Connecting Gravitation and Electromagnetism In the Dirac Gauge of the Electromagnetic Field

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Abstract. Motivated by the observation made by R. H. Dicke in 1957 that the speed of light seems to be correlated with the gravitational potential of the entire universe, we develop a spatially-variable speed of light theory of gravity based on a gravitational wavefunction. The gravitational wavefunction is treated as a fundamental property of matter, while the gravitational potential and resulting forces derived from it are emergent properties which are critically dependent on the distribution of mass on both local and global scales throughout the universe. We then show that the theory leads naturally to an association of gravitation with zero-point fluctuations of the electromagnetic field in the Dirac gauge, with the amplitude and frequency of the fluctuations constrained by the gravitational potential. The theory is free of gravitational singularities. It is capable of reproducing the Schwarzschild metric and is therefore consistent with tests of general relativity. It allows a first-principles calculation of the numerical value of the cosmological constant and the size of the universe using only the speed of light and the average density of ordinary matter in the universe which agrees closely with the observed values. Finally, it numerically predicts the approximate critical acceleration in modified Newtonian dynamics theory (MOND), providing an alternative explanation for dark matter. This shows that the theory may have value in explaining cosmological observations which are currently attributed to dark matter and dark energy.

1. The Gravitational Wavefunction

There is a serious problem with general relativity which is never discussed in textbooks or the scientific literature. Namely, the theory predicts that we should be surrounded by an abundance of black holes. In 1957 R. H. Dicke published "Gravitation Without a Principle of Equivalence" in which he made the observation that twice the gravitational potential of the observable universe is approximately equal to the speed of light squared [1]. This is precisely the event horizon condition. Small variations in the gravitational potential due to inhomogeneities in the distribution of matter should therefore cause the formation of black holes in abundance throughout our universe. In addition, despite measuring a speed of light of $2.99 \times 10^8 m/s$ in a local inertial frame, when we look out into the surrounding universe the speed of light should appear to be nearly zero. Clearly this is not consistent with observations of the local universe. In general relativity the gravitational potential of the universe is typically neglected and only the massive bodies of interest are taken into account, but there is no logical reason for doing this. The gravitational potential in the Einstein equation should represent the potential generated by all massive bodies in the universe. Dicke did not come to this conclusion in his paper. This is because he was considering the fact that the deflection of light around a gravitating body can be modeled as a local change in the refractive index of the form

$$n = \frac{c_0}{c} = 1 + \frac{2GM}{r{c_0}^2}. \tag{1}$$

He suggested that the number 1 on the right-hand side of this equation has its origin in the gravitational potential of all matter in the universe. However, this conclusion is also not logical because if it is the gravitational potential that determines the square of the speed of light then the speed of light ought to increase in the vicinity of a gravitating body instead of decrease. Some physicists would ascribe Dicke's observation to numerology or coincidence. Whether we call this numerology or not, the fact remains that general relativity predicts that our universe should appear much different than we observe when the potential of the rest of the universe is taken into account. This fact alone supports the idea that the speed of light is not a universal constant but varies spatially in a manner which all observers can objectively agree upon, as outlined in the variable speed of light theories first posited by Einstein, Dicke, and others [1–4]. This is not in violation of special relativity. Instead of saying that any two inertial observers moving relative to each other must measure the same speed of light, we must include the condition that they be in the same gravitational potential.

Let us make the assumption that Dicke's observation is more than just numerology and see where it leads. We are interested in determining the gravitational potential in the vicinity of mass M, and assume that the rest of the universe is very far away from this mass. How we define "in the vicinity" and "very far away" will be discussed later. The crux of the problem is to figure out why matter which is "very far away" should have a gravitational potential of opposite sign when compared to the gravitational potential of mass which is "nearby". We begin with the relativistic energy-momentum relation

$$E^2 = p^2c^2 + {m_0}^2c^4. \tag{2}$$

In the vein of a variable speed of light theory of gravity, if we rearrange terms we obtain the following quadratic equation for c^2

$${m_0}^2(c(\mathbf{r})^2)^2 + p^2(c(\mathbf{r})^2) - E^2 = 0. \tag{3}$$

Solving for $c(\mathbf{r})^2$ we obtain

$$c(\mathbf{r})^2 = -\frac{p^2}{2{m_0}^2} \pm \sqrt{\left(\frac{p^2}{2{m_0}^2}\right)^2 + \left(\frac{E}{m_0}\right)^2}. \tag{4}$$

If we choose the + sign in this expression we can see immediately that we may be on the correct path, as c^2 is determined by two terms with opposite sign.

