Geometric Origin of the Cepheid Variables: Time Varying Light Cones

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Abstract

Cepheid variable stars are among the most important standard candles in observational cosmology, exhibiting periodic variation in brightness. In this work, we propose a novel geometric model to explain their pulsation using a time-periodic spacetime with modulated causal structure. Specifically, we introduce a time-dependent metric coefficient $g_{tt}(t)$ driven by a brightness-inspired function that mimics asymmetric Planck-like emission. This leads to oscillating light cones, whose angles encode the gravitational basis for observed Cepheid variability, as a consequence of causal dynamics in the spacetime geometry.

1 Introduction

Cepheid variable stars exhibit a well-defined relationship between their pulsation periods and luminosities, making them crucial tools in measuring extragalactic distances and calibrating the Hubble constant [2, 3, 4].

Traditionally, this variability is attributed to stellar interior dynamics governed by fluid mechanics and ionization fronts. In this paper, we explore an alternative, geometric viewpoint rooted in general relativity. By embedding a time-dependent, asymmetric function into the temporal part of the metric $g_{tt}(t)$, inspired by Planck-like emission, we explore causal variations in line with general relativity [5, 6, 7].

We propose that this dynamic geometry encodes brightness modulation, offering a compelling mechanism behind the Cepheid variability. The resulting framework links periodic time, gravitational causality, and astrophysical observations into a single geometric model.

2 Embedding Asymmetric Brightness into the TPM Metric

We now embed the asymmetric brightness function into the temporal component of a time-periodic Minkowski (TPM) metric. The original TPM metric is given by:

$$ds^2 = g_{tt}(t)\, dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2. \tag{1}$$

We propose to model $g_{tt}(t)$ using the asymmetric brightness function inspired by a Planck-like distribution. Define:

$$x(t) = \begin{cases} X \cdot \sin^2\left(\frac{\pi(t-\delta)}{T}\right) & \text{if } 0 \le t - \delta \le T \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Then, the modulated temporal metric component becomes:

$$g_{tt}(t) = A \cdot \frac{[x(t)]^\alpha}{e^{\beta x(t)} - 1}, \tag{3}$$

where:

- $T = 2\pi$ is the time periodicity,
- δ controls the phase shift (early peak),
- X is the domain ceiling of $x(t)$,
- A is a normalization constant (maximum brightness),
- α controls the steepness of the rise,
- β controls the tail decay after the peak.

This model introduces time-asymmetric causal structure while preserving periodicity, making it suitable for exploring pulsating cosmologies and analogies with stellar behavior such as Cepheid variables.

3 Light Cone Angles and Causal Structure

The dynamic temporal metric component $g_{tt}(t)$ leads to time-varying causal structure in the spacetime. Specifically, the slope of light cones changes with time, resulting in a pulsating geometry.

For a radial null trajectory ($ds^2 = 0$) in the metric

$$ds^2 = g_{tt}(t)\, dt^2 - dr^2,$$

we find that light rays satisfy

$$\left(\frac{dr}{dt}\right)^2 = g_{tt}(t).$$

Hence, the angle $\theta(t)$ that the light cone makes with the time axis is given by:

$$\tan\theta(t) = \sqrt{g_{tt}(t)} \quad \Rightarrow \quad \theta(t) = \tan^{-1}\left(\sqrt{g_{tt}(t)}\right). \tag{4}$$

Substituting our model:

$$g_{tt}(t) = A \cdot \frac{[x(t)]^\alpha}{e^{\beta x(t)} - 1}, \quad x(t) = X \cdot \sin^2\left(\frac{\pi(t-\delta)}{T}\right),$$

the light cone angle becomes:

$$\theta(t) = \tan^{-1}\left(\sqrt{A \cdot \frac{[x(t)]^\alpha}{e^{\beta x(t)} - 1}}\right).$$

This allows us to visualize how the light cone opens and closes over each time cycle. Where $g_{tt}(t)$ is large (near the brightness peak), the light cone widens; where $g_{tt}(t)$ is small (early and late in the cycle), the light cone narrows.

Such modulated cone structure is reminiscent of causal horizons that fluctuate with energy emission, providing a geometric analogy to pulsed astrophysical sources such as Cepheid variables.

4 Application to Schwarzschild Geometry with Time-Modulated Light Cones

We now extend our asymmetric brightness-driven causal structure to curved spacetime, particularly the Schwarzschild geometry. The standard Schwarzschild metric is given by:

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2. \tag{5}$$

To introduce temporal pulsation analogous to the Time-Periodic Minkowski case, we multiply the time-time component g_{tt} by the same brightness modulation factor. Let:

$$g_{tt}(t,r) = \left(1 - \frac{2GM}{r}\right) \cdot \left[A \cdot \frac{[x(t)]^\alpha}{e^{\beta x(t)} - 1}\right], \tag{6}$$

where:

$$x(t) = \begin{cases} X \cdot \sin^2\left(\frac{\pi(t-\delta)}{T}\right) & \text{if } 0 \le t - \delta \le T \\ 0 & \text{otherwise} \end{cases}$$

This generates a *time-periodic Schwarzschild spacetime*, where the gravitational potential is modulated over time, analogously to pulsating stars.

