion (as outlined by the titles of the following subsections) are not supposed to be mutually exclusive but they rather represent various views from various angles to the subject of non-Newtonian fluid mechanics and hence there are considerable overlaps (although this should not be considered as repetition due to the differences in perspectives and considerations). We should also note that although we try to be as comprehensive as possible in this discussion (within the scope and size of the present paper), our investigation is far from being thorough and hence the following items of discussion should be considered as a good and large sample of the issues and items in this regard.

2.1 Non-Newtonian Flow in Geometries of Complex Morphology

Most of the past and current research in non-Newtonian fluid mechanics focuses on idealized flows in simple geometries, such as the flow of simple fluids (like power law fluids) in circular pipes (or thin slits or channels of simple geometry) under simple and idealized conditions. However, many real-world applications involve geometries of complex morphology (such as complex porous media as exemplified by geological structures, irregular industrial pipes, medical devices, biological tissues, and so on). As indicated, this issue is not limited to the geometry of the flow containers and conduits but to the geometry plus the attached conditions (regarding for example the flow, fluid and environment) as well as the fluid models which are supposed to represent the rheology of the fluid which flows in these geometries. For example, when we talk in the following about the flow in conduits of regular non-circular cross sections we should consider flow of different non-Newtonian fluid models (e.g. power law, Ellis, Herschel-Bulkley, Carreau, Cross, etc.) with different conditions (e.g. with and without slip, compressible and incompressible, with and without external body forces and fields, with different boundary conditions, and so on).[1]

A sample of suggested areas of research in this field is:

1. Flow in conduits and pipes of regular non-circular cross sections such as elliptical, rectangular, triangular, circular sector, half moon (or crescent) and half round, limacon, polygon (convex or concave, and simple or complex), polygon with curved sides, star polygon, and so on.

[1] In fact, these considerations (as well as their combinations) are what makes these areas of research virgin or under-investigated. Otherwise, most of the areas that we will discuss in the subsections of this section are investigated in general (although with limitations in these considerations) and hence we can find considerable amounts of literature in most of these areas.

2. Flow in conduits and pipes of irregular cross sections such as some of the previous geometries (e.g. irregular polygons), totally irregular geometry (with and without variation in the cross section along the conduit length), and so on.
3. Flow in conduits of multiple (or non-simple) connectivity such as circular concentric annulus, circular eccentric annulus, circular annulus with non-circular hole or holes (e.g. elliptical or square or triangular hole), some of the previous geometries with concentric/eccentric circular/non-circular holes (e.g. rectangular with concentric/eccentric hole or holes of various regular/irregular geometries), and so on.
4. Flow in converging-diverging (or diverging-converging) and corrugated regular geometries. Some irregularities in these geometries can also be considered (such as variations in the convergence-divergence and corrugation parameters). In fact, convergence-divergence and corrugation should be considered with conduits of various general geometries and hence it should include circular and non-circular conduits of various geometric shapes as given in the previous points.
5. Flow in conduits and channels with non-constant cross-sectional area or/and non-constant cross-sectional shape. In fact, convergence-divergence and corrugation (which we mentioned in the previous point) may be considered as instances or special cases of this point.
6. Flow in curved conduits of the previous geometries (as well as circular geometry) such as curved pipes of elliptical or rectangular cross section.
7. Flow in conduits with combinations of the considerations of the previous points such as the flow in a curved pipe of triangular cross section with non-constant cross-sectional area.

2.2 Non-Newtonian Flow in Non-Rigid Structures

Again, we repeat what was said in the last subsection that is this issue is not limited to the non-rigid structures of the flow containers and conduits but to the non-rigid structures plus the attached conditions (regarding for example the flow, fluid and environment) as well as the fluid models which are supposed to represent the rheology of the fluid flowing in these structures.[2]

A sample of suggested areas of research in this field is:

1. Flow in elastic conduits of various geometries, conditions and models (as discussed in

[2] In fact, this should apply to all the issues discussed in this section and hence we will not repeat this note in the following subsections.

the previous subsection).

