An argument against quantum computers (or against certain decoherence models)

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Abstract We present a fundamental theoretical obstacle that prevents quantum computers obeying 4 axioms from being more than a constant factor more powerful than classical ones. All presently known quantum error correction and quantum fault tolerant circuit ideas are defeated by this obstacle. The crucial question is whether "axiom #4" (concerning the nature of decoherence) is true in our universe. There are numerous previous decoherence models that both obey and disobey axiom 4.

Previous arguments against quantum computers were of the form "although quantum computers may be valid theoretically, they seem extremely difficult to build in practice." In constrast, the present obstacle is theoretical, but may leave plenty of room for engineers to build quantum computers. That is because it merely shows there is some maximum number $N_{\max}$ of qubits in which one can hope to maintain coherence – but this upper bound might be enormous.

Our argument also may be thought of as a partial explanation of why the macroscopic world appears classical. For that purpose, even a bound as weak as $N_{\max} < 10^{30}$ still seems interesting.

Keywords — Quantum computers, quantum error correcting codes, quantum fault tolerance techniques, decoherence, Church's thesis, quantum-classical transition, gravitational decoherence.

CLAIM: *Any quantum computer made of qubits and qugates satisfying axioms 1-4 below, cannot have more than a constant factor more computational power than a classical computer.*

Axioms:

1. Suppose qubits cannot be smaller than some size (e.g. the size of an atom).
2. Suppose they cannot move faster than some speed (e.g. $c = 299792458$ meters/sec).
3. Suppose no quantum gate can operate faster than some constant time delay (e.g. 1 Angstrom divided by c).
4. Suppose any qubit with Δx "self-separation" will decohere at a rate R which goes to infinity as $\Delta x \to \infty$.

Remarks about the axioms: Axiom 1 may be weakened: it will suffice if the radius $f(N)$ of the smallest ball that can contain N qubits obeys $\liminf_{N\to\infty} f(N) = \infty$. Rigorous upper bounds on bit density are available [15]a. Axiom 2 may similarly be replaced by (weaker) bounds on information flux [15]b. Something like Axiom 3 is a consequence of the rigorous form of the energy-time uncertainty principle [15]c. Axiom 4 seems the crucial one and is discussed below.

The argument: Any quantum computer (satisfying axioms 1-4) more than a constant factor more powerful than a conventional computer must by axioms 1 and 2 have qubits which enter superpositions of being in two locations which are widely separated (indeed further separated than any constant distance). Because: if not, each constant-size chunk of the quantum computer could be classically simulated, with only constant simulation slowdown. Similarly the number of such "wide" qubits must be greater than any constant. To make matters simple, consider some computation in which there must be qubits involved in state superpositions of the form: with probability $\approx 1/2$ this qubit is *here*, and with probability $\approx 1/2$ it is *there* (further than any constant distance apart). In that case, by axiom 4 any such qubit would decohere arbitrarily more quickly than any constant time delay. Meanwhile, "quantum error correction" circuits [11] require a constant number at least of gate delays to perform a correction. Hence these errors will be uncorrectable. Q.E.D.

Another way of looking at this: The rate of destruction of quantum information stored at large self-separation (axiom 4) outpaces, by arbitrarily large factors, the rate at which correction information can flow into the appropriate region (bounded by the speed of light, packing density limitations for quantum gates and wires, in axioms 1,2) and be used (axiom 3). Beyond some size limit, it becomes impossible for correction to outpace destruction. Q.E.D.

Discussion: Aharonov and Ben-Or [1] gave a somewhat similar, but longer and more complicated, argument. Their result states that in a certain model of qubits and their decoherence, any system in which all qubits decohere at some, sufficiently high, constant rate is simulable by a classical machine, with at most polynomial simulation:reality slowdown. (They proved

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this for .97 decoherences per timestep per qubit, and gave experimental evidence .64 would suffice.) This statement neither logically subsumes nor is subsumed by mine, and more importantly they were not trying to draw any conclusion that quantum computers are impossible. (In contrast [11], in the same model but with their constant sufficiently *small* [$\lesssim 10^{-6}$] their decohering quantum systems can efficiently simulate decoherence-free noiseless qubit systems, and hence presumably [14] cannot be simulated efficiently by classical computers.)

I have had versions of this paper since the early days of quantum computing ($\approx$ 1997). The present argument seems concisest. I showed it to Peter Shor and William Unruh. Shor argued in personal correspondence that "Mach's principle" that all physics is local should prevent decoherence from being faster the further apart you go, i.e. my axiom 4 is false. That argument influenced me not to publish. However, I now point out that quantum mechanics (even for only one particle and without "measurements") *is* somewhat nonlocal in the sense that the Aharanov-Bohm effect [13] depends on magnetic vector potential and not on magnetic *field*. That *would* appear to permit arbitarily faster dephasing of a more widely separated qubit. Furthermore, the new simple "gravitational decoherence" model presented at the end of this paper should make it clear that decoherence *can* unboundedly speed up with increased qubit self-separation. Several other (more complicated) decoherence models also obey axiom 4 (see list at end). In contrast, in all the models of decoherence used by the computer scientists (including Shor) in work [11] on quantum error correction and fault tolerance, qubits decohere at a *constant* rate at worst, *not* increasing unboundedly with distance. This computer science model of decoherence is, we claim, *essential* to make quantum error correction work, and indeed my argument and [1] show that. Such models, however, stand in direct contradiction to axiom 4.