Light Cone Angles in the Modulated Schwarzschild Metric

For radial null trajectories, we impose $ds^2 = 0$ and $d\theta = d\phi = 0$. Then,

$$0 = g_{tt}(t, r)\, dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2, \tag{7}$$

which implies:

$$\left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{2GM}{r}\right)^2 \cdot \left[A \cdot \frac{[x(t)]^\alpha}{e^{\beta x(t)} - 1}\right]. \tag{8}$$

Thus, the light cone angle becomes:

$$\theta(t, r) = \tan^{-1}\left(\sqrt{\left(1 - \frac{2GM}{r}\right)^2 \cdot \left[A \cdot \frac{[x(t)]^\alpha}{e^{\beta x(t)} - 1}\right]}\right). \tag{9}$$

This formulation reveals how spacetime curvature and temporal modulation combine to dynamically shape the light cones. Near the Schwarzschild radius, the factor $1 - 2GM/r$ becomes small, narrowing the cone even during peak brightness, highlighting gravitational redshift and horizon effects.

5 Extension to Kerr Geometry: Rotating Time-Modulated Spacetime

We now generalize our model to a rotating black hole by embedding the brightness-modulated temporal component into the Kerr metric. The standard Kerr line element in Boyer–Lindquist coordinates (t, r, θ, ϕ) is:

$$ds^2 = \left(1 - \frac{2GMr}{\Sigma}\right) dt^2 + \frac{4GMar\sin^2\theta}{\Sigma} dt\, d\phi - \frac{\Sigma}{\Delta} dr^2 - \Sigma\, d\theta^2 - \left(r^2 + a^2 + \frac{2GMa^2 r\sin^2\theta}{\Sigma}\right)\sin^2\theta\, d\phi^2, \tag{10}$$

where:

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 - 2GMr + a^2.$$

Brightness-Modulated Time Component

To embed time-periodicity and asymmetric brightness structure, we propose modifying the g_{tt} term as:

$$g_{tt}(t, r, \theta) = \left(1 - \frac{2GMr}{\Sigma}\right) \cdot \left[A \cdot \frac{[x(t)]^\alpha}{e^{\beta x(t)} - 1}\right], \tag{11}$$

with $x(t)$ defined as before:

$$x(t) = \begin{cases} X \cdot \sin^2\left(\frac{\pi(t-\delta)}{T}\right) & \text{if } 0 \le t - \delta \le T \\ 0 & \text{otherwise} \end{cases}$$

Light Cone Angle in Kerr Background

In the equatorial plane ($\theta = \pi/2$), we examine radial null trajectories, setting $d\theta = d\phi = 0$. The Kerr metric reduces to:

$$0 = g_{tt}(t,r)\,dt^2 - \frac{\Sigma}{\Delta}dr^2. \tag{12}$$

Then, the radial speed of light becomes:

$$\left(\frac{dr}{dt}\right)^2 = \frac{\Delta}{\Sigma}\cdot g_{tt}(t,r,\theta = \pi/2), \tag{13}$$

yielding the light cone angle:

$$\theta(t,r) = \tan^{-1}\left(\sqrt{\frac{\Delta}{\Sigma}\cdot\left(1 - \frac{2GMr}{\Sigma}\right)\cdot\left[A\cdot\frac{[x(t)]^\alpha}{e^{\beta x(t)} - 1}\right]}\right). \tag{14}$$

Interpretation

This result captures how rotation, curvature, and temporal modulation interact to determine the local causal structure. The combination of $a \neq 0$ and $g_{tt}(t)$ breaks both staticity and spherical symmetry, generating light cone oscillations that depend on both r and t. The result may be especially relevant to modeling pulsating stars with angular momentum and modulated emission.

6 Curvature and Matter Analysis of the Time-Periodic Minkowski (TPM) Spacetime

To understand the geometric origin of Cepheid variable behavior, we investigate the curvature of the modulated time-periodic Minkowski (TPM) spacetime. We analyze whether time modulation introduces nontrivial spacetime curvature and whether the associated Einstein tensor implies a nonzero energy-momentum content.