Now we come back to Dicke's observation. According to expression (4) we require the gravitational potential of the entire visible universe to be associated with the square root term. The kinetic term $p^2/(2{m_0}^2)$ represents the gravitational potential only of matter which is nearby. Next, we must define, and if possible quantify, what we mean by nearby or far away. We may gain some insight about how to accomplish this from the Einstein equation of general relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{5}$$

The cosmological constant Λ is generally moved to the right-hand side of (5) and treated as a source of gravitation with positive energy density and negative pressure. This has become known as "dark energy" which could be considered the energy of the vacuum. We will consider a different interpretation here. Given that $G_{\mu\nu}$ contains second derivatives of the metric $g_{\mu\nu}$, it is interesting to note the similarities between (5) and the inhomogeneous Helmholtz equation having the general form

$$\nabla^2\phi(\mathbf{r}) + k^2\phi(\mathbf{r}) = 8\pi G\rho(\mathbf{r}). \tag{6}$$

In our interpretation, $\Lambda^{1/2}$ serves as the magnitude of a wavevector for the gravitational potential. While Λ is considered to be a scalar in general relativity, in its most general form the wavevector will be a complex number

$$\mathbf{k} = \boldsymbol{\beta} + i\boldsymbol{\alpha}. \tag{7}$$

Equation (6) has a solution of the form

$$\begin{aligned} \phi(\mathbf{r}) &= -\frac{2GM}{r}e^{i\mathbf{k}\cdot\mathbf{r}} \\ &= -\frac{2GM}{r}e^{-\alpha r}\left[\cos(\beta r) + i\sin(\beta r)\right]. \end{aligned} \tag{8}$$

In keeping with general relativity in the following we will assume $\alpha = 0$ so that the cosmological constant remains a scalar and (8) simply becomes

$$\phi(\mathbf{r}) = -\frac{2GM}{r}\left[\cos(\beta r) + i\sin(\beta r)\right]. \tag{9}$$

We will refer to (9) as the gravitational wavefunction, distinct from the gravitational potential which we now define below. Returning to equation (4), we propose the following relationships

$$\frac{p^2}{2{m_0}^2} = -Re\left\{\sum_i \phi_i\right\} = \sum_i \frac{2GM_i}{r_i}\cos\left(\beta r_i\right) \tag{10}$$

$$\frac{E}{m_0} = -Im\left\{\sum_i \phi_i\right\} = \sum_i \frac{2GM_i}{r_i}\sin\left(\beta r_i\right). \tag{11}$$

Using these expressions in (4) yields

$$c^2(\mathbf{r}) = -\sum_i \frac{2GM_i}{r_i}\cos(\beta r_i) + \sqrt{\left[\sum_i \frac{2GM_i}{r_i}\cos(\beta r_i)\right]^2 + \left[\sum_i \frac{2GM_i}{r_i}\sin(\beta r_i)\right]^2}. \tag{12}$$

We will also define the gravitational potential $\Phi(\mathbf{r})$ according to $2\Phi(\mathbf{r}) = c^2(\mathbf{r})$.

Next, we would like to determine the magnitude of the wavevector β in (9). We can do this by writing the sums in (12) as integrals and assuming an average mass density $\rho = 4.08\times10^{-28}kg/m^3$. This is 4.6% of the critical density in the ΛCDM model which is an estimate of the percentage of ordinary matter in the universe based on measurements of the inhomogeneity of the cosmic microwave background performed by the WMAP experiment [5]. Therefore, we are neglecting dark matter and dark energy in our model.

If we choose to integrate over a full wavelength the integrals take the following form

$$\begin{aligned}\sum_i \frac{2GM_i}{r_i}\cos(\beta r_i) &= \int_0^{2\pi/\beta} \frac{2G(4\pi r^2\rho)}{r}\cos(\beta r)dr \\ &= 8\pi G\rho \int_0^{2\pi/\beta} r\cos(\beta r)dr \\ &= 0\end{aligned} \tag{13}$$

$$\begin{aligned}\sum_i \frac{2GM_i}{r_i}\sin(\beta r_i) &= \int_0^{2\pi/\beta} \frac{2G(4\pi r^2\rho)}{r}\sin(\beta r)dr \\ &= 8\pi G\rho \int_0^{2\pi/\beta} r\sin(\beta r)dr \\ &= -\frac{16\pi^2 G\rho}{\beta^2}.\end{aligned} \tag{14}$$

Using (13) and (14) in (12) yields

$$\beta = \sqrt{\frac{16\pi^2 G\rho}{{c_0}^2}} = 6.92\times10^{-27}m^{-1}. \tag{15}$$

The wavelength of the gravitational wavefunction is given by

$$\lambda = \frac{2\pi}{\beta} = 9.08\times10^{26}m. \tag{16}$$

Note that the diameter of the observable universe is $8.8\times10^{26}m$. Thus, we have a remarkable coincidence which is very likely not a coincidence at all - the wavelength of the gravitational wavefunction corresponds nearly exactly to the diameter of the observable universe.

