ON SOME RESULTS ON INFINITE, UNDER CERTAIN ASSUMPTIONS

JAYANT SHARMA

ABSTRACT. In this document, we formally initiate the study on the mathematical properties of divergent infinity, under certain assumptions, made in the paper. While under these assumptions the paper, certainly proposes a better framework, but doesn't fits for certain standards of mathematical rigour.

We define, under an assumption, the Φ, which later, is shown to be beyond any mathematical descriptions yet permeates, zero and infinite. In the conclusion, we also present a conclusion *not to be sought under mathematical rigour, but rather as a potential field for future study.*

CONTENTS

1. ASSUMPTIONS OF THE STUDY

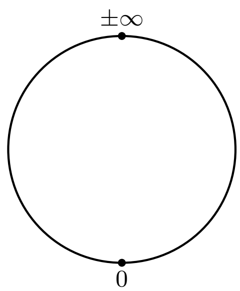
For the sake of study, we assume, $\mathbb{T}$ is the extended number system including the real numbers as well as the *infinities.* That is,

$$\mathbb{T} = [-\infty, \infty]$$

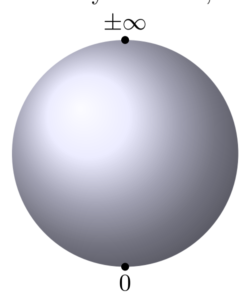
Since, we will be working under $\mathbb{T}$, it follows as another assumption that, under this study, we no longer consider $\pm\infty$ an idea, but rather as a quantity, such that,

$$\pm\infty > \lim_{h \to \pm\infty} h$$

And thirdly we assume and embrace a new model of numbers under the study within $\mathbb{T}$, that is,



$\mathbb{T}$ for real numbers



$\mathbb{T}_i$ for imaginary numbers

That is, formally stated equivalent to,

$$\lim_{h \to \infty^+} h = \lim_{h \to \infty^-} h$$

which is equivalent to,

$$\lim_{h \to 0^+} \frac{1}{h} = \lim_{h \to 0^-} \frac{1}{h}$$

Date: May 1, 2025.

And both such limits exist.

2. Preliminaries

Under the three assumptions made in the earlier section, we now, formally state our first lemma,

Lemma I. *Under* $\mathbb{T}$*,*

$$\frac{k}{0} = \pm\infty \quad k \in \mathbb{R}$$

holds true.

Proof. If *left hand side limits* and *right hand side limits*, both agree and exist for a certain value, then we can continue the function for that certain point, even if it seems undefined at first sight.

That is if,

$$f(a) \neq k \quad \text{(and is undefined)}$$

but if,

$$\lim_{h \to a^+} f(h) = \lim_{h \to a^-} f(h) = k \quad \text{(both do exist)}$$

then,

$$\tilde{f}(x) = \begin{cases} f(x), & x \neq a, \\ k, & x = a \end{cases}$$

Applying this, we continue the real function,

$$g(x) = \frac{k}{x} \quad k \in \mathbb{R}$$

Under the number model of $\mathbb{T}$, and using our third assumption,

$$\lim_{h \to 0^+} \frac{k}{h} = \lim_{h \to 0^-} \frac{k}{h} = \pm\infty$$

And both do exist, under the assumptions made earlier. So, we can continue it to,

$$\frac{k}{0} = \pm\infty \quad k \in \mathbb{R}$$

(end of proof)

Similarly, we state,

Lemma II. *Under* $\mathbb{T}$*,*

$$\frac{k}{\pm\infty} = 0 \quad k \in \mathbb{R}$$

holds true.

Proof. By $\mathbb{T}$,

$$\lim_{h \to \pm\infty^+} \frac{k}{h} = \lim_{h \to \pm\infty^-} \frac{k}{h} \quad k \in \mathbb{R}$$

And both limits do exist, by the smooth behaviour of $\mathbb{T}$ at the infinities, and zeroes.

Hence we continue it to,

$$\frac{k}{\pm\infty} = 0$$

(end of proof)

Lemma III. *Under* $\mathbb{T}$*,*

$$-\infty = +\infty$$

holds true.

Proof. This is as a direct consequence of the assumptions and the structure of *number model in* $\mathbb{T}$. *(end of proof)*

3. Results On $\mathbb{T}$

In this section, we use the previously proved lemmas to build some results on the model of ,$\mathbb{T}$.

Theorem I. *The expression,*

$$0 \times (\pm\infty) = \Delta$$

holds true. *Δ denotes indeterminacy.