2. Flow in viscoelastic conduits of various geometries, conditions and models (as discussed in the previous subsection).
3. Pulsatile flow in distensible conduits in general (elastic or viscoelastic).
4. Peristaltic flow in distensible conduits in general.
5. Pressure wave propagation through distensible conduits in general.

2.3 Non-Newtonian Flow in Porous Media

In fact, this could be included in the previous subsections (mainly § 2.1) but because of its importance we put it in a separate subsection. Although there are many investigations about this subject, there are many gaps and virgin areas of research in the investigation of non-Newtonian fluid flow in porous media.[3]

A sample of suggested areas of research in this field is:

1. Flow in various ordered and disordered porous media such as 2D and 3D porous media of regular and irregular structure and morphology (e.g. fractal or lattice-based with and without fixed geometry or/and size of throats and their connectivity).[4]
2. Flow in distensible porous media of elastic or viscoelastic properties.[5]
3. Flow in fractured porous media.
4. Flow in heterogeneous porous media.
5. Flow in anisotropic porous media.
6. Multi-phase flow in porous media (also see § 2.9).
7. Interaction between flowing fluids and porous media (especially distensible porous media).

2.4 Non-Newtonian Flow in Micro- and Nano-Scale Systems

Most of the past and current research in non-Newtonian fluid mechanics is about macroscopic fluid systems. In fact, this is understandable since micro- and nano-scale studies

[3] Most of the past and current investigations in the fluid flow through porous media are either about the flow of Newtonian fluids, or they use rather simplistic methodologies such as the use of network modeling with simple geometries and topologies for the porous media elements (mainly nodes and throats). Moreover, they generally employ simple flow and ambient conditions and usually ignore rather complex considerations (such as heterogeneity and anisotropy of porous media or its interaction with the flowing fluid or its distensibility).

[4] In fact, even entirely irregular (or chaotic) porous media should be considered in this context.

[5] Biological tissues (or some of their types) can be considered as distensible porous media (noting that biological fluid systems will be discussed later on; see § 2.8).

(whether in fluid mechanics or in general) are relatively new. So in short, while the behavior of non-Newtonian fluids in macroscopic systems is widely studied, the behavior of such fluids at micro- and nano-scales is generally under-researched.

A sample of suggested areas of research in this field is:

1. Investigating how non-Newtonian characteristics and attributes (such as shear-thinning, shear-thickening, viscoelasticity, thixotropy, yield stress, and so on) change or adapt at reduced scales, especially under varying structure and ambient conditions (such as confinement, temperature, and surface effects).
2. Investigating how non-Newtonian behavior can be exploited in new scientific and technological applications at microscopic and sub-microscopic scales.[6]
3. Investigating the validity of the continuum (or large-scale or bulk) fluid models (such as power law or Herschel-Bulkley or Carreau) in micro- and nano-scale systems (noting, for instance, that most non-Newtonian fluids, such as polymeric liquids, have molecular chain structures within fluid mediums).

2.5 Non-Time-Independent Non-Newtonian Flow

So far, most of the investigations and research in non-Newtonian fluid mechanics and dynamics are focused on the time-independent fluid flow. This is understandable due to the relative simplicity of time-independent non-Newtonian flow and the huge complexities associated with non-time-independent[7] flow (largely classified under the two broad terms of thixotropy and viscoelasticity). In fact, non-time-independent non-Newtonian effects and attributes (such as memory effects) are very important in many natural phenomena and technological applications and hence they require more attention and investment is research.

A sample of suggested areas of research in this field is:

1. Investigation of various aspects, attributes and phenomena of viscoelasticity such as the rate of strain in viscoelastic fluids (and its dependencies especially on time), hysteresis in stress-strain correlations, storage modulus, loss modulus, creep, stress relaxation, memory effects in general, and so on.

[6] In fact, fluid flow (and non-Newtonian in particular) at micro- and nano-scales is important for many traditional and newly-emerging high-tech technologies. For example, understanding how non-Newtonian fluids behave in confined environments (such as microfluidic devices or nano-scale channels) is essential for advancing technologies in drug delivery, lab-on-a-chip devices, and miniaturized systems.

[7] We use "non-time-independent" instead of "time-dependent" to avoid potential suggestion of limitation to thixotropy (which is usually labeled as time-dependent).