To gain further insight into the physics implied by equation (12) we plot the gravitational potential and acceleration for the mass of the Earth in an otherwise empty universe. The potential is given by

$$\Phi(r) = \frac{1}{2}c(r)^2 = \frac{GM}{r}\left[1-\cos(\beta r)\right]. \tag{17}$$

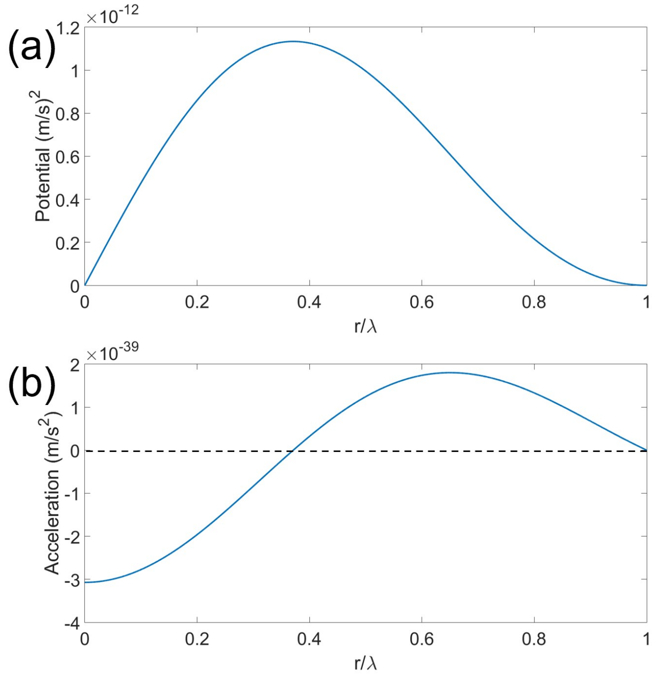

Figure 1. (a) Gravitational potential and (b) gravitational acceleration near an isolated mass equal to that of the Earth.

Figure 1(a) shows a plot of (17). There are two important points to be learned from this plot. The first is that the potential is everywhere positive, which it must be since the potential represents $c^2(r)$. This is the first departure from general relativity and Newtonian gravity. The second point is that there is no singularity at the origin. Instead, the origin is the location of the event horizon of general relativity where the speed of light goes to zero. Third, the speed of light increases until reaching a maximum at a distance of 0.37λ, where it then begins decreasing.

Figure 1(b) shows the gravitational acceleration as derived from (12)

$$a(\mathbf{r}) = -\frac{1}{2}\nabla c^2(\mathbf{r}) = \frac{GM}{r^2}\left[1 - \cos(\beta r)\right] - \beta\frac{GM}{r}\sin(\beta r). \tag{18}$$

The most important point from this plot is that the acceleration transitions from being attractive to repulsive at 0.37λ. Again, this is a drastic departure from both Newtonian gravity and general relativity where gravity is only attractive. In addition, the acceleration approaches a finite value at the origin instead of tending to infinity. In general, the acceleration approaches $-\beta^2 GM/2$ at the origin which can be obtained by applying L'Hôpital's rule to (18). Therefore, there is nothing particularly special about a black hole. The gravitational acceleration approaches a finite limit. Physics does not break down because there is nothing beyond the event horizon which lies at the origin.

It is notable that the gravitational acceleration near the surface of the Earth in an otherwise empty universe is not the familiar $-9.8m/s^2$ but $-3\times 10^{-39}m/s^2$ according to Fig. 1(b). This highlights an important aspect of our theory, which is that gravitational forces and accelerations are not fundamentally created by mass. Rather, gravitational forces emerge from the cooperative interactions between the gravitational wavefunctions of all masses in the universe. When we take into account the matter in the rest of the universe, (12) reduces to the familiar result from general relativity for the speed of light in the vicinity of a mass M.

