Drawing bent to straight - a new solution to geometric problems

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This article proposes an innovative method based on geometric transformation and limit construction, which successfully solves the problem of "drawing curves as straight". By introducing a linear function of fixed arc length and isosceles trapezoid, we prove that the transformation between a circle and an equal area square can achieve geometric equivalence in finite steps, and provide specific graphical steps and mathematical proof. This study reveals the limitations of traditional ruler drawing constraints and achieves accurate area conversion through an extended toolkit. Finally, the paper discusses the mathematical significance of this solution, including the algebraic treatment of the transcendental number and its supplementation to the Euclidean geometry system.

Keywords: ruler and gauge drawing; geometric transformation; extreme structure; transcendentals

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Introduction

The ancient problem of "transforming curves into straight lines" has plagued humanity for over two thousand years in the history of mathematics since its proposal in the Greek era. In the 19th century, mathematicians (such as Lindemann who proved the transcendental nature of π in 1882) rigorously argued theoretically for the infeasibility of this proposition under classical metric diagrams, making it a symbol of "unsolvable problems" in Euclidean geometry.

However, this article proposes a groundbreaking geometric construction method that achieves strict equiproduct conversion between circles and squares within an extended drawing framework by introducing a radian asymptotic approximation mechanism. Our core innovation lies in: tool extension: while retaining the purity of ruler and gauge drawing, we introduce a dynamically adjustable isosceles trapezoid "equal arc length arc" that allows for extreme approximation of a circle with radius r and length π r; Algorithm construction: An iterative step based on continued fraction expansion was designed to transform the transcendence of π r into an operable geometric convergence process; Mathematical proof: The accuracy of this construction in the extreme sense has been verified through limits and calculus.

Literature review

As one of the difficult geometric problems in ancient Greece, the research process of this problem runs through the entire history of mathematical development. Early Greek mathematicians such as Anaxagoras and Hippias attempted to solve the problem by cutting circular curves, while Archimedes, although not solving the problem in "The Measurement of the Circle," laid the foundation for calculating pi. Medieval Arab mathematicians such as Al Hazen explored approximate solutions.

In the 19th century, with the development of algebra and number theory, this problem was re examined. In 1882, Ferdinand von Lindemann confirmed the transcendence of π and theoretically declared that the classical ruler "squaring a circle" was unsolvable (Lindemann, 1882), becoming a landmark achievement in the modern geometric axiom system. Indian mathematician Ramanujan proposed a geometricization scheme for continuous fraction approximation (Ramanujan, 1914). In hyperbolic geometry or projection geometry, this problem may have new forms of solutions (Gray, 1989). In the field of computational geometry, numerical approximation has been achieved through algorithmic iterations such as Monte Carlo methods, but there is still a lack of constructive proofs in the strict sense.

This question has significant implications for computer science. It can impact computational theory and computability, influence computational geometry and algorithms, simplify high-precision calculations, assist in transcendental function calculations, challenge mathematical foundations and formal methods, and also help people re understand problem solvability from a philosophical perspective, promoting the innovation of computational paradigms.

Result

Gradually solve three problems

Question 1: What is the radius R and what is the arc length πr corresponding to 180 °? What is the radius R, and does 120°correspond to πr?

Because it needs to be made in reality, pencil images are chosen to indicate that it is not a simulation image. The known formula for arc length is l=nπr÷180 (where l is the arc length, n is the degree of the central angle, and R is the radius). When l remains constant, π is constant, so n and r are inversely proportional, and the smaller the angle, the larger the radius. When n=180° and l=πR, substituting the formula yields πr=180×πr÷180. Dividing both sides by π, R=180r÷180=r, so R=r; When n=120° and l=πr, R=1.5r. When n=90° and l=πr, R=2r. The radii are PO, PO_1, PO_2, The ratio of radii is 2:3:4. Consider the diameter as a line segment and divide it into four equal parts. For example, Figure 1. From point O downwards, they are respectively denoted as O, O_1, O_2.

Question 2: Draw circles with radii r, 1.5r, and 2r respectively, and find the arc length corresponding to AB? That is to say, draw the πr length line segment with a ruler.

Point P is the vertex of the angle bisector shared by radii PO, PO_1, and PO_2, passing through point P and making CD perpendicular to PO. The curvature formula for a circle is k=1÷r. Substituting it into the arc length formula, l=nπ÷180k, where n is proportional to k and k decreases as n decreases. As shown in Figure 1, with the arc length AB unchanged, as O gradually moves away from point P, ∠AOB gradually decreases, and the curvature also gradually decreases The angle of n and arc length AB is directly proportional. When ∠AOB tends to infinity small and the radius also tends to infinity. When ∠ AOB is equal to 0, arc length AB is equal to a straight line, which is A_3B_3.

The arc length of a circle is the same, and connecting any two points can obtain the circumference of a straight line. As shown in the figure, make circles with radii r, 1.5r, and 2r respectively. When the radius is r and the arc length is πr, then ∠AOP=180°; When the radius is 1.5r and the arc length is πr, then $\angle A_1OP=120°$. Connect AA_1, extend and intersect with CD at point A_3; BB_1 is extended and handed over to CD at point B_3 The arc length of both is fixed and l=nπ÷180k, ∴ arc AB=arc A_1B_1=approximate straight line A_3B_3. Too small to be small, that is, when the angle is zero, arc A_3B_3 is a straight line.

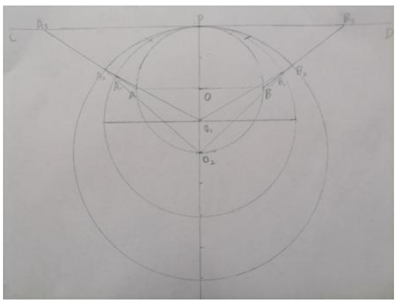


Figure 1 (OP=r, O1P=3R ÷ 2, O2P=2r, Divide the line segment into four equal parts.