2. Investigation of various aspects, attributes and phenomena of thixotropy such as the rate of strain in thixotropic fluids (and its dependencies especially on time),[8] reversibility in thixotropic cycles, the factors that influence the thixotropic index, memory effects in general, and so on.
3. Investigation of non-time-independent non-Newtonian effects and attributes (mainly related to viscoelasticity and thixotropy) at microscopic and sub-microscopic levels to understand the underlying mechanisms of these phenomena and their aspects and attributes.
4. Employing and developing more realistic fluid models in the investigation and modeling of non-time-independent non-Newtonian fluid flows.
5. Investigating various aspects related to non-time-independent non-Newtonian effects and attributes (such as the influence of memory effects on heat transfer and flow stability).
6. Investigating how non-time-independent non-Newtonian behavior can be exploited in new scientific, medical and technological applications.[9]
7. In fact, even some topics of yield stress fluids[10] (such as the flow of these fluids in complex structures where yield can have complex time-dependent behavior pattern) could be included (although yield stress is generally considered as a time-independent effect).

2.6 Some Types of Time-Independent Non-Newtonian Fluids and Behavior

In the previous subsection we discussed the research areas of non-time-independent non-Newtonian fluid mechanics that require more attention and investment. This may wrongly suggest that the topics of time-independent non-Newtonian fluid mechanics are properly and sufficiently investigated and researched. However, this is not true in general since some of the topics of time-independent non-Newtonian fluid mechanics are not given sufficient attention in comparison to other (or corresponding) topics of time-independent

[8] In fact, time dependencies can be classified into two main types: intrinsic due to the thixotropic nature and extrinsic due to external variation of applied stress by the acting agent. This should also apply to time dependencies of viscoelastic fluids which we discussed in the previous point.

[9] Some potential applications that can benefit from the investigation of non-time-independent non-Newtonian effects and attributes are: biomedical applications (such as blood flow and related aiding devices), 3D printing, oil extraction and refinement, and industrial manufacturing in general.

[10] Actually, this may apply to non-Newtonian aspects and attributes other than yield stress which are commonly and primarily classified as time-independent behavior.

non-Newtonian fluid mechanics.

For example, shear-thickening fluids and behavior are not given the same level of attention as shear-thinning fluids and behavior. Although this may be justified by the fact that shear-thinning fluids and behavior are more common, this is not an excuse from a theoretical viewpoint (and even from some practical viewpoints where shear-thickening fluids and behavior can potentially have some important uses and applications that can match or even exceed the importance of uses and applications of shear-thinning fluids).

Another example is yield stress fluids and behavior. Although there is a considerable amount of literature about yield stress fluids (and materials in general) and their behavior and attributes (as well as uses and applications), there are still many aspects of yield stress and related phenomena which are not well understood and hence yield stress and related aspects and phenomena require further attention and investment in research.[11]

In short, leveling up in research in these topics (and other similar topics) of time-independent non-Newtonian fluid mechanics is required despite the fact that time-independent non-Newtonian fluid mechanics was (and is still) an area of active and extensive research (at least since the mid-twentieth century).

2.7 Non-Newtonian Behavior Under High Strain Rates

Most of the existing studies of non-Newtonian fluid mechanics are related to low and medium rates of strain. This is due first to the relative simplicity of these investigations in comparison to the corresponding investigations at high strain rates (where many complications arise at high strain rates), and second to the fact that non-Newtonian fluid systems at high strain rates are relatively rare in comparison to those at low and medium strain rates.

Anyway, the behavior of non-Newtonian fluids at high strain rates (such as during impacts or shock loading) especially with regard to shear-thickening and memory fluids is an area of research that requires more attention and investment. This demand is amplified by the fact that these investigations are essential for understanding impacts, crash dynamics, flows and ejections at high rates of strain, and some advanced manufacturing processes (as well as many other natural phenomena and industrial processes).

A sample of suggested areas of research in this field is:

[11] For example, more research is required in modeling yield stress fluids and behavior in ordered and disordered porous media so that the yield point (for instance) of flow in such media can be predicted. Also, the development (and time dependencies) of flow after yield should be investigated thoroughly so that they can be predicted and comprehended.