$$c^2(r) = {c_0}^2 - \frac{2GM}{r} \tag{19}$$

and the acceleration reduces to the familiar Newtonian result

$$a(r) = \frac{-GM}{r^2}. \tag{20}$$

It is possible to calculate the net radial acceleration our galaxy experiences due to the surrounding matter in the universe using $a(r) = -\nabla\Phi(r) = -1/2\nabla c^2(r)$ and the general form of the potential given in (12). Evaluating the cosine term in front of the square root we obtain

$$\begin{aligned}
&-\frac{1}{2}\nabla\left[-\sum_i \frac{2GM_i}{r_i}\cos(\beta r_i)\right] = \sum_i \nabla\left[\frac{GM_i}{r_i}\cos(\beta r_i)\right] \\
&= \sum_i -\frac{GM_i}{{r_i}^2}\cos(\beta r_i) + \sum_i -\beta\frac{GM_i}{r_i}\sin(\beta r_i) \\
&= -4\pi G\rho\int_0^{2\pi/\beta}\cos(\beta r)\,dr - 4\pi G\rho\beta\int_0^{2\pi/\beta} r\sin(\beta r)\,dr \\
&= -4\pi G\rho\beta\left[-\frac{2\pi}{\beta^2}\right] = \frac{8\pi^2 G\rho}{\beta}.
\end{aligned} \tag{21}$$

Evaluating the square root term we obtain

$$\begin{aligned}
&-\frac{1}{2}\nabla\sqrt{\left[\sum_i \frac{2GM_i}{r_i}\cos(\beta r_i)\right]^2 + \left[\sum_i \frac{2GM_i}{r_i}\sin(\beta r_i)\right]^2} \\
&= -\frac{1}{4}\left[\left[\sum_i \frac{2GM_i}{r_i}\cos(\beta r_i)\right]^2 + \left[\sum_i \frac{2GM_i}{r_i}\sin(\beta r_i)\right]^2\right]^{-1/2} \\
&\times\left\{\begin{array}{l}\left[\sum_i \frac{4GM_i}{r_i}\cos(\beta r_i)\right]\times\left[\sum_i -\frac{2GM_i}{{r_i}^2}\cos(\beta r_i) - \sum_i \beta\frac{2GM_i}{r_i}\sin(\beta r_i)\right] \\ +\left[\sum_i \frac{4GM_i}{r_i}\sin(\beta r_i)\right]\times\left[\sum_i -\frac{2GM_i}{{r_i}^2}\sin(\beta r_i) + \sum_i \beta\frac{2GM_i}{r_i}\cos(\beta r_i)\right]\end{array}\right\} \\
&= -\frac{1}{4}\left[\left[8\pi G\rho\int_0^{2\pi/\beta} r\cos(\beta r)\,dr\right]^2 + \left[8\pi G\rho\int_0^{2\pi/\beta} r\sin(\beta r)\,dr\right]^2\right]^{-1/2} \\
&\times\left\{\begin{array}{l}\left[16\pi G\rho\int_0^{2\pi/\beta} r\cos(\beta r)dr\right]\times\left[-8\pi G\rho\int_0^{2\pi/\beta}\cos(\beta r)\,dr - 8\pi G\rho\beta\int_0^{2\pi/\beta} r\sin(\beta r)\,dr\right] \\ +\left[16\pi G\rho\int_0^{2\pi/\beta} r\sin(\beta r)dr\right]\times\left[-8\pi G\rho\int_0^{2\pi/\beta}\sin(\beta r)\,dr + 8\pi G\rho\beta\int_0^{2\pi/\beta} r\cos(\beta r)\,dr\right]\end{array}\right\}.
\end{aligned} \tag{22}$$

From the following identities

$$\int_0^{2\pi/\beta} r\cos(\beta r)dr = 0$$
$$\int_0^{2\pi/\beta} \sin(\beta r)dr = 0 \tag{23}$$

we find that (22) is identically zero, leaving only the contribution from (21). Using $\rho = 4.08 \times 10^{-28} kg/m^3$ and $\beta = 6.92 \times 10^{-27} m^{-1}$, we obtain an inward (positive) acceleration acting toward the center of the galaxy of $a = 3.1 \times 10^{-10} m/s^2$. The order of magnitude of this acceleration should be familiar to astrophysicists who study dark matter, as it is approximately the critical acceleration in Modified Newtonian dynamics (MOND) [6–8]. For accelerations smaller than approximately $1.2 \times 10^{-10} m/s^2$, neither Newtonian mechanics nor general relativity can account for the flat velocity versus distance curves for the outer regions of the Milky Way galaxy and other galaxies. Equation (12) tells us that the flat velocity curves are due to the fact that the acceleration becomes dominated by the rest of the universe pushing inward on the outer regions of the galaxy, instead of the inward pull from the galaxy's center.