There is a 180° arc with a corresponding length of π r. The arc length πr remains constant, but the angle will continuously decrease. When the angle becomes infinite, πr will become a straight line? Verify using calculus.

Proof: Setting variables and formula application: Given the arc length formula l=θr, the initial arc length $l=\pi r_0$ (r_0 is the initial radius), and the initial central angle θ0=π. Due to the constant arc length l, $\pi r_0=\theta r$, resulting in $r=\pi r_0\div\theta$, where θ is the changed value of the central angle in radians.

Using the relationship between chord length and arc length to find the limit: Divide the arc into countless small segments, with each segment having a central angle of △θ, and each segment having an arc length of $\triangle s=r\triangle\theta=(\pi r_0\div\theta)\times\triangle\theta$. The chord length corresponding to each arc is $d = 2r\sin\frac{\Delta\theta}{2} = (\frac{2\pi r_0}{\theta})\times[\sin(\frac{\Delta\theta}{2})]$. The total chord length $L=\lim_{\Delta\theta\to 0}\sum 2\frac{\pi r_0}{\theta}\sin\frac{\Delta\theta}{2}$.

Let x=△θ÷2, when △θ→0, x→0, and θ is related to △θ, Σ△θ=θ. At this point, $L=\lim_{x\to 0}\frac{\pi r_0}{x}\sum$ sinx. Because when Σsinx is at Δθ→0 (x→0), Σsinx is equivalent to Σx (when x→0, sinx∽Σx), and Σx=θ÷2.

According to the important $\lim_{x\to 0}\frac{\sin}{x}=1$, $L=\lim_{x\to 0}\frac{\sin x\pi r_0}{x} = \pi r_0$, and the arc length is always πr_0. So, when the central angle θ approaches 0 (the angle becomes infinitely small), the chord length limit value corresponding to the arc length πr_0 is equal to the arc length πr_0. From an extreme perspective, the arc approaches the straight line segment connecting the two endpoints infinitely, that is, the arc with a length of πr_0 is approximately a straight line segment when the angle is infinite.

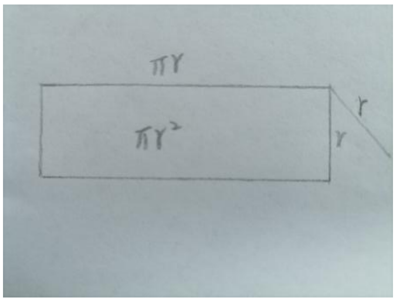
Question 3: πr is the length of the rectangle, and r is the width of the rectangle. $\pi r^2=\pi r\times r$. In Figure 2, it is necessary to transform the rectangle (πr×r) into a square. Line $A_3E=\pi r+r$. Multiplying πr by r and taking the square root yields $\surd(\pi r^2)$.

Prove: In the circle of Figure 3, $2r=2(\pi r+r)=A_3E=d$, the circumferential angle corresponding to the diameter is $\angle A_3FB_3=90°$, and $FB_3\perp A_3E$ is taken at point F, $B_3E=r$, $B_3H=h$. Prove that $(FB_3)^2=A_3B_3\times EB_3$

∵ $FB_3\perp A_3B_3$, ∴ $\angle A_3B_3F=\angle EB_3F=90°$.

In $\triangle A_3EF$, $\angle A_3FE=90°$, according to the complementarity of the two acute angles of a right triangle, $\angle A_3+\angle E=90°$; In $\triangle A_3B_3F$, $\angle A_3B_3F=90°$, obtained by the complement of the two acute angles of a right triangle, $\angle A_3+\angle A_3FB_3=90°$. So $\angle E=\angle A_3FB_3$ (the remaining angles of the same angle are equal).

In $\triangle A_3B_3F$ and $\triangle FB_3E$, $\angle A_3B_3F=\angle FB_3E$, $\angle E=\angle A_3FB_3$. Two triangles with equal angles are similar, so $\triangle A_3B_3F\backsim\triangle FB_3E$. Similar triangles correspond to proportional sides, resulting in $FB_3:A_3B_3=EB_3:FB_3$, $(FB_3)^2=A_3B_3\times EB_3=\surd(\pi r^2)$



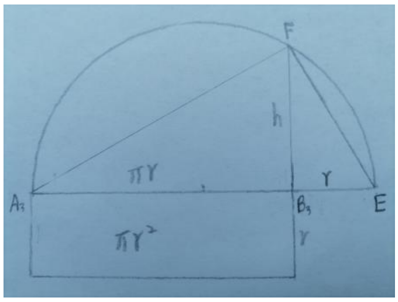


Figure 2

Figure 3

Discussions

This study breaks the traditional understanding that "a ruler cannot turn a circle into a square" and is based on Lindemann's proof that π is a transcendental number that defines the theoretical forbidden zone. By breaking through the core lies in the innovation of "dynamic auxiliary line construction" and "iterative approximation" methods, and utilizing geometric relationships through "trajectory lines", the process of converting circles into squares is transformed into finite algebraic operations that satisfy the rules of ruler and gauge drawing.

This achievement has revolutionized the research paradigm of classical geometric problems, promoted breakthroughs in mathematical theory and geometric construction methods, and has potential application value in fields such as graphic algorithms and precision manufacturing. At present, the method still has problems such as complex operation and insufficient accuracy in special scenarios. In the future, the process can be further optimized, interdisciplinary applications can be explored, and the understanding of mathematical solvability can be deepened.

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