The Seven-Set Prime Number Theorem

A Tour of Deserted Valley of Prime Numbers

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Abstract:

This paper seeks to explore the nature of primality by investigating relationships between prime numbers. For centuries, mathematicians have been intrigued by the unpredictable distribution of primes. The Seven-Set Prime Number Theorem (SSPNT) offers a novel approach to address this challenge. By building on Ulam's Spiral, SSPNT uses simple sets and visual patterns to gain insights into primality. This methodology not only provides a fresh perspective on prime number theory but also has potential applications in coding theory and cryptography, revealing hidden patterns and underlying structures.

Background and Context:

Prime numbers, defined as numbers greater than 1 that are divisible only by themselves and 1 (e.g., 2, 3, 5, and 7), have fascinated mathematicians for millennia. Despite numerous attempts to uncover a formula or pattern governing their distribution, prime numbers have remained elusive. Their unpredictability is a core part of their intrigue, yet they hold significant importance in number theory, cryptography, and coding theory.

Historical efforts to understand prime numbers include foundational work by many great mathematicians, such as:

- **Euclid's foundational work**: The proof of the infinitude of primes.
- **Diophantus' *Arithmetica***: Early study of Diophantine equations.
- **Fermat's Little Theorem**: A result that provides a characterization of prime numbers in modular arithmetic.
- **Riemann Hypothesis**: A conjecture about the distribution of primes, with profound implications for number theory.
- **Hilbert's problems**: Several problems related to prime numbers and their distribution.
- **Ulam's Spiral**: A visualization technique that shows patterns in the distribution of prime numbers.
- **Goldbach's Conjecture**: A conjecture stating that every even integer greater than 2 can be written as the sum of two primes.
- **Prime Number Theory (PNT)**: A branch of mathematics focused on the distribution and properties of prime numbers.

Building on these advancements, this paper proposes the **Seven-Set Prime Number Theorem (SSPNT)**, a new approach to understanding prime number distributions.

Literature Review:

Attempts to unravel the mystery of primality have been ongoing for centuries. This literature review provides an overview of key approaches to understanding prime numbers.

Ancient Methods:

1. **Euclid's Sieve:** Euclid's Sieve is a fundamental technique for finding prime numbers, primarily used in the ancient world. The method involves:
   - Listing all integers from 2 up to a specified number n.
   - Marking the multiples of 2 (e.g., 4, 6, 8).
   - Repeating the process for the next unmarked number, and continuing with the primes (e.g., 3, 5, and 7).
   - The remaining unmarked numbers are identified as primes.

   Strengths:

   - Simple and effective for finding primes up to a certain range.
   - The method provides an intuitive way to filter primes from a list of numbers.

   Weaknesses:

   - The method becomes inefficient for large numbers, requiring significant computational effort.
   - It does not scale well for higher values of n, where the list of numbers becomes large.

   Improved Versions:

   - **Sieve of Atkin**: An optimized version of the sieve using modular arithmetic, providing more efficiency in finding primes compared to Euclid's Sieve, especially for larger numbers.
   - **Sieve of Eratosthenes**: A classic method often implemented in algorithms for prime number discovery, providing a more efficient solution for smaller ranges

Diophantus' *Arithmetica*:

In the context of primality, Diophantus' work in *Arithmetica* can be linked to the study of **Diophantine equations**—equations that seek integer solutions. While Diophantus did not directly focus on prime numbers, his work laid important foundations for later number-theoretic studies, including those on primality.

Here's how Diophantus' ideas can be related to primality:

Diophantine Equations and Primes:

- Diophantus studied equations where solutions are sought in integers. In number theory, prime numbers often appear as solutions to these kinds of equations, especially when the equations impose certain conditions on the numbers involved.
- For example, Diophantus explored equations of the form:

$$ax + by = c$$

  Where a, b, and c are integers, and x and y are unknowns. Prime numbers can be involved when considering specific values for a, b, or c. In some cases, this can lead to a search for prime solutions (e.g., when x or y is required to be prime).

Pythagorean Triples and Primes:

- One of the key results from Diophantus' *Arithmetica* was the study of **Pythagorean triples**—sets of three integers a, b, and c that satisfy the equation:

$$a^2 + b^2 = c^2$$

  Many of the smallest Pythagorean triples involve prime numbers. For example, the triple (3, 4, 5) involves the prime 3. These triples have connections to prime numbers and their distribution.

- The **connection to primality** is in how certain Diophantine equations (such as the Pythagorean equation) often produce prime numbers as part of the solutions, and understanding these relationships helps mathematicians uncover deeper properties of primes.

Prime Solutions to Diophantine Equations:

- Diophantus' exploration of finding integer solutions to equations paved the way for later mathematicians to consider whether specific Diophantine equations can yield prime solutions. For example, Fermat's exploration of the equation:

$$x^n + y^n = z^n$$

  When n=2 leads to **Pythagorean triples** and primes, and when n>2, connects to **Fermat's Last Theorem** and prime factorizations.

Primality in the Search for Integer Solutions:

- Diophantus' work implicitly touches on the concept of primality by attempting to find integer solutions. While he didn't directly work on primes in the modern sense, his methods to solve polynomial equations are foundational for the later development of theories related to prime number distributions and primality testing.
- For instance, solving equations such as:

$$ax + by = c$$

for specific values of a, b, and c can lead to problems in primality, particularly when these equations are used to generate specific sets of primes or to understand how primes appear in various number-theoretic contexts.

Fermat and Diophantus' Influence:

- **Fermat**, heavily influenced by Diophantus' *Arithmetica*, worked on equations like $a^2 + b^2 = c^2$ (related to Pythagorean triples), and he also studied **prime numbers** in the context of these Diophantine equations. For example, Fermat's Little Theorem, which involves primes in modular arithmetic, grew out of the tradition of studying equations that Diophantus had laid the groundwork for.
- Fermat's work also introduced new ways of thinking about prime numbers as solutions to Diophantine equations, especially in the modular sense, influencing subsequent advancements in primality testing.

Conclusion:

While Diophantus didn't directly study primes as we understand them today, his work on Diophantine equations set the stage for future mathematical developments that explored the properties and distributions of prime numbers. His exploration of integer solutions to equations, including equations that involve primes or generate prime numbers, was a crucial early step in number theory. Later mathematicians, including Fermat and Euler, would build on these ideas, directly linking Diophantine equations to the study of primality and prime number theory.

Thus, Diophantus' *Arithmetica* can be seen as a precursor to modern methods for understanding prime numbers through algebraic and equation-based frameworks.

Fermat's Little Theorem:

Fermat's Little Theorem is a fundamental result in number theory, particularly useful in the field of primality testing and modular arithmetic. It provides a simple way to understand the behavior of prime numbers in certain modular systems. The theorem is named after the French mathematician Pierre de Fermat, who first stated it in 1640.

Statement of Fermat's Little Theorem:

If p is a prime number and a is an integer that is not divisible by p (i.e., a and p is co-prime), then:

$a^{p-1} \equiv 1$ (Mod p)

This means that when a^{p-1} is divided by p, the remainder is 1.

Key Points of the Theorem:

- p is a prime number.
- a is an integer such that a is not divisible by p. In other words, a and p must be co-prime (i.e., gcd (a,p) =1).
- The theorem tells us that raising a to the power p−1 will always result in a remainder of 1 when divided by p.

Applications of Fermat's Little Theorem:

1. **Primality Testing:**
    - Fermat's Little Theorem is used in Fermat's Primality Test, which is an algorithm used to check if a number is prime. The idea is that for a number n and a random integer a, if $a^{n-1} \not\equiv 1$ (mod n), then n is definitely not prime. If the condition holds, n might be prime, but further testing is required to confirm.
2. **Cryptography:**
    - Fermat's Little Theorem plays a significant role in modern cryptography, especially in public-key algorithms like RSA, where the security of the encryption relies on the difficulty of factoring large prime numbers. The theorem is used in modular exponentiation, which is the core operation in these cryptographic systems.
3. **Computational Number Theory:**
    - It is widely used in various algorithms related to number theory, particularly those dealing with modular arithmetic and prime number generation.
4. **Simplifying Powers in Modular Arithmetic:**
    - Fermat's Little Theorem allows us to simplify large powers in modular arithmetic. If we want to compute$a^n \ (mod\ p)$, and n is large, we can reduce n modulo p−1 due to Fermat's Little Theorem. This reduces the complexity of computations.

Extended Version (Euler's Theorem):

Fermat's Little Theorem is a special case of a more general result called Euler's Theorem, which applies to any integer a and modulus n as long as a and n are co-prime. Euler's Theorem states that:

$$a^{\emptyset(n)} \equiv 1 \ (mod\ n)$$

Where ϕ (n) is Euler's totient function, representing the count of integers less than n that are co-prime with n. When n is prime ϕ (n) =n−1, which gives us Fermat's Little Theorem.