TPM Metric

We recall the TPM metric:

$$ds^2 = g_{tt}(t)\,dt^2 - dr^2 - r^2 d\theta^2 - r^2\sin^2\theta\,d\phi^2, \tag{15}$$

where

$$g_{tt}(t) = A\cdot\frac{x(t)^\alpha}{e^{\beta x(t)} - 1}, \quad x(t) = X\cdot\sin^2\left(\frac{\pi(t-\delta)}{T}\right). \tag{16}$$

Only g_{tt} depends on time, and the spatial part of the metric remains flat.

Christoffel Symbols and Riemann Tensor

The only non-zero Christoffel symbols involving time are:

$$\Gamma^t_{tt} = \frac{1}{2g_{tt}} \frac{dg_{tt}}{dt}. \tag{17}$$

As all spatial components are flat, the only potentially non-zero curvature arises from the time variation of g_{tt}.

Ricci Tensor and Scalar

The only non-zero component of the Ricci tensor is:

$$R_{tt} = -\frac{1}{2}\left[\frac{g''_{tt}}{g_{tt}} - \frac{1}{2}\left(\frac{g'_{tt}}{g_{tt}}\right)^2\right]. \tag{18}$$

This arises purely due to the time-dependence of $g_{tt}(t)$. All other components vanish. The Ricci scalar becomes:

$$R = g^{tt} R_{tt} = \frac{1}{g_{tt}} R_{tt}. \tag{19}$$

Einstein Tensor and Effective Matter Content

Using the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{20}$$

we find that $G_{tt} \neq 0$, and all other components vanish. Thus, the spacetime has an effective energy-momentum tensor:

$$T_{\mu\nu} = \frac{1}{8\pi G} G_{\mu\nu}, \tag{21}$$

with non-zero T_{tt} interpreted as a time-localized energy density.

Interpretation

While the spatial geometry remains flat, the time-varying g_{tt} induces a nonzero Ricci curvature and Einstein tensor. This implies the existence of a non-zero energy density, concentrated in time. The TPM spacetime thus represents a scenario where periodic brightness is a manifestation of localized geometric energy pulses, offering a geometric and gravitational analog of stellar pulsation in Cepheid variables.

7 Curvature and Effective Matter in TPS and TPK Geometries

We now extend our curvature and matter analysis to the Time-Periodic Schwarzschild (TPS) and Time-Periodic Kerr (TPK) geometries, which embed the same asymmetric brightness modulation into the temporal metric components of curved spacetimes.

TPS Metric and Curvature

In the TPS spacetime, the Schwarzschild metric is modified as:

$$g_{tt}^{\mathrm{TPS}}(t, r) = \left(1 - \frac{2GM}{r}\right) \cdot \left[A \cdot \frac{x(t)^\alpha}{e^{\beta x(t)} - 1}\right], \tag{22}$$

while the spatial components remain those of the Schwarzschild geometry.

Because both g_{tt} and g'_{tt} now depend on time and radius, the curvature is no longer purely static. Differentiation reveals:

$$\frac{\partial g_{tt}}{\partial t} \neq 0, \tag{23}$$

$$\frac{\partial g_{tt}}{\partial r} \neq 0. \tag{24}$$

Thus, the Riemann tensor gains additional components. More importantly, the Ricci tensor $R_{\mu\nu}$ becomes non-zero, signaling a departure from the vacuum condition of the Schwarzschild solution.

The corresponding Ricci scalar R and Einstein tensor $G_{\mu\nu}$ also become non-zero. Hence, the modified metric supports an effective energy–momentum tensor:

$$T_{\mu\nu}^{\mathrm{TPS}} = \frac{1}{8\pi G} G_{\mu\nu}, \tag{25}$$

representing a matter distribution sourced by the time-periodic modulation.

TPK Metric and Curvature

In the TPK geometry, the Kerr metric is modified similarly:

$$g_{tt}^{\mathrm{TPK}}(t, r, \theta) = \left(1 - \frac{2GMr}{\Sigma}\right) \cdot \left[A \cdot \frac{x(t)^\alpha}{e^{\beta x(t)} - 1}\right], \quad \text{where } \Sigma = r^2 + a^2 \cos^2 \theta. \tag{26}$$

The resulting metric varies both in time and angular direction. Consequently, the metric exhibits non-trivial dependence on all spacetime coordinates:

$$\frac{\partial g_{tt}}{\partial t} \neq 0, \quad \frac{\partial g_{tt}}{\partial r} \neq 0, \quad \frac{\partial g_{tt}}{\partial \theta} \neq 0.$$

This leads to a highly dynamic Riemann tensor and a fully non-zero Ricci tensor. The Einstein tensor $G_{\mu\nu}$ becomes complex and angle-dependent, suggesting that the spacetime supports an anisotropic and time-dependent energy–momentum distribution:

$$T_{\mu\nu}^{\mathrm{TPK}} = \frac{1}{8\pi G} G_{\mu\nu}. \tag{27}$$

Physical Interpretation

Unlike in the static Schwarzschild or Kerr cases (which are Ricci-flat), the TPS and TPK geometries possess time-modulated curvature arising from the embedding of periodic brightness-like functions. The Einstein tensor is no longer zero, implying the presence of matter or energy encoded in spacetime geometry. This reinforces the central thesis: that *geometrically encoded time-periodicity can manifest as effective stellar pulsation behavior*, offering a gravitational basis for observed Cepheid variability.