Proof. For any $k \in \mathbb{T}$,

$$k = \frac{k}{c} \times c$$

now, by the definition of $\mathbb{T}$ these limits exist as,

$$\lim_{c \to \pm\infty^+} \frac{k}{c} \times c = \lim_{c \to \pm\infty^-} \frac{k}{c} \times c = k$$

and also,

$$\lim_{c \to 0^+} \frac{k}{c} \times c = \lim_{c \to 0^-} \frac{k}{c} \times c = k$$

so we can continue, this analytically as,

$$\frac{k}{\pm\infty} \times (\pm\infty) = k$$

and also,

$$\frac{k}{0} \times (0) = k$$

which, by using the *Lemma 1* and *Lemma 2*, follows in,

$$0 \times (\pm\infty) = k$$

and since, k can be anything in $\mathbb{T}$, due to this indeterminacy, we propose,

$$0 \times (\pm\infty) = \Delta$$

(end of proof)

Theorem II. *Let, $f : A \to B$ such that,*

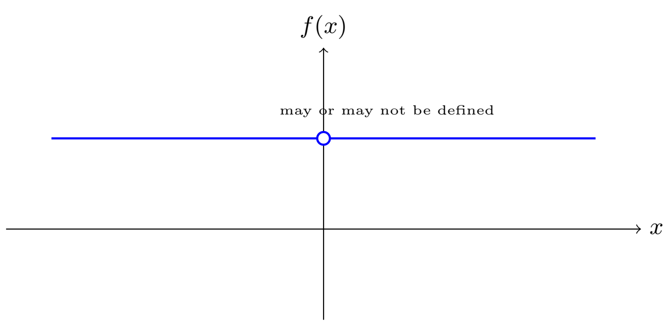
$$B = \{c\} \forall x \in A \in \mathbb{R}$$

That is,

$$f(x) = \mathit{constant} \neq \pm\infty, 0 \quad \forall x \in \mathbb{R}, \quad x \neq 0$$

Then,

$$f(\pm\infty) = \Delta \quad \textit{(Indeterminate)}$$



may or may not be defined on zero

Proof. The *graph* of such a function resembles a like parallel to one of the axes.

And since by *Theorem 1*,

$$g(x) = 0 \times x$$

is indeterminate at $x = \pm\infty$, so will be the function,

$$g(x) + c = 0 \times x + c = f(x)$$

and since, $g(\pm\infty) = \Delta$ it follows that, $f(\pm\infty) = \Delta$ as well. And since, $f(x)$ is the general form for any constant function, our *Theorem 2* is hence proven. *(end of proof)*

Principle I (Principle of Dependency)**.** *If a function $f : A \to B$ is independent of a certain set of quantities $X = \{x_0, x_1, x_2, \ldots, x_n\}$, then under $\mathbb{T}$, the function is transformed to have a zero-dependency at $A \cap \mathbb{R}$.*

**This principle clarifies that, if a function is independent of a certain quantity, then, under $\mathbb{T}$, it isn't actually independent, but rather carries a zero-dependency over the real numbers.*

Proof. Assuming that, the function $f : A \to B$ can be written as,

$$f(k_0, k_1, k_2, \ldots, k_n)$$

And let, $\mathfrak{Z}_f(x_0, x_1, x_2, \ldots, x_n)$, be the *change in f as the quantities X change.*

Then, since f is independent of X, the function $\mathfrak{Z}_f$ is a constant function of X, and hence, by *Theorem 2*

$$\mathfrak{Z}_f = \Delta \quad \text{(When any quantity} \in X \text{ equals } \pm\infty)$$

Hence, if the function f isn't transformed to have a *Zero Dependency*, it will violate the above statement, as it won't evaluate itself *having an indeterminate dependency at infinite*, hence proving our principle. *(end of proof)*

4. On Φ and conclusion

In the last section, by the *Principle of Dependency*, we saw that, every function is dependent upon every quantity, however, in the real numbers system, this dependency further evaluates to *Independency*.

However, this section, comparatively less rigorous to others, still provides a direction towards the future formalizations in the field. Hence to begin, we define formally,

Definition I. Φ *denotes what is common in both zero and infinities.*

In other words, Φ is what permeates both zero and infinite.

In the earlier philosophies, this is what matches the *Sāṃkhya Model*, where the Φ is proposed to be the fabric of existence and inexistence, from which they blossomed, and which is present in both of them and is beyond them. Since, Φ in itself is the *subtler existence common in both existence and inexistence* it serves as the ground on which any properties and values are assigned and thus itself possesses no property or value. This idea was firmly believed, and hence, later the first seperation of witness and witnessed is best described by 0 being the witness and $\pm\infty$ being the witnessed and both when interact possess the property to birth the universe.

Hence, this model above best describes the ancient knowledge.

In the mathematical planes, *Uncountably Infinite*, lines stack together to give birth to a plane and this continues forever higher dimmensions too.

Which later raises these conjectures

Conjecture I. *Is the $\pm\infty$ considered in $\mathbb{T}$ (and in the paper) uncountably infinite, or it can be countably infinite as well?*

Conjecture II. *Can those assumptions used for the model of $\mathbb{T}$ in this paper be reduced to only one axiom?*