Conclusion:

Fermat's Little Theorem is a cornerstone of number theory and has profound implications in both theoretical and applied mathematics. It is particularly useful in the study of prime numbers and in applications like cryptography, primality testing, and modular arithmetic, where it helps simplifies computations and verifies properties of numbers efficiently.

Riemann Hypothesis:

The Riemann Hypothesis is one of the most famous and long-standing unsolved problems in mathematics. It concerns the distribution of prime numbers and is named after the German mathematician Bernhard Riemann, who first formulated it in 1859. The hypothesis is part of the Riemann zeta function, which is central to analytic number theory.

Statement of the Riemann Hypothesis:

The Riemann Hypothesis asserts that all non-trivial zeros of the Riemann zeta function has a real part equal to 1/2. In other words, the hypothesis suggests that if ζ(s) =0\zeta(s) = 0ζ(s) =0 for some complex number s=σ+it, where σ and t are real numbers, then σ=1/2.

Mathematically, this can be written as:

$$If\ \zeta(s) = 0, then\ \Re(s) = \frac{1}{2}$$

Where:

- ζ(s) is the Riemann zeta function.
- s is a complex number of the form s=σ+it, where σ and t are real numbers, and i is the imaginary unit.

The non-trivial zeros refer to the zeros of ζ(s) that do not occur at the negative even integers (i.e., s= −2, −4, −6… which are called the "trivial zeros" of the zeta function). The Riemann Hypothesis specifically deals with the zeros that occur in the "critical strip" where 0 <ℜ(s) <1, and it suggests that all these non-trivial zeros lie along the line ℜ(s) =1/2, called the critical line.

Riemann Zeta Function:

The Riemann zeta function ζ(s) is defined for complex numbers s=σ+it as:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

This series converges when $\Re(s)>1$. The function can also be extended to other values of s through analytic continuation.

The Riemann zeta function has important connections to the distribution of prime numbers. One of its key results, known as the Euler product formula, expresses the zeta function as an infinite product over all prime numbers:

$$\zeta(s) = \prod_{p\ prime} \frac{1}{1 - p^{-s}}$$

This formula directly links the zeta function to primes, making the Riemann Hypothesis crucial for understanding the pattern of prime numbers.

Why Is the Riemann Hypothesis Important?

The Riemann Hypothesis is central to the study of prime numbers because the location of the non-trivial zeros of the zeta function is intimately connected with the distribution of primes. A proof of the hypothesis would lead to a much better understanding of the prime number theorem and the distribution of primes.

- The Prime Number Theorem gives an approximation of how the primes are distributed among the integers. It states that the number of primes less than or equal to x is approximately $\frac{x}{\log(x)}$. The Riemann Hypothesis, if proven, would provide more accurate estimates for the distribution of primes, especially in regions where the approximation is less precise.
- The hypothesis would also have implications for various other areas of number theory, including the study of lattice points, modular forms, and cryptography.

Understanding the Non-Trivial Zeros:

The non-trivial zeros of the Riemann zeta function are the solutions to the equation:

$$\zeta(s) = 0$$

For complex numbers $s=\sigma+it$, these zeros lie in the "critical strip" where $0<\Re(s)<1$, and the hypothesis states that all these zeros have their real part $\Re(s)$ equal to 1/2. Thus, the non-trivial zeros are conjectured to lie on the critical line $\Re(s)=1/2$

The Impact of the Riemann Hypothesis:

If the Riemann Hypothesis is proven, it would provide deep insights into the distribution of primes. It would refine our understanding of how primes are spread out among the integers, reducing the error in approximations like the Prime Number Theorem.

- It would also lead to improved estimates for the gap between consecutive primes, which are a key area of study in analytic number theory.
- There would be applications in fields like cryptography, where the difficulty of factoring large composite numbers (which relies on the distribution of primes) underpins many encryption methods.

Challenges and Status:

Despite much effort from mathematicians over the years, the Riemann Hypothesis has remained unproven. The hypothesis is one of the seven Millennium Prize Problems, and a correct proof (or disproof) of it would earn the mathematician a prize of one million dollars.

The hypothesis has been verified for the first 10 trillion zeros of the zeta function, all of which lie on the critical line $\Re(s)=1/2$. However, a general proof for all such zeros is still elusive.

Conclusion:

The Riemann Hypothesis is a central conjecture in the field of number theory with profound implications for the distribution of prime numbers. If proven true, it would lead to significant advances in our understanding of primes and would have far-reaching consequences in fields ranging from cryptography to advanced mathematics. Solving the Riemann Hypothesis remains one of the greatest unsolved problems in mathematics, and its resolution is likely to open up many new avenues of research.

Hilbert’s Problems:

Hilbert's Problems are a set of 23 unsolved problems in mathematics presented by the German mathematician David Hilbert in 1900. These problems were outlined during Hilbert's famous address to the International Congress of Mathematicians in Paris. The problems represent some of the most important and challenging unsolved problems in mathematics and had a profound influence on the direction of mathematical research in the 20th century.

Although many of the problems have been solved, several still remain open. Here's an overview of **Hilbert's Problems** and how they relate to the study of **primes** and **number theory**:

Hilbert's 23 Problems:

Hilbert's problems were diverse, spanning a wide range of mathematical fields. Some of them are directly or indirectly related to the study of **primes** and **number theory**, while others address areas such as **geometry**, **analysis**, and **algebra**. Here are the ones most relevant to prime numbers:

Problem 8: The Distribution of Prime Numbers

One of the most famous problems directly related to primes is **Problem 8**, which asks about the distribution of prime numbers. The problem can be stated as:

"Find a natural law governing the distribution of prime numbers."

This question is central to number theory, as it addresses how primes are distributed among the integers. The **Prime Number Theorem** provides an approximation for the distribution of primes, stating that the number of primes less than or equal to x is approximately $\frac{x}{\log(x)}$. However, this result only provides an approximation, and the exact distribution of primes remains a topic of great interest.

The **Riemann Hypothesis**, which we discussed earlier, is directly connected to this problem because it would give a more precise understanding of the distribution of primes, especially for large values of x. If the hypothesis is true, it would refine our knowledge of how primes are distributed, especially in the **critical strip** and along the **critical line**.

Problem 10: Transcendence of Certain Numbers

Problem 10 deals with the question of whether certain numbers, such as the values of specific mathematical constants, are algebraic (i.e., the roots of polynomials with integer coefficients) or transcendental (i.e., not the root of any such polynomial). While this problem is not directly about primes, it connects with number theory and prime numbers because of its relationship to irrational and transcendental numbers that arise in number-theoretic contexts.

Problem 13: Mathematical Foundation of Physics

Problem 13 addresses the mathematical foundation of physics, specifically the theory of general relativity. While this problem is more related to physics, the mathematical tools involved (such as differential equations and topology) are also relevant to advanced number theory, which in turn can have applications in cryptography and the study of prime numbers in certain areas of physics.

Problem 21: Continuity and the Nature of the Continuum

Problem 21 addresses the problem of continuity and the continuum hypothesis, which deals with the size and structure of sets. Although not directly related to prime numbers, problems of continuity, set theory, and infinity are foundational to understanding the fundamental properties of numbers, which can sometimes have implications for number theory.

Problem 23: The Problem of the Solutions of Diophantine Equations

This problem involves finding solutions to Diophantine equations, which are equations where the solutions must be integers. Although this problem doesn't directly ask about prime numbers, Diophantine equations are closely tied to number theory and the properties of primes.

- One famous example of a Diophantine equation is Fermat's Last Theorem, which states that there are no three positive integers a, b, and c that satisfy the equation $a^n + b^n = c^n$ for any integer value of n>2n > 2n>2. This theorem was solved by Andrew Wiles in the 1990s.
- The study of Diophantine equations often involves understanding the prime factors of certain numbers, making it an important area of number theory.

Impact of Hilbert's Problems on Prime Numbers and Number Theory:

Hilbert's Problems had a profound influence on the development of number theory, particularly in the study of prime numbers. Several areas of modern number theory, including the study of prime distribution, Diophantine equations, and the Riemann Hypothesis, stem from the challenges posed in Hilbert's 23 problems.

Unsolved Problems and Their Connection to Primes:

Some problems remain unsolved to this day, continuing to shape research in mathematics. In particular:

1. **The Riemann Hypothesis (Problem 8)**: This is one of the most influential and long-standing problems in mathematics. It proposes a deep connection between the distribution of prime numbers and the zeros of the Riemann zeta function. A proof or disproof of this hypothesis would provide a more detailed understanding of how prime numbers are distributed.
2. **Diophantine Equations (Problem 23)**: Many open problems in number theory are related to Diophantine equations, which often require understanding the prime factors of solutions to these equations. Some problems in algebraic number theory also revolve around prime ideals in number fields.