1. Developing more accurate and realistic models for these extreme conditions since most of the existing non-Newtonian fluid models (whether time-independent or not) are not capable of dealing with the complexities of non-Newtonian behavior at high rates of strain.
2. Investigating the transitions between different flow regimes to develop a better understanding of these transitions (and related processes and phenomena).
3. Investigating how non-Newtonian behavior at high rates of strain can be exploited in new scientific and technological applications (as well as in improving our understanding of many natural physical phenomena and processes).[12]

2.8 Non-Newtonian Fluids in Biological and Biomedical Systems

Most of the past and current research in biological and biomedical fluid mechanical systems (such as blood circulation system and related devices) is based on Newtonian fluid model and assumptions. Although this is valid in many instances (either exactly in some cases and circumstances or approximately in many cases and circumstances) it is not valid in general especially in some circumstances, systems and under certain conditions. In fact, the rheology of most biological fluids (such as blood, mucus, saliva, lymph and synovial fluids) as well as many fluids used in biomedical applications and devices (such as gels and oils) is non-Newtonian. While this is well-known, there is still much to be explored in the context of how these fluids behave in varying normal and pathological conditions (e.g. diseases, mechanical stress, varying strain rates, varying temperature and pressure, and so on).

A sample of suggested areas of research in this field is:

1. Developing and adopting (and even adapting existing) non-Newtonian fluid models that can cope with the specialties of biological fluid systems (noting that most of the existing non-Newtonian fluid models are developed for industrial and non-biological fluid systems).
2. Investigating how non-Newtonian behavior can be exploited in improving our understanding of biological phenomena and medical situations.[13]

[12] For example, a better understanding of non-Newtonian behavior at high rates of strain can help in industries ranging from automotive to aerospace, where non-Newtonian fluids are used for damping, shock absorption, or structural protection (as well as many other processes). It can also help in understanding many natural physical phenomena such as lava flow dynamics, eruption styles and magma transport in volcanoes (as well as some geological and astronomical processes related to non-Newtonian fluid dynamics).

[13] An example is investigating how the rheological properties of non-Newtonian biological fluids change in

3. Investigating how non-Newtonian behavior can be exploited in biomedical applications such as developing medical devices and medical procedures.[14]

2.9 Multi-Phase Non-Newtonian Flow

Multi-phase non-Newtonian flow[15] (and actually multi-phase flow in general) is one of the areas of research that is not investigated properly and sufficiently (due mainly to its difficulty and complexity)[16] and hence it requires further attention and investment. Multi-phase flow in general and non-Newtonian in particular have important scientific instances and technological applications such as in biological and biomedical systems, food manufacturing and processing, and oil recovery and refinement.

A sample of suggested areas of research in this field is:

1. Developing novel models and effective strategies (whether analytical or numerical and computational) for tackling multi-phase flow in general and non-Newtonian flow in particular.
2. Giving observational and experimental investigations about multi-phase flow more attention and resources (by encouraging and funding these investigations).
3. Investigating multi-phase flow in various contexts and situations such as in biological versus non-biological systems and in porous media versus conduits of rather simple geometries (such as pipes of circular and elliptical cross sections, thin slits, and open channels of rectangular cross section).
4. Investigating multi-phase flow under various conditions and with various associations such as at extreme ambient conditions (e.g. temperature and pressure) or with and without flow stimulation (e.g. by mechanical vibration).
5. Investigating various issues and situations associated with and related to multi-phase flow such as heat transfer and interfacial interaction in multi-phase flow.

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pathological states (like in the case of blood during atherosclerosis or mucus during cystic fibrosis) and how this change could have important implications and impacts on pathological symptoms, medical diagnostics and treatment design.

[14] An example is investigating how certain non-Newtonian attributes (such as shear-thinning, yield stress and viscoelasticity) can be exploited maximally and beneficially in the drug delivery devices and procedures.

[15] We should note that "multi-phase non-Newtonian flow" should include the cases where only some phases are non-Newtonian as well as the cases where all phases are non-Newtonian.

