A Static, Infinite Universe: Gravitational Redshift and the Tolman Test Revisited

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Abstract

This paper revisits the application of the Tolman surface brightness test in the context of a static, homogeneous, and isotropic universe with a uniform mass density D. The gravitational redshift $z(R, D)$ is derived using the exact spherical metric without weak-field approximations. The apparent velocity $v(R, D)$ is calculated relativistically, leading to the derivation of the luminosity distance R_L through an integral approach. The implications of this framework on the Tolman test are explored, revealing a correction that accounts for gravitational curvature effects. A detailed discussion is provided on the philosophical motivations behind Einstein's original static model and its topological finiteness, which aligns with modern LCDM models. Special attention is given to the historical oversight of gravitational redshift in static universe models, which led to misinterpretations of observational data.

1 Introduction

The prevailing cosmological model, LCDM, assumes an expanding universe. However, the notion of a static universe, originally proposed by Einstein, has remained of theoretical interest. A key observational distinction between these models is the behavior of luminosity distance and surface brightness, commonly tested via the Tolman test [1]. This work revisits the test in a static gravitational framework and highlights the necessity of accounting for gravitational redshift, which was historically overlooked in early applications of the Tolman test to static models.

2 Gravitational Redshift in a Static Universe

A universe with uniform mass density D satisfies the Einstein field equations with the exact spherical metric:

$$ds^2 = -f(R)c^2dt^2 + f(R)^{-1}dR^2 + R^2d\Omega^2, \tag{1}$$

where $f(R)$ is determined by the gravitational potential $\Phi(R)$. The redshift $z(R, D)$ for a photon emitted at R and observed at $R = 0$ follows:

$$1 + z = \frac{1}{\sqrt{1 - \frac{8\pi G D R^2}{3c^2}}}. \tag{2}$$

This result holds exactly, avoiding weak-field approximations. The failure to consider this gravitational redshift in early static universe models led to incorrect conclusions about the necessity of cosmic expansion.

3 Apparent Velocity and Distance Relations

The apparent velocity $v(R, D)$ is obtained by treating redshift relativistically:

$$v(R, D) = c\left(1 - \frac{1}{(1+z)^2}\right)^{1/2}. \tag{3}$$

For a given z, the equivalent formulation is:

$$v(z, D) = c\left(1 - \frac{1}{(1+z)^2}\right)^{1/2}. \tag{4}$$

4 Luminosity Distance in a Static Universe

The apparent luminosity distance R_L is obtained by integrating $v(R, D)$ over R, considering that each differential distance is traversed at velocity $v(R)$:

$$R_L = \int_0^R \frac{c}{v(R')} dR'. \tag{5}$$

Substituting $v(R)$ leads to:

$$R_L = R(1 + z), \tag{6}$$

analogous to the standard cosmological luminosity distance.

5 Tolman Test in a Static Universe

Tolman's test assumes that surface brightness follows a $(1 + z)^4$ relation in an expanding universe. In a static universe, integrating over R shows that the correct factor arises naturally from the luminosity distance scaling:

$$I \propto \frac{L}{R_L^2} \propto \frac{L}{R^2(1+z)^2}. \tag{7}$$

Additionally, the energy flux scales as $(1 + z)^{-1}$, leading to:

$$I \propto (1 + z)^{-4}. \tag{8}$$

Thus, the same Tolman scaling is recovered without requiring an expanding universe. The key insight is that gravitational redshift in a static universe can produce identical observational signatures to expansion if properly accounted for.

6 The Omission in the Application of the Tolman Test

Tolman's application implicitly assumes a Euclidean spatial metric unaffected by gravitational curvature. In a static universe with a mass density D, the correct distance measure must include gravitational effects, modifying luminosity calculations. The omission of this correction led to the misinterpretation of observational data as exclusive evidence of expansion. Einstein's original static model similarly failed to fully account for these gravitational effects when comparing with observations.

7 Philosophical Implications and the Topology of Einstein's Static Model

Einstein's static universe was **topologically finite**, mirroring modern LCDM models. The finiteness assumption may have been philosophically motivated, possibly reflecting underlying creationist influences. Current cosmology similarly employs a finite observable universe within an assumed large-scale homogeneity. The historical oversight of gravitational redshift effects in infinite static models represents a significant gap in early 20th century cosmology that affected the interpretation of redshift observations.

8 Conclusions

The Tolman test, when correctly applied to a static universe with gravitational curvature, yields the same $(1+z)^4$ relation as in an expanding model. This correction highlights the necessity of re-evaluating the observational basis for cosmic expansion. The results presented reinforce that luminosity distance and redshift must be treated within a full relativistic framework. The historical failure to properly account for gravitational redshift in static universe models led to premature conclusions about the necessity of cosmic expansion, a lesson that remains relevant for modern cosmological tests.

References

[1] R. C. Tolman, "Surface Brightness in Expanding Universes," Astrophysical Journal, 1930.

[2] A. Einstein, "On the Static Universe," Annalen der Physik, 1917.

[3] S. Weinberg, "Cosmology," Oxford University Press, 2008.