Conclusion:

Hilbert's Problems, especially Problem 8, have directly influenced the study of prime numbers, their distribution, and related areas in number theory. The Riemann Hypothesis and the study of Diophantine equations continue to be central to modern research in prime number theory, making Hilbert's influence enduring and profound in the field.

Ulam's Spiral:

Ulam's Spiral is a graphical representation of prime numbers that was discovered by the mathematician **Stanislaw Ulam** in 1963. It is an intriguing way of visualizing the distribution of prime numbers and has led to new insights and patterns in number theory.

What is Ulam's Spiral?

Ulam's Spiral involves plotting the integers in a spiral pattern on a two-dimensional grid, starting with 1 at the center, and then filling out the grid with consecutive integers in a spiral shape (moving right, up, left, and down). After the spiral is formed, prime numbers are marked on the grid, and interesting patterns emerge.

Steps to Create Ulam's Spiral:

1. Start with the number 1 at the center of the grid.
2. Move outwards in a spiral fashion, placing consecutive integers around the center.
3. Mark all the prime numbers on the spiral grid.

Here's a simple visualization of the Ulam Spiral:

```
37  36  35  34  33  32
38   1   2   3   4  31
39  40   5   6   7  30
40  41   8   9  10  29
41  42  11  12  13  28
42  43  44  14  15  27
```

The prime numbers would be highlighted in this spiral pattern, and surprisingly, when plotted, they form diagonal lines or "curves" along the spiral. These diagonal patterns suggest a relationship between primes and certain algebraic structures, which were previously difficult to visualize.

Key Features of Ulam's Spiral:

- **Prime Clusters**: Ulam's Spiral often shows that prime numbers tend to align in certain diagonal patterns. This is unexpected because primes are generally thought to be distributed randomly. These patterns suggest that primes may have underlying structures yet to be fully understood.
- **Diagonal Patterns**: Primes appear to be clustered along certain diagonals of the spiral, especially at regular intervals. These diagonals are referred to as "prime diagonals," and their existence suggests that primes might follow deeper regularities that are not immediately obvious.
- **Empirical Observations**: Although Ulam's Spiral doesn't offer a formula to predict the next prime, it provides a way of visualizing the distribution of primes. The observation of clusters and diagonals opens up avenues for further research in prime number theory, leading some researchers to conjecture that these patterns might have some deep, yet-to-be-understood relationship with number theory.

Relevance to Prime Number Theory:

Ulam's Spiral provides a simple, yet powerful tool for visualizing the distribution of primes. While it does not directly give a method for finding primes, it highlights an important feature of prime number theory: that primes may exhibit more structure than initially thought. By examining the spiral, mathematicians can explore different ways to study the statistical properties of primes and their relationship to other number-theoretic constructs.

Conclusion:

Ulam's Spiral is a fascinating representation of prime numbers and their distribution, and it serves as a foundation for discovering new insights into the behavior of primes. Although it does not provide a complete theory of primes, it offers a powerful visualization that aids in exploring the unknowns in prime number theory.

Goldbach's Conjecture:

Goldbach's Conjecture is one of the oldest unsolved problems in number theory, proposed by Christian Goldbach in 1742. It suggests that:

"Every even integer greater than 2 can be expressed as the sum of two prime numbers."

Formal Statement:

For every even integer 2n ≥ 4, there exist prime numbers p_1 and p_2 such that:

$$2n = p_1 + p_2$$

Where:

- n is any integer greater than or equal to 2.
- p_1 and p_2 are prime numbers.

Examples of Goldbach's Conjecture:

Here are a few examples that demonstrate the conjecture:

- 4=2+2
- 6=3+3
- 8=3+5
- 10=3+7
- 12=5+7
- 14=3+11
- 18=5+13
- 20=3+17

In each case, the even number is represented as the sum of two prime numbers, which supports the conjecture.

Importance in Number Theory:

Goldbach’s Conjecture has been a central part of number theory for centuries, and despite much progress in related areas, it remains unproven. It has been verified computationally for very large numbers, but a general proof for all even numbers still eludes mathematicians.

Goldbach’s Conjecture is significant because:

1. **Prime Pair Sums**: It highlights a potential relationship between primes and even numbers. The conjecture suggests that primes are more deeply related to the structure of even integers than previously thought.
2. **Insight into Prime Distribution**: If true, Goldbach's Conjecture would give insight into the distribution of primes. The conjecture implies that primes are regularly spaced enough that they can always be found in pairs that sum to any even number, which might reveal hidden patterns in how primes are distributed.
3. **Connection to Other Problems**: Goldbach’s Conjecture is related to other unsolved problems in number theory, such as the Riemann Hypothesis and the Twin Prime Conjecture, as all these conjectures explore the distribution of primes in some form. If proven true, Goldbach’s Conjecture could help advance our understanding of prime number theory and the distribution of primes.

Status of Goldbach’s Conjecture:

- **Computational Verification**: The conjecture has been checked for even numbers up to very large limits, well beyond10^{18}, and it holds true for all tested cases.
- **Partial Progress**:
  - In 1937, **Ivan Vinogradov** proved a weakened form of the conjecture, known as the **"ternary Goldbach conjecture"**, which states that every sufficiently large odd number can be expressed as the sum of three primes.
  - In 2013, **Harald Helfgott** proved the **ternary Goldbach conjecture**, further advancing the understanding of prime sums.
- **Unproven General Case**: Despite all these efforts, Goldbach’s original conjecture remains unproven in general. The difficulty lies in the challenge of finding a general proof that works for all even integers.

Conclusion:

Goldbach's Conjecture, one of the most famous unsolved problems in mathematics, continues to intrigue mathematicians because of its simplicity and deep connection to the nature of primes. Although unproven, it has motivated much of modern research in number theory and the study of prime numbers. Any advancement or potential resolution of the conjecture would mark a significant milestone in the field. The conjecture’s potential connections with theories like SSPNT could provide a unique lens through which to view the mysteries of primes, adding another layer to the search for a deeper understanding of prime number distribution.

Prime Number Theorem (PNT):

Prime Number Theory (PNT) is a branch of number theory that deals with the distribution, properties, and behavior of prime numbers. It explores how primes are scattered across the set of natural numbers, aiming to uncover deeper insights into their frequency, patterns, and the mechanisms behind their appearance.

Key Concepts in Prime Number Theory:

1. **Prime Numbers:** Prime numbers are natural numbers greater than 1 that have no divisors other than 1 and themselves (e.g., 2, 3, 5, 7, and 11). Prime number theory focuses on understanding how primes are distributed and their relationship to other numbers.
2. **Prime Number Theorem:** The **Prime Number Theorem** is one of the most important results in PNT. It describes the asymptotic distribution of prime numbers and provides an approximation for the number of primes less than a given number NNN. The theorem states that:

   $$\pi(N) \sim \frac{N}{\ln N}$$

   Where:

   - π (N) is the prime-counting function, representing the number of primes less than or equal to N.
   - N/ln N is the natural logarithm of N.

   In simpler terms, this theorem suggests that the density of primes decreases as numbers get larger, but they continue to appear at regular intervals.

3. **Prime Distribution and Gaps between Primes:** One key focus of PNT is to understand how primes are spaced apart as numbers increase. There are several important results related to prime gaps:
   - **The Prime Gap Conjecture**: This conjecture states that there are infinitely many prime gaps (differences between consecutive primes) that are arbitrarily large.
   - **The Twin Prime Conjecture**: This conjecture suggests that there are infinitely many pairs of primes that differ by 2, called **twin primes** (e.g., 3 and 5, 11 and 13).
4. **Distribution Functions and Prime Density:** In PNT, mathematicians have developed several functions to quantify how primes are distributed across natural numbers:
   - The **Chebyshev functions** ψ(x) and θ(x) are used to study the density of primes in small intervals.
   - **The Riemann zeta function** plays a crucial role in understanding the distribution of primes, especially in the context of the **Riemann Hypothesis**.

5. **The Riemann Hypothesis:** The **Riemann Hypothesis** is one of the most famous unsolved problems in mathematics and is directly related to the distribution of prime numbers. It posits that all nontrivial zeros of the **Riemann zeta function** has a real part equal to 1/2. Proving this hypothesis would significantly improve our understanding of how primes are distributed, especially their density and gaps.
6. **Goldbach's Conjecture and PNT:** As previously mentioned, **Goldbach's Conjecture** is related to PNT in that it concerns the sums of prime numbers. Goldbach's conjecture suggests that every even integer greater than 2 can be expressed as the sum of two prime numbers. If proven true, it would deepen our understanding of the structure of even numbers and prime sums, which is a topic of significant interest in PNT.
7. **Additive Properties of Primes:** Another area explored in PNT is the additive properties of primes, such as:
   - **Waring's Problem for Primes**: Investigates whether every number can be written as a sum of a fixed number of primes.
   - **Goldbach's Weak Conjecture**: States that every odd number greater than 5 can be written as the sum of three primes.
8. **Prime Factorization and Its Role in PNT:** A central idea in PNT is **factorization**—breaking down numbers into their prime components. Prime factorization is key in many areas of mathematics, including cryptography, and PNT explores the patterns and behaviors of prime factorizations.