[16] In fact, multi-phase flow in general and non-Newtonian in particular are notorious for their difficulties and complexities. This may be inferred from the fact that most of the existing studies about multiphase flow (including non-Newtonian) are of numerical and computational nature (e.g. lack of decent and sufficient analytical and experimental approaches and efforts in this regard) and they are based on many simplifications and approximations.

6. Investigating how non-Newtonian effects (such as shear-thickening, yield stress and viscoelasticity) affect multi-phase flow (by comparing Newtonian and non-Newtonian flows in corresponding situations to assess the impact of the non-Newtonian effects on multi-phase flow).
7. Investigating how non-Newtonian behavior in multi-phase flow can be employed scientifically and exploited technologically and industrially.

2.10 Flow Stability and Transition in Non-Newtonian Fluid Dynamics

There are many virgin (or under-explored) issues and areas of research related to the stability of flow and the transition between its different phases and stages. For example, the transition between laminar and turbulent flow in non-Newtonian fluids is still not fully understood, particularly when complicated non-Newtonian effects (such as shear-thickening or viscoelasticity or thixotropy) come into play in this transition. In fact, these issues and areas of research have many theoretical and practical benefits and applications. For instance, investigating flow instabilities and transition mechanisms in non-Newtonian fluids can shed light on many scientific issues and industrial processes, such as mixing, pumping, and heat transfer which often rely on and are affected by flow stability and phase transitions.

2.11 Rheology of Non-Newtonian Fluids Under External Fields

There is an emerging interest in how non-Newtonian fluids behave under the influence of external fields (such as gravitational or electric or magnetic fields). In fact, this research is still in its infancy. Investigating how external fields influence the micro-structures, rheological properties, and flow behavior of non-Newtonian fluids could have theoretical and practical applications in areas like astronomy, smart fluids, sensors, and new material design.

2.12 Other Areas

There are many other virgin and insufficiently-explored areas of research in non-Newtonian fluid mechanics which may not be included primarily under the broad titles of the previous subsections. Some examples of these areas are:

1. Non-Newtonian effects in turbulent flow. In fact, turbulent flow in general (even of Newtonian fluids) is a very challenging and subtle subject and hence it requires more attention and investment in research.
2. Interaction of Non-Newtonian effects with other fluid dynamics effects and situations such as slip at fluid solid interface and multi-phase flow.
3. Technological and industrial applications. There are many technological and industrial applications of non-Newtonian flow behavior which are not properly and sufficiently explored and hence they require more attention and investment in research. An example is non-Newtonian fluids behavior in additive manufacturing (which is commonly known as 3D printing) where the use of non-Newtonian fluids in these manufacturing processes (such as shear-thinning, shear-thickening and viscoelastic materials) is still an emerging field. The flow behavior during extrusion, deposition, and solidification in these processes (and similar industrial processes) is not fully understood. Understanding the interplay between rheology and printing parameters (such as speed, temperature, and layer thickness) could lead to the design of more efficient 3D printing techniques, especially for complex materials (such as hydro-gels and bio-materials).
4. Non-Newtonian fluids in high energy systems. Non-Newtonian fluids are used in some high energy technologies and processes (such as geothermal drilling and hydraulic fracturing), but the fluid dynamics under extreme conditions (mainly temperature and pressure) and at large-scale is not fully understood. Developing analytical and computational models to describe and simulate non-Newtonian fluid behavior in such high energy systems and under extreme environmental conditions (such as in deep-sea and deep-well environments) could provide significant advancements in these high energy technologies. Experimental work in these areas should also be beneficial to such objectives.

3 Introducing Novel Tools and Methods

In this section we briefly discuss (in broad and general terms) the novel tools and methods that could and should be introduced in the research of non-Newtonian fluid mechanics in general (and in the research of virgin and under-explored areas in particular). In fact, introducing such tools and methods is not only beneficial to this research but in many cases it is a necessity and demand.

3.1 Novel Analytical Methods and Strategies

It is important to consider developing novel analytical methods and strategies to deal with the difficult problems and subtle issues of non-Newtonian fluid mechanics. In fact, this should include the adoption and use of traditional analytical methods and strategies which are ignored or not given sufficient attention in the past and current research of non-Newtonian fluid mechanics. For example, more attention should be given to the use of variational approaches and optimization methods in tackling some of the problems and issues of non-Newtonian fluid mechanics in general and the novel and under-investigated ones in particular (see for example § [3, 4]).