Before concluding this section, we must address the assumptions we have made in our model. For example, in all of our integrals over the matter in the universe, we chose to integrate over a radius of a full wavelength $\lambda = 2\pi/\beta$. This decision may appear arbitrary. We could have integrated over a half wavelength or a quarter wavelength. This would change the numerical value we obtain for λ which seems to be correlated with the size of the universe, as well as the numerical value and direction of the MOND critical acceleration computed above. When we perform these calculations using different limits of integration we find that integration over a full wavelength yields the simplest form of the integrals and also agrees most closely with the known size of the universe and the empirically measured value for the critical acceleration. However, integration over a finite volume of the universe in what may very likely be an infinite universe is not very satisfying from a philosophical perspective.

We could also consider a model in which we change the upper limit of integration to infinity and incorporate a finite attenuation factor α. The wavefunction then exhibits an exponential decay

$$\phi(\mathbf{r}) = -\frac{2GM}{r} e^{-\alpha r} \left[\cos(\beta r) + i \sin(\beta r)\right]. \tag{24}$$

We will again use relationship (10) with the addition of the exponential decay factor

$$\frac{p^2}{2{m_0}^2} = -Re\left\{\sum_i \phi_i\right\} = \sum_i \frac{2GM_i}{r_i} e^{-\alpha r_i} \cos\left(\beta r_i\right). \tag{25}$$

However, instead of (11) we will use the following relationship

$$\sqrt{\left(\frac{p^2}{2{m_0}^2}\right)^2 + \left(\frac{E}{m_0}\right)^2} = \sum_i |\phi_i| = \sum_i \frac{2GM_i}{r_i} e^{-\alpha r_i}. \tag{26}$$

The reason for using (26) instead of (11) with the addition of an exponential decay factor is that the latter would result in a MOND critical acceleration of zero which is not consistent with observations. As we will see below, using (25) and (26) leads us again to a MOND critical acceleration very similar to the empirically observed value. From (26) we could also obtain an expression for E/m_0

$$\frac{E}{m_0} = \sqrt{\left[\sum_i \frac{2GM_i}{r_i} e^{-\alpha r_i}\right]^2 - \left[\sum_i \frac{2GM_i}{r_i} e^{-\alpha r_i}\cos(\beta r_i)\right]^2}. \tag{27}$$

Equation (4) then takes the form

$$c(\mathbf{r})^2 = -\sum_i \frac{2GM_i}{r_i} e^{-\alpha r_i}\cos(\beta r_i) + \sum_i \frac{2GM_i}{r_i} e^{-\alpha r_i}. \tag{28}$$

Writing the sums as integrals yields the following closed-form expressions in terms of α and β.

$$\begin{aligned} \sum_i \frac{2GM_i}{r_i} e^{-\alpha r_i}\cos(\beta r_i) &= \int_0^\infty \frac{2G(4\pi r^2 \rho)}{r} e^{-\alpha r}\cos(\beta r) dr \\ &= 8\pi G\rho \int_0^\infty r e^{-\alpha r}\cos(\beta r) dr \\ &= 8\pi G\rho \left[\frac{(\alpha^2-\beta^2)}{(\alpha^2+\beta^2)^2}\right]. \end{aligned} \tag{29}$$

$$\begin{aligned} \sum_i \frac{2GM_i}{r_i} e^{-\alpha r_i} &= \int_0^\infty \frac{2G(4\pi r^2 \rho)}{r} e^{-\alpha r} dr \\ &= 8\pi G\rho \int_0^\infty r e^{-\alpha r} dr \\ &= 8\pi G\rho \left[\frac{1}{\alpha^2}\right]. \end{aligned} \tag{30}$$

Equation (28) then becomes

$$\frac{8\pi G\rho}{\alpha^2} - 8\pi G\rho \left[\frac{(\alpha^2-\beta^2)}{(\alpha^2+\beta^2)^2}\right] = {c_0}^2. \tag{31}$$

Since α and β both have units of m^{-1}, we will assume that both of these factors scale along with the scale factor of the universe a such that $\alpha \propto \beta \propto a^{-1}$. Therefore the ratio $\alpha/\beta = \delta$ is a constant in cosmological time as the size of the universe increases. From (31) if we choose δ to satisfy the following equation

$$\frac{1}{\delta^2} - \frac{\delta^2 - 1}{(1+\delta^2)^2} = 2\pi \tag{32}$$

then (31) becomes identical to (15) so that we again have

$$\beta = \sqrt{\frac{16\pi^2 G\rho}{{c_0}^2}} = 6.92 \times 10^{-27} m^{-1}. \tag{33}$$

From (32) we can solve for δ to obtain $\delta = 0.42$ from which we have $\alpha = 2.90 \times 10^{-27} m^{-1}$.