8 Linking Light Cone Opening to Brightness Variability in Cepheid Stars

A central hypothesis of our model is that the brightness modulation observed in Cepheid variable stars is a macroscopic manifestation of an underlying geometrical mechanism: the periodic opening and closing of the light cone in a time-dependent spacetime.

Light Cone Geometry and Visibility

In general relativity, the local causal structure of spacetime is encoded in the light cone, determined by the metric. For radial null geodesics in a time-periodic geometry, such as:

$$ds^2 = g_{tt}(t)\, dt^2 - dr^2,$$

we find the condition for light propagation:

$$\left(\frac{dr}{dt}\right)^2 = g_{tt}(t), \quad \Rightarrow \quad \theta(t) = \tan^{-1}(\sqrt{g_{tt}(t)}),$$

where $\theta(t)$ is the angle between the time axis and the light ray in spacetime diagrams. As $g_{tt}(t)$ increases, the light cone opens up, allowing light to travel more freely across spatial slices — enhancing visibility from the source.

Standard Brightness Equations

Cepheid variables obey the classical period-luminosity relation:

$$M_V = a \log_{10}(P) + b,$$

where M_V is the absolute magnitude, P the period of pulsation, and a, b empirical constants. The observed (apparent) magnitude m is then connected to M_V via the luminosity distance d_L:

$$m - M_V = 5 \log_{10}(d_L / 10\,\mathrm{pc}).$$

We hypothesize that the luminosity distance d_L is affected by the dynamics of light cone opening via:

$$d_L(t) \propto \frac{1}{\sqrt{g_{tt}(t)}} \propto \frac{1}{\tan\theta(t)}. \tag{28}$$

This expresses how light cone narrowing (smaller θ) corresponds to longer effective distance — dimmer observation — while opening (larger θ) shortens the path, enhancing brightness.

Asymmetric Light Curves from Geometry

By embedding an asymmetric time function into $g_{tt}(t)$ — such as a Planck-distribution-inspired profile — the opening and closing of the light cone itself becomes asymmetric. This naturally leads to asymmetric light curves, such as those seen in Cepheid variables:
- Steep rise in brightness Rapid cone opening - Gradual decline Slow cone closure

Interpretation

Thus, the light cone dynamics provide a geometric interpretation for both the time variation and asymmetry of brightness in Cepheid stars. Our framework directly links this to classical observables like magnitude and distance, implying that such variables might have a geometric-causal origin in modulated spacetime itself.

9 Hubble Tension and Geometric Interpretation of Cepheid Variability

A major outstanding problem in modern cosmology is the *Hubble tension*, a persistent discrepancy between early-universe and late-universe measurements of the Hubble constant H_0. Measurements from the cosmic microwave background (CMB), notably by the *Planck* satellite, suggest a value of $H_0 \approx 67.4 \pm 0.5$ km/s/Mpc [4], while direct distance ladder measurements using Cepheid-calibrated supernovae yield a significantly higher value of $H_0 \approx 73.2 \pm 1.3$ km/s/Mpc ...

The root of this tension may lie in our assumptions about the underlying physics of standard candles such as Cepheid variable stars. In most current treatments, their pulsation and brightness variation are modeled as classical thermodynamic or fluid dynamical processes internal to the star [1]. However, these models do not explain why such variability obeys precise periodicity or links cleanly to luminosity.

In our geometric framework, we interpret the periodic variation in brightness of Cepheid variables as arising from an underlying time-periodic spacetime metric. By embedding an asymmetric Planck-like function into the temporal component of the metric, $g_{tt}(t)$, we construct a causal model where the observed brightness modulation arises from variations in light cone angle and redshift over time.

This perspective introduces a geometric origin for the period–luminosity relation, potentially recalibrating the intrinsic distances assigned to Cepheid variables. If the luminosity is modulated not only by thermodynamic processes but also by spacetime geometry, then distance ladder estimates based on these stars may require revision. Such a shift could resolve part of the Hubble tension by reducing the inferred late-universe expansion rate.

Key Insight: Our model provides a natural physical mechanism for asymmetric light curves and introduces corrections to the Cepheid luminosity calibration, offering a novel path to address the Hubble constant discrepancy.

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