3.2 Novel Computational Methods and Strategies

It is important to consider developing novel computational and numerical methods and strategies (as well as adopting and using traditional computational and numerical methods and strategies which are ignored or not given sufficient attention in the past and current research) to deal with the difficult problems and subtle issues of non-Newtonian fluid mechanics in general and the novel and under-investigated ones in particular. The obvious example in this regard is artificial intelligence where machine learning techniques (for instance) can be used for more efficient description and prediction of flow properties and attributes. In fact, artificial intelligence can be a great aiding tool for dealing with highly complex situations which are difficult or impossible to deal with by using traditional methods. Moreover, it can be an aiding tool even in developing novel analytical methods and strategies (which we discussed in the previous subsection).

3.3 Novel Experimental Methods and Strategies

Again, it is important to consider developing novel experimental methods and strategies (as well as adopting and using traditional experimental methods and strategies which are ignored or not given sufficient attention in the past and current research) to deal with the difficult problems and subtle issues of non-Newtonian fluid mechanics in general and the novel and under-investigated ones in particular. In fact, more investment in experimental work is required in the research of non-Newtonian fluid mechanics in general (noting that the past and current research in this field is overwhelmingly theoretical and computational).

4 Limitations of Past and Current Research

In this section we highlight some of the limitations and shortcomings of the past and current research in non-Newtonian fluid mechanics. These limitations and shortcomings should have a special impact on any potential research in virgin and under-investigated areas in this field.

4.1 Limitations of Fluid Models

There are many limitations in the non-Newtonian fluid models and methodologies which are developed and used in the past and current research in this field. Some examples of these limitations are:

1. Most of the developed and employed non-Newtonian fluid models in the past and current research in this field are simple and rather unrealistic (such as power law model). Novel research should address this issue by developing and using more realistic (and usually more complicated) non-Newtonian fluid models. This particularly true for the investigation of processes and phenomena that involve significant non-Newtonian effects of non-time-independent nature such as viscoelasticity and thixotropy (as will be outlined in the next point).
2. Most of the developed and employed non-Newtonian fluid models in the past and current research in this field are about time-independent flow. Novel research should address this issue by developing and using non-time-independent flow models.
3. Most approaches in tackling non-Newtonian flow phenomena adopt continuum models (where the fluids are generally treated as consisting of a single homogeneous phase). Although these models are good in many cases and circumstances, they are not realistic in general (for example in modeling some polymers or some biological fluids in certain biological systems). Novel research should address this issue by developing and using non-continuum models to deal with situations where the continuum models are not appropriate.

4.2 Limitations of Conditions and Assumptions

Most of the past and current investigations in non-Newtonian fluid mechanics are based on simplistic and rather unrealistic conditions and assumptions with regard to the flow, fluid, conduit and environment (such as laminarity of flow, incompressibility of fluid, and simplicity of geometry of flow conduit and ambient conditions like being steadystate and isothermal). Although such simple conditions and assumptions are sufficient in many situations (especially those of practical use), they are not acceptable in general where the situation is too complicated to be modeled appropriately by such conditions and assumptions. The following are a few examples of complicated cases and situations that require more complex and realistic conditions and assumptions:

1. Non-Newtonian flow under extreme physical conditions (such as high pressure and temperature).
2. Non-Newtonian flow in dynamical mechanical environment and physical conditions (e.g. in vibrating vessels and rotating conduits or in non-adiabatic and non-isothermal conditions).
3. Non-Newtonian flow in conduits with peristaltic movement.
4. Non-Newtonian flow in porous media of complex morphology and topology.
5. Non-Newtonian flow with yield stress.
6. Non-Newtonian flow with non-time-independent non-Newtonian effects.
7. Non-Newtonian flow with total or partial wall slip.
8. Non-Newtonian flow in microscopic and sub-microscopic fluid systems.
9. Cases and circumstances that require consideration of effects and influences beyond the basic flow situation. For example, more complex and realistic conditions and assumptions are required in the investigation of issues and aspects required to develop a proper understanding of the role and impact of boundary layers, flow instabilities, inertial effects, and the effects of surface roughness in and on non-Newtonian fluid dynamics.