We may also compute the local acceleration due to the rest of the universe in analogy with (21) and (22)

$$\begin{aligned}
&-\frac{1}{2}\nabla\left[-\sum_i \frac{2GM_i}{r_i}e^{-\alpha r_i}\cos\left(\beta r_i\right)\right] = \sum_i \nabla\left[\frac{GM_i}{r_i}e^{-\alpha r_i}\cos\left(\beta r_i\right)\right] \\
&= \sum_i -\frac{GM_i}{{r_i}^2}e^{-\alpha r_i}\cos\left(\beta r_i\right) + \sum_i -\alpha\frac{GM_i}{r_i}e^{-\alpha r_i}\cos\left(\beta r_i\right) + \sum_i -\beta\frac{GM_i}{r_i}e^{-\alpha r_i}\sin\left(\beta r_i\right) \\
&= -4\pi G\rho\int_0^\infty e^{-\alpha r}\cos\left(\beta r\right)dr - 4\pi G\rho\alpha\int_0^\infty re^{-\alpha r}\cos\left(\beta r\right)dr - 4\pi G\rho\beta\int_0^\infty re^{-\alpha r}\sin\left(\beta r\right)dr \\
&= -4\pi G\rho\left[\frac{\alpha}{\alpha^2+\beta^2}\right] - 4\pi G\rho\alpha\left[\frac{\left(\alpha-\beta\right)\left(\alpha+\beta\right)}{\left(\alpha^2+\beta^2\right)^2}\right] - 4\pi G\rho\beta\left[\frac{2\alpha\beta}{\left(\alpha^2+\beta^2\right)^2}\right] = 0.
\end{aligned} \tag{34}$$

$$\begin{aligned}
&-\frac{1}{2}\nabla\sum_i \frac{2GM_i}{r_i}e^{-\alpha r_i} = \sum_i \frac{GM_i}{{r_i}^2}e^{-\alpha r_i} + \alpha\sum_i \frac{GM_i}{r_i}e^{-\alpha r_i} \\
&= 4\pi G\rho\int_0^\infty e^{-\alpha r}dr + 4\pi G\rho\alpha\int_0^\infty re^{-\alpha r}dr \\
&= \frac{8\pi G\rho}{\alpha} = 2.35 \times 10^{-10} m/s^2.
\end{aligned} \tag{35}$$

Again, this is very close to the MOND critical acceleration.

Both theories we have presented are free of gravitational singularities. The latter theory which includes an exponential decay factor in the wavefunction has the advantage of not requiring an arbitrary limit of integration. From a philosophical standpoint it would seem more logical that we are gravitationally connected to an infinite universe, rather than arbitrarily imposing a finite limit of integration on the theory. In addition, the latter theory predicts a MOND critical acceleration closer to the locally measured value. However, this comes at the expense of an additional parameter α. As we will see in the next section, other than differences in the predictions of the MOND critical acceleration, from the standpoint of cosmology the two versions of the theory are indistinguishable. In the following, for simplicity we will use the model with no attenuation factor, although we should keep an open mind about which of the two theories is correct. The physics outlined in the rest of this article could easily be adapted to the model with a non-zero attenuation factor.

2. Cosmology

We will now outline the predictions of this theory on cosmology. Modern cosmology is currently based on the Friedmann equations which govern the expansion of the universe in general relativity

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{36}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \tag{37}$$

where k is the spatial curvature, Λ is the cosmological constant, and $\dot{a}$ and $\ddot{a}$ represent first and second time derivatives of the scale factor [9]. The scale factor is equivalent in a sense to what we have described as the wavelength of the gravitational wavefunction, which we have shown is correlated with the size of the universe. In a universe without spatial curvature and a cosmological constant (36) becomes simply

$$\left(\frac{\dot{a}}{a}\right)^2 = {H_0}^2 = \frac{8\pi G\rho}{3} \tag{38}$$

where H_0 is the Hubble parameter defined as $\dot{a}/a$. This equation is generally solved by assuming that the mass density decreases with the volume of the universe so that we may express it as $\rho = \rho_0 a^{-3}$, and the gravitational constant is taken to be constant in time. This equation may be solved to obtain

$$a(t) = \left(\frac{3}{2}\sqrt{\frac{8\pi G\rho_0}{3}}t\right)^{2/3}. \tag{39}$$