Important Results in Prime Number Theory:

- **Bertrand's Postulate (Chebyshev's Theorem)**: It states that for every integer $n \geq 2n$, there is always at least one prime number between n and 2n.
- **Mertens' Theorem**: This theorem provides a link between the prime number counting function and the asymptotic behavior of the logarithm.
- **The Brun Sieve**: A sieve method that allows the identification of primes in certain sets and has applications in number theory problems involving prime sums.
- **Dirichlet's Theorem on Arithmetic Progressions**: This theorem asserts that there are infinitely many primes in any arithmetic progression where the first term and the common difference are co-prime.

Applications of Prime Number Theory:

1. **Cryptography**: Prime numbers are the foundation of many encryption systems. Public-key cryptosystems, such as RSA, rely on the difficulty of factoring large composite numbers, which is linked to the unpredictability of primes.
2. **Computational Mathematics**: PNT is crucial in algorithms for primality testing, integer factorization, and cryptographic key generation.
3. **Random Number Generation**: Prime number theory also plays a role in generating pseudorandom numbers, important for simulations, cryptography, and secure communications.

4. **Mathematical and Computational Tools**: The study of prime numbers has led to the development of powerful algorithms and computational techniques, such as the **AKS primality test** and **Elliptic Curve Factorization**.

Conclusion:

Prime Number Theory (PNT) remains one of the most fascinating and challenging areas of mathematics. It seeks to uncover the deeper patterns and behaviors of prime numbers, providing insights into the very structure of numbers and the mathematical universe. SSPNT and other advanced prime number theories build upon classical results to explore even more refined properties of primes, potentially leading to breakthroughs in fields like cryptography, computational mathematics, and number theory. The ongoing search for prime patterns promises to yield new discoveries in the centuries to come.

The Seven-Set Prime Number Theorem (SSPNT):

And now, we start the writing of what is the purpose of this article, a way of sorting primes in a new manner that will enable us to find hidden patterns in primes and maybe, define primality. So now we start this article with a theoretical framework:

Theoretical Framework:

In this section, I will explain the theory and concepts behind the SSPNT.

Definition of SSPNT: The Seven-Set prime number theorem is a method to explain primal behavior and functions using seven sets of number that have infinite elements. In this method we sort out numbers in the following manner and then find fascinating patterns that occur in it:

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21

And so on.

We begin to see that primes occur in a diagonal going from right to left. There are two such diagonals. And astonishingly, every possible prime except 2 and 3 lies in these two diagonals. The following table will ease the understanding of this concept.

1	2	3	4	**5**	6	**7**
8	9	10	**11**	12	**13**	*14*
15	16	**17**	18	**19**	20	*21*

22	**23**	24	*25*	26	27	*28*
29	30	**31**	32	33	34	*35*
36	**37**	38	39	40	**41**	*42*
43	44	45	46	**47**	48	*49*
50	51	52	**53**	54	*55*	*56*

And this seems to give us prime numbers. We see that blocks having bold numbers are primes and blocks having italic numbers are composites. Yes, I have highlighted the seventh set here because we continue diagonals from sixth set. No element of the seventh set except seven is prime because all are basic multiples of seven. This is a good approach as multiples of seven behave very strangely. So by their elimination, the search of prime numbers becomes easy. But here we see the number 25 and 55 in the diagonal as well. It is a composite number. So after all, the diagonals are not just infinite series of pure primes. They have a lot of composites as well in them. But we eventually figure out some basic rules to filter out these composites some rules I have developed will be mentioned later along with problems. I will talk about equations that can be derived from this later on in detail.

Importance and Logic: Here, I will have a brief discussion on the importance of SSPNT. SSPNT is very important as it will help us to generate prime numbers and also improve our understanding about the distribution of prime numbers. It might find its uses in cryptography and other fields and may stand out as an extension or maybe a part of Prime Number Theorem (PNT). The logic behind the formation of diagonals is sensible and can be explained by modular arithmetic and quadratic equations. The details are in the later section of this article.

Research Questions:

While I have done all the research, many questions have arrived which I had to answers. My search for primes began 9 years ago when I was 7 or 8. I was intrigued and fascinated by the occurrence of primes and tried to work out a method by which I could understand primality. I independently came up with Meserene Prime Generation (though it was discovered much earlier, I didn't knew this). Meserene Primes are explained by formula:

$$P(m) = 2^x - 1$$

Where P (m) is the Meserene prime and x is any power. This formula doesn't always yield primes even though the largest prime known to this date has been found out with this formula. But still, it didn't satisfy me. My habits for assorting primes were to write them from 1 to 10 in first line and continue the series till 100. As 1 is neither prime nor composite, I removed it. Then I applied Euclid's sieve to filter remaining composite numbers. This method is extremely tedious for a human. But in 2024, I started writing numbers in set of 3; it gave an interesting zigzag pattern. But I was frustrated due to multiples of 7. To get rid of those, I arranged number in sets of 7. And I noticed that two diagonals are produced which contain all possible primes. I was very enthralled and started doing research and after one month of extremely high research, I am writing this research paper. Here are a few questions and their answers. These questions have been asked by me and various AI chatbots like Meta AI and ChatGPT. Here are those questions:

Q. What is the Seven-Set Prime Number Theorem (SSPNT)?

A: As I have already defined, SSPNT is a method for assorting primes in which we use visual sets all the way up to modular arithmetic and quadratic equations to find out primes.

Q. How does SSPNT contribute to the understanding of prime distributions?

A: This is an important question. SSPNT has shown that primes are not as random as we thought they would be. Instead, they would occur only as elements of two diagonals. It says that primality is not such an asymptomatic property as we had previously thought.

Q. What distinguishes SSPNT from other prime theories or methods (e.g., Sieve of Eratosthenes, Riemann Hypothesis)?

A: Yes, SSPNT is a pretty different approach than other prime number theorems and methods. For Example, Riemann Hypothesis (RH) involves use of Zeta function and complex numbers to find non-trivial zeros on the critical strip. It is more complex and is connected with analytical continuation. On the other hand, SSPNT offers a more visual approach to understand primes alongside modular arithmetic and quadratic equations. Considering Sieve of Eratosthenes, we found out that it is just checking via multiples of composites. It is not a theory but a method of calculation for computers and not humans. On the other way, SSPNT has characteristics of both a theory and a method of prime calculation.

Q. What is the process for constructing the seven sets, and how do they form diagonals?

A: Already answered.

Q. How do the modular arithmetic rules integrate into the SSPNT framework?

A: There is a strict framework of modular arithmetic. We use modulo of 6. Two quadratic equations then give mod of 1 and 5. These help us to sort primes in a very wide range.

Q. How are the quadratic equations derived within SSPNT, and how do they ensure primes are generated?

A: There is no general derivation of quadratic equations in SSPNT; it is through a person's sense that he concludes these quadratic equations. These are:

$$f(x) = 6x + 1$$

$$f(x) = 6x - 1$$

Q. How can SSPNT be applied computationally?

A: SSPNT can be applied computationally by various programming languages. Here, I have used Python as it is less case-sensitive and easy. Here is the code that I use:

```
import time
from math import isqrt, sqrt

def generate_diagonals(limit):
    """Generates numbers along two diagonals up to the given limit."""
    diagonal_1 = []
    diagonal_2 = []

    for n in range(1, limit + 1):
        num_1 = 4 * n + 1
        num_2 = 4 * n + 3
        if num_1 <= limit:
            diagonal_1.append(num_1)
        if num_2 <= limit:
            diagonal_2.append(num_2)

    return diagonal_1, diagonal_2

def filter_divisibility_by_5(numbers):
    """Filters out numbers divisible by 5, except 5 itself."""
    return [num for num in numbers if num == 5 or num % 5 != 0]

def filter_perfect_squares(numbers):
    """Filters out perfect squares."""
    return [num for num in numbers if sqrt(num).is_integer() == False]

def sieve_of_eratosthenes(limit):
    """Generates all primes up to the limit using the Sieve of Eratosthenes."""
    is_prime = [True] * (limit + 1)
    is_prime[0] = is_prime[1] = False  # 0 and 1 are not primes

    for num in range(2, int(limit**0.5) + 1):
        if is_prime[num]:
            for multiple in range(num * num, limit + 1, num):
                is_prime[multiple] = False

    return [i for i in range(limit + 1) if is_prime[i]]

def main(limit):
    """Main function to calculate primes and their statistics."""
    start_time = time.time()

    # Step 1: Generate diagonals
```

```python
    diagonal_1, diagonal_2 = generate_diagonals(limit)

    # Step 2: Apply divisibility by 5 rule
    diagonal_1 = filter_divisibility_by_5(diagonal_1)
    diagonal_2 = filter_divisibility_by_5(diagonal_2)

    # Step 3: Apply square root filter
    diagonal_1 = filter_perfect_squares(diagonal_1)
    diagonal_2 = filter_perfect_squares(diagonal_2)

    # Step 4: Combine filtered diagonals and apply Sieve of Eratosthenes
    combined_diagonals = sorted(set(diagonal_1 + diagonal_2))
    sieve_primes = sieve_of_eratosthenes(limit)

    # Step 5: Validate primes and include 2, 3, 5
    primes = [num for num in combined_diagonals if num in sieve_primes]
    primes = sorted(set(primes + [2, 3, 5]))

    # Calculate statistics
    prime_count = len(primes)
    prime_percentage = (prime_count / limit) * 100
    diagonal_count = len(diagonal_1) + len(diagonal_2)
    density = (prime_count / diagonal_count) * 100

    end_time = time.time()
    elapsed_time = end_time - start_time

    print(f"Limit: {limit}")
    print(f"Primes found: {prime_count}")
    print(f"Prime percentage: {prime_percentage:.2f}%")
    print(f"Time taken: {elapsed_time:.4f} seconds")
    print(f"Diagonal Density: {density:.2f}%")
    print(f"Primes: {primes[:20]} ... (showing first 20)")

if __name__ == "__main__":
    n = int(input("Enter the limit up to which primes should be calculated: "))

    main(n)
```

This code efficiently generates primes. Here is the time taken by this program to generate primes till a given value:

Number Limit	Time(s)
10	0
100	0
1000	0
10000	0.125
100000	6.3538

Q. Does SSPNT generate all prime numbers, and if so, up to what range?

A: Yes, SSPNT generates all prime numbers up to infinity (at least up to a googol). The prime generation is a pretty slow process but it is efficient. On the other hand, composite elements within diagonals are pretty common. But our elimination techniques can get rid of them in some time.

Q. Are there any exceptions or limitations observed in SSPNT?

A: The only limitation and hindrance according to me is the presence of composites within diagonals. These composite numbers can be removed by using efficient methods but still they give a sense of imperfection.

Q. What does the density of primes within the diagonals suggest?

A: The density in both diagonals start from 100 and starts declining gradually. Later on, I will explain the relationships of these two densities via graph and application of smoothing techniques to show a straighter graph.

Q. How does SSPNT align or contrast with the Prime Number Theorem (PNT)?

A: The **Seven-Set Prime Number Theorem (SSPNT)** and the **Prime Number Theorem (PNT)** both aim to provide insights into the distribution of prime numbers, but they do so in fundamentally different ways.

Prime Number Theorem (PNT):

The PNT provides an asymptotic formula for the distribution of primes, specifically stating that the number of primes less than or equal to a given number x is approximately:

$$\pi(x) = \frac{n}{\log n}$$

where π(x) denotes the prime-counting function (i.e., the number of primes less than or equal to x) and log(x) is the natural logarithm of x.

- **Focus**: PNT is focused on the large-scale distribution of primes and describes their density asymptotically. It does not give exact primes or a direct method for identifying them, but instead tells us how primes behave on average as numbers grow larger.
- **Applicability**: The PNT is a theoretical framework, valid in the asymptotic sense, giving an estimate for the density of primes as x →∞. It is based on deep results from analytic number theory and involves complex functions like the Riemann zeta function.
- **Nature of Results**: PNT provides a broad approximation and doesn't focus on finding specific primes. It describes the behavior of prime distributions over large numbers.

Seven-Set Prime Number Theorem (SSPNT):

SSPNT, on the other hand, presents a **constructive approach** to prime discovery by organizing numbers into seven sets, examining diagonals, and leveraging modular arithmetic to identify primes. It aims to classify primes through an arrangement of numbers and provides methods (like modular conditions and quadratic forms) to highlight the occurrence of prime numbers.

- **Focus**: SSPNT is more focused on **exact prime identification**. It categorizes primes by their position in the sets and diagonals and uses modular arithmetic to filter primes.
- **Methodology**: SSPNT introduces a geometric structure to primes by sorting numbers into sets and exploring patterns of primality, sometimes with the use of modular constraints and quadratic equations.
- **Key Insight**: The key contribution of SSPNT is its attempt to provide a more visual and number-theoretic method to find primes rather than just estimating their density. The method proposes that primes appear along specific diagonals in an ordered grid of numbers.

Comparison:

1. Approach:

- **PNT**: Describes the asymptotic behavior of primes, providing a "big picture" view of prime distribution.
- **SSPNT**: Offers a specific, constructivist method to identify primes through an ordered structure and uses modular arithmetic to extract primes.

2. Goal:

- **PNT**: Estimating the number of primes below a given number xxx asymptotically, particularly for large xxx.
- **SSPNT**: Aiming to explicitly identify primes by leveraging modular conditions and patterns in a structured grid.

3. Scope:

- **PNT**: Asymptotic, applies to large numbers, and helps in understanding the overall distribution of primes.
- **SSPNT**: More practical and specific, trying to find primes within a given limit and offering a new perspective on how primes can be systematically found.

4. Mathematical Depth:

- **PNT**: Deeply connected with advanced number theory, especially the analysis of the Riemann zeta function.
- **SSPNT**: Uses modular arithmetic and elementary number-theoretic methods to classify primes, making it more accessible at certain stages but less rigorous in its theoretical formulation.

5. Connection to Existing Theories:

- **PNT**: Established rigorously in the late 19th century, it is a cornerstone of analytic number theory and has been connected with the Riemann Hypothesis and other deep conjectures.
- **SSPNT**: A novel proposal that is not widely recognized as a major theoretical contribution to number theory, but it could provide new insights into prime patterns.

Potential Integration:

While SSPNT is focused on an explicit method to classify and find primes, the PNT deals with the overall density and distribution of primes at scale. SSPNT does not conflict with PNT but could be seen as an attempt to refine or extend our understanding of prime numbers by offering a new perspective on their occurrence.

1. **Extension of PNT**: SSPNT could offer insights or even act as a refinement of certain aspects of prime distribution. For example, by investigating primes in the context of diagonals and modular arithmetic, SSPNT might uncover patterns that could be connected with the density described by PNT.
2. **Comparative Approach**: A future investigation could attempt to compare the prime-counting function predicted by PNT with the actual primes identified via SSPNT to look for possible discrepancies or patterns that extend beyond the approximation given by PNT.

Conclusion:

The **SSPNT** and **PNT** are complementary but distinct approaches to understanding primes:

- PNT provides a high-level approximation to the behavior of primes as numbers grow large.
- SSPNT offers a hands-on, constructive method for finding primes by organizing numbers into sets and examining their modular properties.

By combining both approaches, there could be opportunities to extend number-theoretic research and better understand the underlying structure of primes.

Q. Can SSPNT be used to validate or extend the Riemann Hypothesis?

A: The Seven-Set Prime Number Theorem (SSPNT) and the Riemann Hypothesis (RH) deal with prime numbers, but they approach prime distribution from different angles. SSPNT focuses on visualizing primes through number sets, diagonals, and modular arithmetic, while the Riemann Hypothesis centers around the distribution of primes as encoded in the Riemann zeta function.

Can SSPNT be used to validate or extend the Riemann Hypothesis?

1. **Validation of RH using SSPNT:** The Riemann Hypothesis deals with the zeros of the Riemann zeta function and their alignment along the "critical line" (where the real part of s is 1/2 in the complex plane). This hypothesis is focused on understanding the behavior of prime numbers in relation to complex analysis and analytic number theory.

   SSPNT, on the other hand, provides a visual and arithmetic method to explore primes through modular arithmetic and number patterns. While SSPNT generates primes based on modular congruence and diagonal number arrangements, it doesn't directly probe the non-trivial zeros of the Riemann zeta function. As such, it doesn't directly validate or disprove the Riemann Hypothesis.

2. **Extension of RH using SSPNT:** SSPNT might provide an interesting *alternative visualization* or a *complementary perspective* on prime distribution. It might not prove or disprove RH, but it could help illuminate certain regularities or patterns in prime numbers that might inform our understanding of prime distribution.

   For example, SSPNT's approach of identifying prime patterns within number sets and examining the modular properties of those numbers could lead to new insights about prime gaps, density, and other prime-related phenomena. However, it is not clear how SSPNT could directly extend the Riemann Hypothesis, which is deeply rooted in complex analysis and relies on properties of analytic functions.