4.3 Limitations of Topics and Phenomena

There are many limitations of the topics and phenomena in the past and current research in the field of non-Newtonian fluid mechanics. A small sample of those topics and phenomena which require further attention to address these limitations and gaps are:

1. Non-Newtonian fluid-structure interactions.
2. Non-Newtonian flow in astronomical and astrophysical systems.
3. Non-Newtonian flow in turbulent situations.
4. Non-Newtonian flow in many special settings and circumstances such as non-Newtonian flow phenomena and effects at branching junctions or around certain geometric objects (e.g. immobile/vibrating/rotating cylinder of various cross sectional shapes or sphere or other 3D geometric objects).
5. Shear banding in non-time-independent non-Newtonian fluid flow.

4.4 Limitations of Experimental Investigations

As indicated earlier, there are relatively few experimental studies in non-Newtonian fluid mechanics in comparison to theoretical and computational studies. This is due to the practical difficulties and inconveniences which are usually associated with experimental projects as well as their extra costs and expenses in comparison to corresponding theoretical and computational studies.[17] Anyway, leveling up (or at least reducing the gap) is required in this regard where more efforts and resources should be allocated to the experimental (including observational) work in this field. For example, more funding should be allocated to experimental PhD programs and post-doctoral research projects.[18]

5 Conclusions

We outline in the following points the main achievements and conclusions of the present paper:

1. There are many virgin and insufficiently-investigated areas of non-Newtonian fluid mechanics. In this investigation we drew the attention of the research community in the subject of non-Newtonian fluid mechanics to those research areas and zones of investigation that require more attention and further investment of efforts and resources in the future research in this subject.
2. In this investigation we identified the main virgin and under-investigated areas of research in the field of non-Newtonian fluid mechanics (see § 2). We also discussed briefly the novel tools and methodologies that should be considered and introduced in the research of these virgin and under-investigated areas (and actually in non-Newtonian fluid mechanics in general; see § 3). Similarly, we discussed briefly the main limitations and shortcomings of the past and current research in non-Newtonian fluid mechanics (see § 4). All these investigations and discussions should help in tackling the deficiencies and addressing the shortcomings of the past and current research in non-Newtonian fluid

[17] Actually, there is another important reason for this bias against the experimental studies that is experimental work is generally seen (or "stereotyped") as inferior to theoretical and computational work and hence "great scientists should not get their hands dirty" as they should leave dirty tasks to lower-rank scientists or even technicians. This silly stereotype should be changed for the benefit and advancement of science.

[18] In fact, we suspect that there are more experimental studies in this field (as well as many other fields and branches of science and technology) within the industry and certain governmental departments and agencies but theses studies are not published due to their sensitive nature or commercial competition or lack of interest in publicity or lack of interest in any benefit beyond the direct practical benefit especially when such benefit requires unduly extra work (or other similar reasons and factors).

mechanics.

3. Our investigation in this paper should help to level up the research in this field and fill the gaps and vacancies in this subject. It should also help to avoid rumination in the research of this field where some researchers repeat what have already been done or add very little novelty to previous research (or "reinvent the wheel").
4. Our investigation should also help PhD students and their advisers (as well as researchers of higher ranks) in identifying PhD programs (and research projects) that are worthy of investigation and worthwhile for investment of efforts and resources for the best outcome and advancement of science and technology (and knowledge and practical benefits in general) as well as for the best outcome and advancement of their career.
5. Our investigation should also motivate and inspire those within the research community who are looking for breakthroughs and new discoveries in the field of non-Newtonian fluid mechanics (and even beyond). In fact, many of the aforementioned areas of research offer a mix of fundamental challenges (theoretical, computational and experimental) and practical applications where there is considerable room for new contributions. Exploring these areas (with traditional and novel tools and methodologies and with elimination or minimization of the aforementioned limitations) could lead to groundbreaking research that advances the understanding and application of non-Newtonian fluids across many fields of science, medicine and technology.

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