We note here that the gravitational constant has units of $m^3/(s^2 kg)$. A question that naturally arises is why ρ which has units of kg/m^3 decreases as $\rho = \rho_0 a^{-3}$ with the expansion of the universe, and the wavelength of light which has units of meters increases with the expansion of the universe, yet we treat $G \propto m^3$ in our theories as a constant. If we allow G to expand along with the universe then the product $G(t)\rho(t) = G_0\rho_0$ remains constant in time. The solution to (38) then becomes

$$a(t) = a_0 e^{H_0 t} \tag{40}$$

where

$$H_0 = \sqrt{\frac{8\pi G_0\rho_0}{3}} = \sqrt{\frac{8\pi G(t)\rho(t)}{3}}. \tag{41}$$

In a universe which consists only of dark energy with zero curvature and zero mass density equation (36) becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = {H_0}^2 = \frac{\Lambda c^2}{3}. \tag{42}$$

Since $H_0 = \sqrt{\Lambda c^2/3}$ is a constant, the solution to (42) is

$$a(t) = a_0 e^{H_0 t}. \tag{43}$$

Therefore, a universe filled with only dark energy evolves in exactly the same manner as a universe filled only with ordinary matter but where the gravitational constant is allowed to increase along with the scale factor. In other words, the dark energy problem is a problem created through the imposition of our preference for a constant gravitational constant on Nature.

We return now to equation (15) which was derived from our expression for c^2 in (12)

$$\beta = \sqrt{\frac{16\pi^2 G\rho}{c^2}}. \tag{44}$$

We may rewrite this equation as

$$c^2 = \frac{16\pi^2 G\rho}{\beta^2} = 4G\rho\left(\frac{2\pi}{\beta}\right)^2 = 4G\rho\lambda^2. \tag{45}$$

Taking the square root of both sides yields

$$c = \sqrt{4G\rho}\lambda. \tag{46}$$

Noting the similarity between (45) and (38), we associate λ with the scale factor a, which is not surprising given that we have already shown how λ is equal to the size of the observable universe. Given that $\sqrt{4G\rho}$ is a constant if we allow G to increase in time as the universe expands, we may interpret this quantity as a definition of cosmological time since $1/\sqrt{4G\rho}$ has units of seconds. When interpreted in this way, (46) is telling us that the speed of light is equal to one unit of cosmological length (the size of the universe λ) divided by one unit of cosmological time. Note also that the speed of light increases in cosmological time along with the size of the universe. This implies that when measured in units of light years, we will always measure the size of the universe to be unchanging in time. It is only increasing in size relative to meter sticks of fixed length on Earth.

Equations (45) and (38) also imply an association between c and $\dot{a}$. We therefore propose that the radius of the universe is increasing at a rate equal to the speed of light, which would imply the diameter is increasing at a rate equal to twice the speed of light, or $\dot{\lambda} = 2c$. We then have

$$\dot{\lambda} = 2c = 2\sqrt{4G\rho}\lambda \tag{47}$$

Note that this model of the universe is spatially flat by default, so there is no fine-tuning needed to make the universe flat. From this equation we may solve for the critical density

$$\rho_{crit} = \frac{\left(\dot{\lambda}/\lambda\right)^2}{16G} = \frac{{H_0}^2}{16G}. \tag{48}$$

Before evaluating this equation, we must point out another important implication of (46). Due to the fact that c is increasing in time along with the size of the universe, our measurements of H_0 which are based on the measurements of redshift are overestimated by a factor of 3. The redshift is related to the scale factor by

$$1 + z = \frac{a_{now}}{a_{then}} = e^{\sqrt{4G\rho}(t_{now}-t_{then})} \tag{49}$$

where t_{now} is the time at which the light from a distant object is measured by our telescopes and t_{then} is the time the light was emitted. For $\Delta t = (t_{now} - t_{then}) << 1/\sqrt{4G\rho}$ we may approximate (49) as

$$z = \sqrt{4G\rho}\Delta t. \tag{50}$$

However, we must also account for the fact that the Rydberg energy is given by $\alpha^2 m_e c^2/2$ where α is the dimensionless fine structure constant and m_e is the electron mass. If c is increasing in time, the energy of a photon emitted at time t_{then} would appear to be smaller, implying a longer wavelength, when compared to a photon emitted at time t_{now}. Equation (49) must therefore be modified to account for the additional apparent redshift caused by a time-varying c