Calling all this a "proof that quantum computers are dead" may be an overstatement in which case it would be more correct to regard it merely as an attention-focuser – identifying the crux theoretical question as axiom 4 re the nature of decoherence. The trouble is, despite occasional claims [23] by physicists that the interpretation of quantum mechanics, and the understanding of what "measurement" is, and how "decoherence" works, all are essentially solved problems, nevertheless from the rigorous viewpoint of a mathematician, these problems are not solved at all. Essentially, one is unable to find a single theorem in the area. (In contrast, quantum mechanics without measurement, and ignoring issues of interpretation, is fully rigorous [15]d.) Several physicists have suggested that quantum mechanics is incomplete and *additional* laws about decohorence need to be brought in. Depending on the nature of those new laws, quantum computers will be able or unable to defeat the Extended Church Thesis. Several physicists have made some simplifying approximations that hopefully do not greatly alter the gross behavior of quantum mechanics, and thereby got a "model" of decoherence. Many such models have been proposed [2][3][4][5][6][7][8][17][12][19][22][21][23]. They then tried to analyse their model (usually inexactly and with no rigorous understanding of the approximations involved) to get bounds or approximations to decoherences of certain kinds in certain kinds of environments.

Unfortunately, the models approximating physical reality the most closely [4] seem the hardest to analyse. The models with more complete analyses [20][7][17][18][19] sometimes output claims that contradict one another.

Physicist's models of decoherence incompatible with powerful quantum computers: Several decoherence models support axiom 4. In fact, some of the same physicists who proposed these models later wrote papers on quantum computers, apparently without noticing any contradiction. The Unruh-Zurek model [20] leading from quantum field theory to the so-called "master equation" [23] supports axiom 4, indeed indicating that *decoherence rate behaves like* $|\Delta x|^2$ for large $|\Delta x|$. Such dependence proportional to $|\Delta x|^2$ (or $|\Delta p|^2$) also arises in the simplest form (where $\mathcal{L}$ is a linear function of x and p; see §2.3.1 of [21]; the original reference is [8]), or the Diosi form [3], or several other forms [22] (including what [22] calls the "standard quantum Brownian motion equation" due to Caldiera and Leggett [2]) of the Lindblad master equation. Also, dependence proportional to $|\Delta x|$ (or $|\Delta p|$) arises in N.Gisin et al.'s Hilbert-space stochastic dynamics approach (§2.4 of [21], same $\mathcal{L}$s; original reference [5]). Here x is position and p is momentum. Most reasonable "background fields" in the model of Stern et al [17] also support axiom 4. Any and all of these kinds of decoherence behavior would destroy quantum computers with superclassical power.

"Gravitational decoherence:" new, simple, and fundamental mechanism obeying axiom 4

Suppose a mass m is somehow completely shielded from neutrinos, cosmic rays, et cetera[1], and is sitting in the center of a perfectly symmetric radius-r sphere of perfect vacuum cooled near absolute zero. (We are aiming to maximize drama by making a scenario with the minimum possible amount of decoherence; then showing that even then, decoherence – of the computer-killing axiom-4 type – happens; and that this can be seen via physical arguments so simple as to be undeniable.) Suppose outside that sphere, but inside a much larger sphere of radius R, is the rest of the universe, consisting of matter of density ρ, i.e. total mass $M = 4\pi(R^3 - r^3)\rho/3$. Assume the rest of the universe is localized to within a Compton wavelength $h/(Mc)$. (Actually in this formula it might be more realistic to use a *thermal* velocity well below c, but that would only affect our discussion by a constant factor.) Assume Newton's laws of gravity and motion.

Our mass, if it were displaced by $\pm\Delta x$, would exert different gravitational forces on the rest of the universe[2] which eventually would pull different parts of it different distances, and different by more than the universe's Compton wavelength. Thus, the

[1] But it is believed that gravity cannot be shielded. Arguments against graviton shields were made by Smolin [16].

[2] Actually, in this spherically symmetric scenario, the net gravitational force on our mass from the rest of the universe would be *zero* regardless of its location. ("Birkhoff's theorem" in general relativity; known to Newton for Newtonian gravity.) Nevertheless, our mass does attract the rest of the universe radially inward, and if it is located non-centrally, it will attract the closer parts of the outside universe more strongly than than the further parts. That suffices.

rest of the universe would "measure" the fact that our mass had been displaced by $+\Delta x$ versus $-\Delta x$, thus decohering it.