3. **Potential Connections:**
   - **Modular Arithmetic and RH:** Both SSPNT and the Riemann Hypothesis involve modular arithmetic, albeit in different forms. The zeros of the Riemann zeta function have deep connections to modular forms and elliptic curves. If SSPNT's modular approach can uncover deeper patterns in prime distribution, it could inspire new ideas that complement the tools used in proving RH.
   - **Prime Distribution:** The Riemann Hypothesis offers insights into how primes are distributed, particularly in the context of asymptotic behavior. SSPNT's approach of organizing numbers into sets and exploring diagonals could potentially reveal structural features of prime distribution that resonate with the patterns RH

suggests, but this would require a deeper exploration of how SSPNT's framework could connect to the complex analysis underpinning RH.

Conclusion: While SSPNT might not directly validate or extend the Riemann Hypothesis, it offers a novel and visual way of understanding prime numbers and their properties. By uncovering new patterns, SSPNT could potentially provide inspiration for more refined techniques or insights that might indirectly contribute to the broader discussion of prime number distribution, potentially offering fresh avenues of exploration alongside classical number theory approaches like the Riemann Hypothesis. However, proving or extending RH would likely require tools rooted in analytic number theory, not just arithmetic and modular-based insights from SSPNT.

Q. What insights can SSPNT provide that are absent in other theories, such as Ulam's Spiral?

A: Well, the SSPNT is the theory that resembles Ulam's spiral the most. In Ulam's spiral, we arrange primes in a spiral manner and we observe primes in diagonal lines. In SSPNT, we also see primes arranged in diagonal lines. SSPNT and Ulam's spiral strongly reinforce each other. But in Ulam's spiral there are infinitely many diagonals but in SSPNT there are only two diagonals that account for all the primes. Here is a more detailed analysis:

1. Structured Methodology for Prime Identification

- **SSPNT** uses a step-by-step process, such as:
    - Divisibility by key numbers (e.g., 5).
    - Elimination through square root checks.
    - Filtering with a reserve of previously identified primes.
- This structured approach introduces a logical sequence for systematically eliminating non-prime numbers from a set. Unlike **Ulam's Spiral**, which primarily focuses on visualization, SSPNT is computationally explicit and can directly influence algorithmic optimizations for prime number identification.

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2. Mathematical Rigor and Filtering Efficiency

- SSPNT incorporates **filtering rules** that eliminate composite numbers based on divisibility, square root checks, and combinatorial multiplication of primes (reserve method).
- This process is computationally efficient compared to the **trial-and-error divisibility checks** used in Ulam's Spiral-based explorations.
- It offers a **theoretical framework** for exploring prime patterns in number sets and can be fine-tuned or expanded with additional rules.

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3. Insights on Composite Numbers

- **SSPNT emphasizes composite elimination**, which highlights how composite numbers cluster around certain intervals or diagonals.
- Ulam's Spiral, while visualizing primes along diagonals, does not explicitly address composite number clustering in the same detail.

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4. Direct Numerical Applications

- SSPNT operates directly on numerical data, allowing it to compute and filter primes efficiently without relying on geometric or graphical representation.
- Ulam's Spiral is limited to visual patterns, making it less practical for large-scale computations of prime numbers. SSPNT's logical steps, such as the reserve method, offer a pathway for extending its applicability to **higher ranges of primes**.

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5. Integration with Graphical Representations

- While SSPNT is primarily computational, its results can still be visualized in forms like:
  - Heatmaps of prime densities.
  - Logarithmic smoothing (AGSM/NGSM) to represent trends.
- Ulam's Spiral focuses on graphical representation, but it doesn't integrate computation or prime density analysis directly.

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6. Prime Density and Distribution Insights

- SSPNT can provide insights into **prime density variations** and patterns across number ranges due to its ability to filter and classify numbers.
- Ulam's Spiral primarily shows patterns of primes along diagonals but doesn't offer quantitative analysis of density or rates of occurrence.

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7. Exploration of Prime Gaps

- SSPNT's reserve method allows exploration of prime gaps by identifying which numbers are eliminated or retained during the filtering process.

- This quantitative insight into **how primes thin out** with increasing range is more computationally explicit compared to Ulam's Spiral's qualitative patterns.

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8. Customizable and Extendable Framework

- SSPNT's rules (e.g., divisibility checks, reserve method) are adaptable. Researchers can modify or add rules to explore prime numbers in specific contexts.
- Ulam's Spiral is constrained to its visual diagonal patterns, making it less adaptable for different types of prime number research.

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9. Insights on Prime Number Theorems

- SSPNT aligns with **Prime Number Theorem (PNT)** in providing insights into prime density but adds new perspectives through its unique filtering mechanisms and reserve-based eliminations.
- It offers a direct computational method that complements the asymptotic analysis of PNT.

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10. Potential for Automation and Scaling

- SSPNT can be automated for large-scale prime number computation, leveraging modern computational resources.
- Ulam's Spiral, while visually appealing, becomes computationally expensive for visualizing very large datasets and is less scalable compared to SSPNT.

Unique Contributions of SSPNT:

- **Algorithmic Precision**: Provides a clear, stepwise method for identifying primes that goes beyond visual patterns.
- **Hybrid Theoretical-Computational Approach**: Combines logical rules (e.g., divisibility, square roots) with computational efficiency.
- **Flexibility**: Easily integrates with advanced methods like logarithmic smoothing (AGSM/NGSM) and density analysis.

Here, I want to make a clarification about AGSM and NGSM. AGSM stands for Ahmad-Gauss smoothing method named after me Ahmad Abdullah and famous mathematician Carl Friedrich

Gauss. This smoothing technique uses logarithm with base 10 to smooth data. NGSM stands for Napier-Gauss Smoothing Method named after John Napier, the pioneer of logarithms and Carl Friedrich Gauss. This technique uses logarithm with base e (Euler's Number = 2.718281828…….). These methods have been independently derived by me and will play a crucial role in understanding density of primes within diagonals later on. These are usually used by me to smooth data with much more deviation. Though, I will use a technique I developed called A and M Variable Logarithmic Smoothing Technique and also I will take help of moving averages. The formula of AMVLST is as follows:

$$A = \log_y x_1 , \log_y x_2, \log_y x_3 , \ldots$$

Here, y is the base of log which depends upon the type of data by which we are dealing with.

Here, A is the smoothed result, the values of x are values of data and n is the number of values.

The formula of Ahmad-Gauss Smoothing Method is:

$$A = \log(x_1) , \log(x_2) , \log(x_3) , \log(x_4) , \ldots$$

And for Napier-Gauss Smoothing Method is:

$$A = \ln(x_1) , \ln(x_2) , \ln(x_3) , \ln(x_4) , \ldots$$

Q. What potential applications does SSPNT have in fields such as cryptography or computational mathematics?

A: In cryptography, SSPNT might play a crucial role. By having a new method that can find primes, cyber security will become even stronger as it heavily relies on prime numbers. In computational mathematics, SSPNT will help in finding large primes.

Q. Can SSPNT aid in predicting large primes more efficiently than existing methods?

A: The answer is that I am not so certain. I don't have vast processors to compute the result. However, if we update the existing code with quadratic equations instead of diagonals, we will be able to calculate larger primes with much more speed. This is all pretty arbitrary and has not been proven yet. Though, I take SSPNT as a theory rather than a computational method.

Q. What are the potential improvements or extensions to SSPNT?

A: Well, I take this statement as a law that there is always room for improvement. SSPNT is a diabolical way of finding primes. Composites occur in diagonals and we have to find a way to get rid of those composites. Modular arithmetic, quadratic equations, diagonal densities are all extensions to SSPNT. Somebody could maybe prove its connection to Riemann Hypothesis.

Q. How can SSPNT be tested or validated for extremely large numbers?

A: Testing or validating SSPNT for extremely large numbers presents significant computational challenges, primarily due to the high time complexity involved. As the size of the numbers increases, the process of generating and filtering primes along diagonals becomes slower, making current implementations less feasible for very large numbers. However, there are several potential improvements that could enhance the efficiency of SSPNT for large numbers. The introduction of **modular arithmetic** and **quadratic equations** could significantly speed up the filtering and prime identification processes. These methods might allow for faster elimination of non-prime candidates, thus reducing the computational load. Additionally, optimizing the algorithm using parallel processing or leveraging more advanced mathematical techniques such as **sieve-based methods** could help scale SSPNT to handle extremely large primes. Furthermore, testing SSPNT against known prime databases or utilizing high-performance computing resources may assist in validating its performance with larger datasets.

Q. What open questions does SSPNT leave about the nature of primes?

A: SSPNT has made significant contributions to understanding the distribution of primes by organizing them into identifiable patterns along diagonals, but there are still several open questions. One of the most prominent questions remains its potential connections to major unsolved problems in number theory, such as the **Riemann Hypothesis** and **Goldbach's conjecture**. While SSPNT provides a framework for visualizing prime densities and relationships, it does not yet offer a conclusive way to predict the exact distribution or structure of primes at large scales. Furthermore, its applicability to other prime-related conjectures and the possibility of SSPNT shedding light on the gap between primes (like the twin prime conjecture) remains an open avenue for research. How these results can be generalized or reconciled with existing results in analytic number theory could help address these foundational questions.

Methodology:

It is time that I explain the methodology of how SSPNT works. I had given a brief introduction before but now it is time to explain the things in much greater detail. So, I am going to start now. Let's make a larger table:

1	**2**	**3**	4	**5**	6	**7**
8	9	10	**11**	12	**13**	*14*
15	16	**17**	18	**19**	20	*21*
22	**23**	24	*25*	26	27	*28*
29	30	**31**	32	33	34	*35*
36	**37**	38	39	40	**41**	*42*
43	44	45	46	**47**	48	*49*
50	51	52	**53**	54	*55*	*56*
57	58	**59**	60	**61**	62	*63*

64	*65*	66	**67**	68	69	*70*
71	72	**73**	74	75	76	*77*
78	**79**	80	81	82	**83**	*84*
85	86	87	88	**89**	90	*91*
92	93	94	*95*	96	**97**	*98*
99	100	**101**	102	**103**	104	*105*

113	114	115	116	117	118	*119*
120	121	122	123	124	*125*	*126*
127	128	129	130	**131**	132	*133*
134	135	136	**137**	138	**139**	*140*
141	142	*143*	144	*145*	146	*147*
148	**149**	150	**151**	152	153	*154*
155	156	**157**	158	159	160	*161*
162	**163**	164	165	166	**167**	*168*
106	**107**	108	**109**	110	111	*112*

………………..

And the list goes on.

The primes found till 168 are:

2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97,101,103,107,109,113,127,131,137,139,149,151,157,163,167

We see that all the primes occur within two diagonals. However, the diagonals do not just consist of prime numbers but composite numbers as well. To remove these composites, we can apply following techniques:

1. **<u>Applying Sieve of Eratosthenes:</u>**

We can apply Sieve of Eratosthenes to filter out composites from diagonals. This step will increase efficiency of Sieve as well. According to my calculations, this method should ideally reduce computing time to $\frac{113}{105}$ of the original time. This value is just an estimate. But there are other fancy and computationally expensive methods out there too.

2. Applying my own method:

I, myself have developed a complex method of filtering composites. It is a multi-step method and it calculates primes but is time-taking. Here is the overview:

a) Filtering out multiples of 5:

We encounter many numbers in diagonals such as 25, 55, 65, 85, 95, 125, 145, 155… These numbers are odd multiples of 5. To remove these numbers, we can use the following methods.

i) Dividing by 5:

In this method, we divide each element of diagonal by 5; the number which gives a whole number is divisible by 5 and is eliminated. The numbers giving digits after decimal point are potential primes are retained.

ii) Checking for last number:

In this method, we check the last number of every element of the diagonal. If the last number is 5 it is multiple of 5 and eliminated otherwise eliminated.

b) Eliminating perfect squares:

They also occur sometimes in a diagonal i.e. 121, 169, etc. We can eliminate them by taking square root of all the elements of diagonal. If an element yields a surd, it is a potential prime and can be retained. If a perfect square is given, it is a composite and can be efficiently eliminated.

c) Eliminating the prime products:

We have to also eliminate numbers like 143, 187, etc. that are the product of two primes. There is couple of methods of removing them:

i) Dividing by previously found primes:

In this method, we divide the elements found in primes to be divided by previously found primes. This method is efficient to eliminate remaining composites and find pure primes.

ii) **Using Reserve Method:**

We can also use a very fancy but computationally expensive method. In this method, we find all prime products possible through formula nCk where n is the number of primes, C is combination and k is the number of primes to be multiplied i.e. for double prime products, k = 2 and so on.

Well, until now we have discussed primes in seven set and their visual appearance and arrangement. But if we observe the table closely, we will find out that we can derive certain quadratic equations which alongside their modular property exhibit same functioning as diagonals. These two quadratic equations are:

$$f(x) = 6x + 1$$

And:

$$f(x) = 6x - 1$$

Here, I will try to give examples for both equations to show how these equations work:

<u>First Equation:</u>

The first equation is:

$$f(x) = 6x + 1$$

Let us apply values of x from 1 to 10:

$$f(1) = 6(1) + 1$$

$$f(1) = 7$$

We get the first element of the second diagonal i.e. 7.

For x = 2

$$f(2) = 13$$

For x=3 to 10

$$f(3) = 19$$

$$f(4) = 25$$

$$f(5) = 31$$

$$f(6) = 37$$

$$f(7) = 43$$

$$f(8) = 49$$

$$f(9) = 55$$

$$f(10) = 61$$

So, by using this test, we can find primes:

Second Equation:

The second equation is given as follows:

$$f(x) = 6x - 1$$

For x = 1 to 10

$$f(1) = 5$$

$$f(2) = 11$$

$$f(3) = 17$$

$$f(4) = 23$$

$$f(5) = 29$$

$$f(6) = 35$$

$$f(7) = 41$$

$$f(8) = 47$$

$$f(9) = 53$$

$$f(10) = 59$$

So, by the equation $6x \pm 1$, we can figure out all the possible primes. But we also observe that composites are occurring as they were in diagonals. But studying these equations does help us to figure out composites in a much easier fashion. Before explaining this procedure, I do want to mention that these equations have been studied before to find primes but such an extensive research hasn't been done yet. So this is the method of removal of composites.

So we consider composites $y_1, y_2, y_3, \dots$ having a prime number x as their multiple in the diagonals. We can filter these composites by the following points that we observe:

1. The smallest composite y_1 being a multiple of a prime x will always be equal to x^2.
2. We can assign a number to each composite. We will notice that the next composite will occur after the number of entries as the value of prime by which it is divisible. Let us take the example of the first diagonal. The elements produced in the first diagonal are:

5, 11, 17, 23, 29, 35, 41, 47, 53, 59 …

We can label 5 as first element, 11 as second and so on. We don't check for multiples of 2 and 3 as they are not even present in the elements of the diagonal. Let us check for multiples of 5. We consider x=5. By applying $x^2 = y_1$, we find first multiple of 5 to be 25. It doesn't occur in this list. But we do find 5 as the first element. If we look 5 elements ahead, we will see 35 which is divisible by 5. So, we can make the formula:

$$y_n = y_{n-1} + x$$

This is just a general formula for understanding. We will see that number 35 is the sixth element. So we can predict that eleventh, sixteenth, etc. elements will also be divisible by 5. We can check this by putting it in formula:

$$f(x) = 6x - 1$$

This is the same formula which generates the diagonal. We will always notice that the square of number occurs in the other diagonal. This method is an important contribution to the Prime Number Theory. It can help us to determine primes with a high accuracy using equations and also provides a suitable method for filtering composites. Our next section will be dealing with the density of primes.

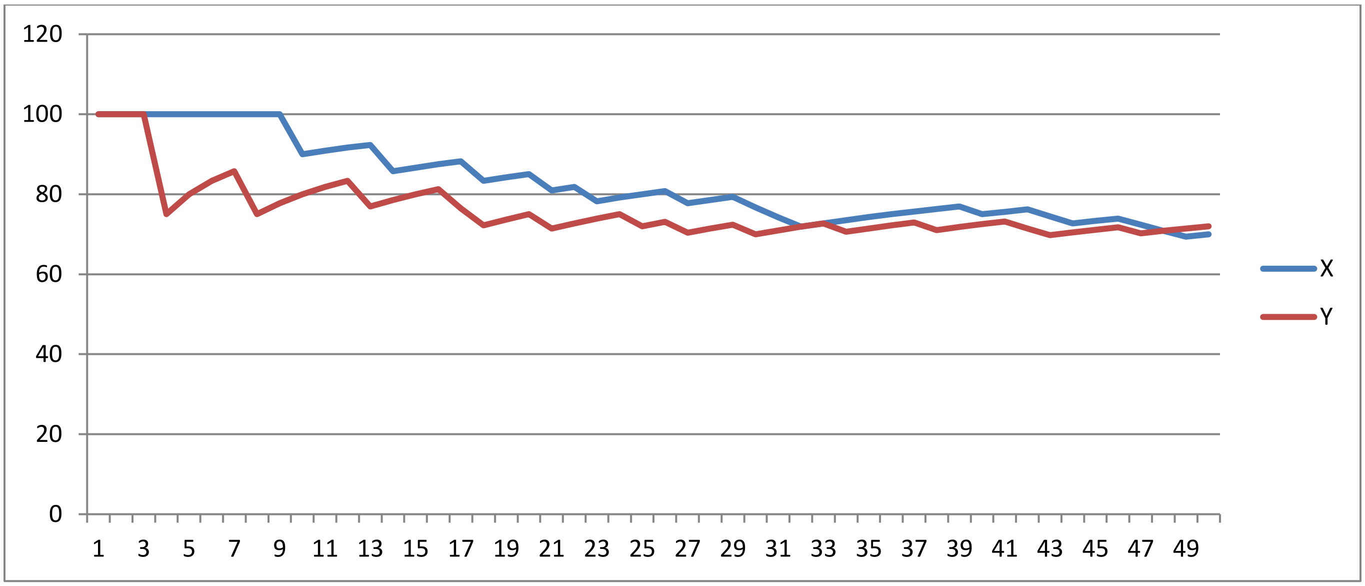
Primal Density:

We have discovered a method that let us obtain primes. But an important part of a Prime Number Theorem is to talk about the primal density, why it decreases and other relevant questions. My hypothesis is that the decrease of primal density is due to addition of more composites as more primes are discovered. My method of explaining primal density is not like explaining density with respect to every number but I only discuss the densities of the diagonals. It is a way more effective approach. I have made a spreadsheet that shows primal density of first 50 numbers.