It is easy to see[3] that force differences are proportional to $Gm\rho\ln(R/r)\Delta x$, so that the "measurement time" τ (i.e. time before a Compton wavelength separation opens up) is of order $\tau \approx \sqrt{h/[cGm\rho\ln(R/r)\Delta x]}$. The "decoherence rate" is $1/\tau$, i.e. proportional to $\sqrt{\Delta x}$. Note that the logarithmic dependence on r means it is exponentially expensive (i.e. infeasible) to try to reduce this decoherence rate. Because this rate increases unboundedly with $|\Delta x|$, this kind of decoherence obeys axiom 4 and disallows super-classical power quantum computers whose qubits are subject to it.

Is this effect, since it is gravitational, extremely weak? Here is a numerical example. Let $m = 10^{-12}$gram and $\Delta x = 1\mu$m (bacterium). Let $r = 10$cm, $R = 6400$km (radius of Earth), and $\rho = 5.5$gram/cc (density of Earth). Then $\tau \approx 20$ nanoseconds. If, instead of c, one were to use 300meter/second (typical thermal velocity), then we would get $\tau \approx 20$ *micro*seconds instead. It might be argued that Newton's law of gravity requires 20 *milli*seconds to act (lightspeed transit time across an Earth radius). In any case (since bacteria and 20msec both are below human perception scales) this mechanism *alone*, despite its weakness, suffices to explain why the world appears classical.

There is an escape hatch. It may be possible to design wide-separated qubits each to have zero "gravitational dipole moment." (Store qubits as rotation states[4] of buckyballs held in fixed locations; never move any particle, including photons, more than a constant distance?) In that case this effect would be evaded. But our argument nevertheless (1) is understandable using the physics of the year 1700, and (2) makes it crystal clear that decoherence mechanisms *do* exist that obey axiom 4.

Further, this paper opens up the possibility that there is a big philosophical difference between quantum and classical computers. Our universe's physical laws (assuming the universe's size, available mass and energy, etc, are suitably infinite) in principle permit a Turing machine, or at least an exponentially large computer [15]e. But it may be that our universe's physical laws do *not* permit constructing a quantum Turing machine.

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[3] From the facts that dipole forces fall off proportionally to distance^{-3}, the dipole moment in our case is proportional to $m\Delta x$, and integration.

[4] However, under general relativity (§19.2 of [9]) rotating objects exert an effect called "frame dragging" on gyroscopes in their surroundings, which, like dipole forces under Newtonian gravity, falls off like distance^{-3}.

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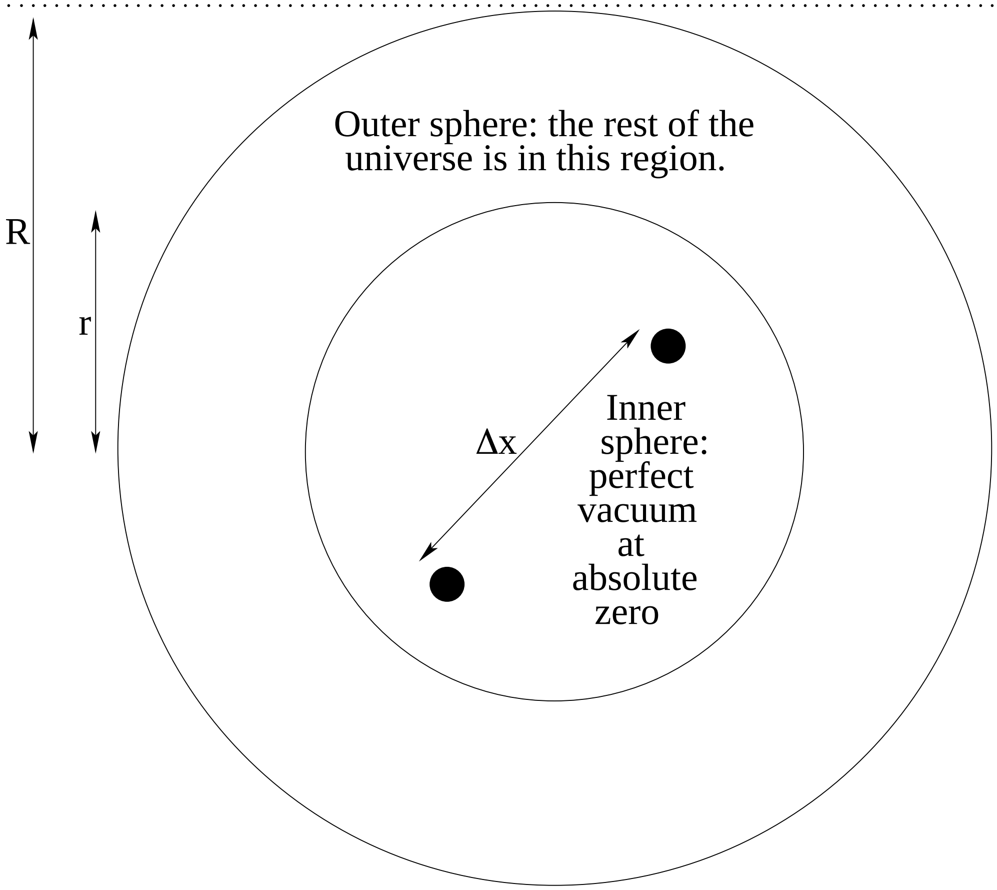

Figure 1.