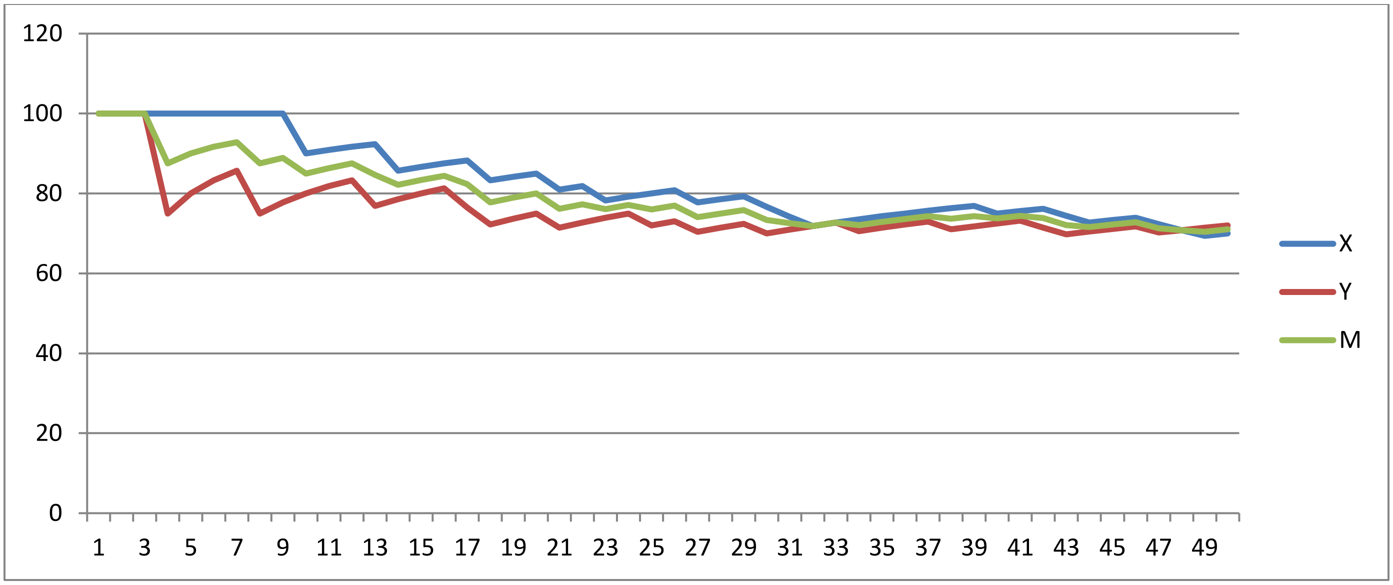
X	Y
100	100
100	100
100	100
100	75
100	80
100	83.33
100	85.71
100	75
100	77.78
90	80
90.91	81.82
91.67	83.33
92.31	76.92
85.71	78.57
86.67	80
87.5	81.25
88.24	76.47
83.33	72.22
84.21	73.68
85	75
80.95	71.43
81.82	72.73
78.26	73.91
79.17	75
80	72
80.77	73.08
77.78	70.37
78.57	71.43
79.31	72.41
76.67	70
74.19	70.97
71.88	71.88
72.73	72.73
73.53	70.59
74.29	71.43
75	72.22
75.68	72.97
76.32	71.05
76.92	71.79
75	72.5
75.61	73.17
76.19	71.43

74.42	69.77
72.73	70.46
73.33	71.11
73.91	71.74
72.34	70.21
70.83	70.83
69.39	71.43
70	72

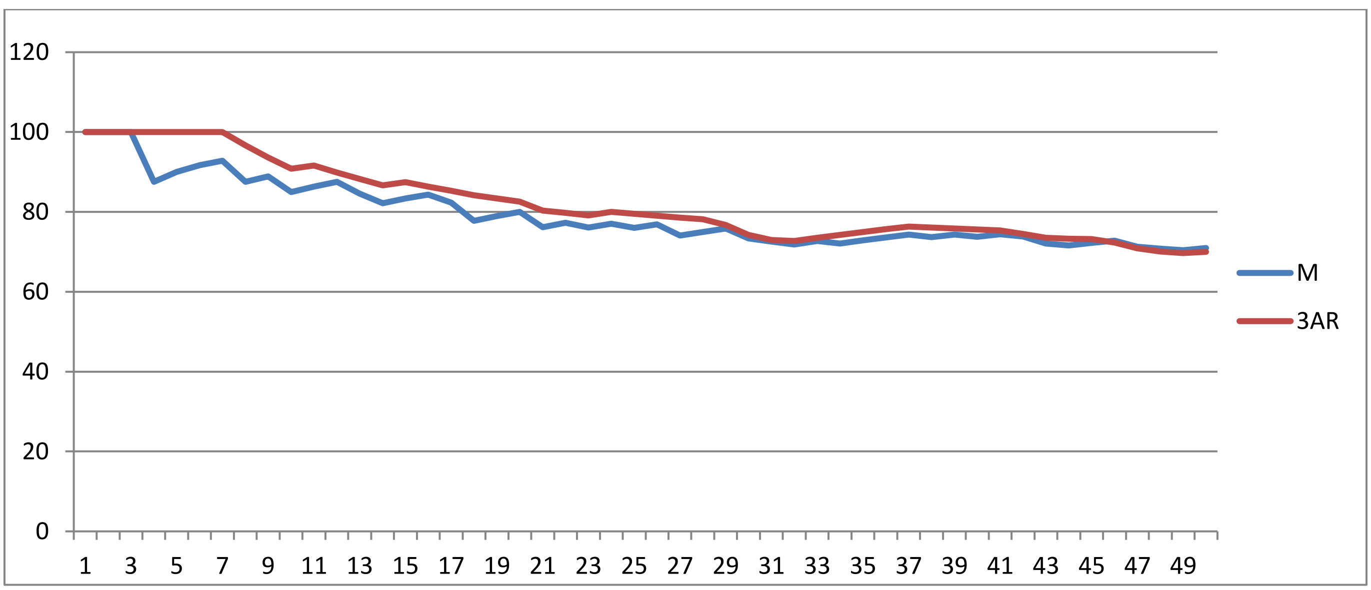
When we draw a graph, it gives the following result:



We notice that the first diagonal seems to have more density in the beginning but slowly the density equalizes the second diagonal and the densities become equal. If we draw a line showing mean density:



The mean line does provide some sort of regularity. Now it is time to use our smoothing techniques. I will use the moving average technique to smoothen line M. I could use AGSM or NGSM or AMVLST. But the problem in using those methods is that they are used to smooth larger sets of data and can just smoother the patterns. But when we approach 1000 or more elements of diagonals, these techniques will just become fundamental.



It is quite visible that the data smoothens. This is done by taking moving averages of first three values. It gives us a curve which keeps sloping downwards. At one point it will approximately touch zero. It gives us an idea of primal density

Conclusion:

After such a lengthy discussion and research, my research paper comes to an end. So in this paper, I presented an approach which not only gives us prime numbers but also a way that governs prime numbers. This can be a pretty naive approach in the Prime Number Theorem. But it might have a lot of implications, Even though I have given SSPNT, I still don't know how it might be related to Riemann Hypothesis, Goldbach's Conjecture (though I did ended up discovering an inverse of Goldbach's conjecture that every even number can be expressed as a difference of two primes) or Twin Prime Conjecture. These are still open questions which have to be solved. But still it has been a fascinating topic for me to explore. I just want to end my research paper with my following quotation:

"As far as I am concerned, primes seem to be captivated by diagonals."

Sources

The following sources have been used by me for writing this research paper.

- Wikipedia

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