$$1 + z = \frac{a_{now}}{a_{then}} \frac{c_{now}^2}{c_{then}^2} = e^{3\sqrt{4G\rho}\Delta t}. \tag{51}$$

which can be approximated by

$$z = 3\sqrt{4G\rho}\Delta t. \tag{52}$$

Equation (48) must be corrected to become

$$\rho_{crit} = \frac{(H_0/3)^2}{16G}. \tag{53}$$

Estimates of H_0 vary depending on the method used to infer it, ranging from the Planck/CMB result of 67.8 $km/s/Mpc$ to the result based on observations of Cepheid variables of approximately 74 $km/s/Mpc$. If we assume an intermediate value of 70 $km/s/Mpc$ we obtain a critical density of $\rho_{crit} = 5.35 \times 10^{-28} kg/m^3$. Since multiple experiments have proven that the observable universe is spatially flat to better than 0.4%, including BOOMERang [10], WMAP [5], and Planck [11], our theory predicts a different average density of ordinary matter in the universe compared to the ΛCDM model ($\rho = 5.35 \times 10^{-28} kg/m^3$instead of the value we used previously $\rho = 4.08 \times 10^{-28} kg/m^3$). If we use this new density in (15) and (16) we obtain $\lambda = 7.93 \times 10^{26} m$ or 83.8 billion light-years.

With this model, we may also compute the deceleration parameter of the universe which determines the rate of acceleration in the case where it is negative

$$q = -\frac{\ddot{\lambda}\lambda}{\dot{\lambda}^2}. \tag{54}$$

This was first measured to be negative by Riess and Perlmutter in the first discovery that the expansion is accelerating instead of decelerating [12, 13]. Subsequent data and processing of that data have placed further constraints on the magnitude of q. Using our expression for $\lambda = \lambda_0 e^{\sqrt{4G_0\rho_0}t}$ we obtain $\dot{\lambda} = \sqrt{4G_0\rho_0}\lambda$and $\ddot{\lambda} = 4G_0\rho_0\lambda$. Using these expressions in (54) gives $q = -1$. One of the most recent analyses which utilizes observations of supernovae beyond the local universe constrained the deceleration parameter to $q = -1.08 \pm 0.29$ which supports our theoretical model [14]. A value of $q = -1$ is only possible in the ΛCDM model if the universe is completely dominated by dark energy, whereas in our model it is possible in a universe with only ordinary matter.

Finally, we must consider the implications of this theory on the age of the universe. Given that the theory predicts an exponential rate of expansion given by (40), if we were to trace back the evolution of the universe to a singularity it would take an infinite amount of time. In addition, the universe only grows by a factor of e every 83.8 billion years. If the universe were only 13.8 billion years old, the universe would have only grown in size by approximately 18% since its birth. There appear to be several possibilities. The first is that the universe is drastically older than our current models predict. The second possibility is that the universe did not begin from a singular point but was much larger and colder at its birth. The third is that inflation resulted in a universe drastically larger than is currently predicted. It is also possible that there was a period of time in the early universe when the speed of light was significantly higher. Finally, it could be a combination of all or some of the above possibilities. We leave it an open question to cosmologists to determine which of the above or combination of the above is the most plausible based on the available cosmological data.

3. Connecting Gravitation and Electromagnetism

Any quantum theory of gravity must necessarily provide a microscopic and local description of the physics which leads to the emergence of the metric tensor of general relativity. We will attempt to provide a path to doing that here. Thus far, our discussion has been primarily mathematical in nature. We have not yet associated a physical intuition or meaning with (4), (10), and (11). These equations suggest that the energy of the gravitational field has both kinetic and potential energy components. A kinetic energy component implies motion. A natural question to ask is what does that motion look like?

We begin by considering a thought experiment consisting of a model universe in which the Earth is at the center. We also have a test particle in the vicinity of the Earth a distance r from its center and having mass m_0. In such a universe we have a background speed of light c_0 given by (12) and (14). Equation (12) can be approximated by

$$c^2(r) = -\frac{2GM_E}{r} + {c_0}^2 \tag{55}$$

where M_E is the mass of the Earth and

$$\frac{p^2}{2m_0^2} = \frac{2GM_E}{r}. \tag{56}$$

If we solve for the momentum in (56) we obtain

$$p = m_0\sqrt{2}\sqrt{\frac{2GM_E}{r}}. \tag{57}$$

The quantity $\sqrt{2GM_E/r}$ is a velocity. In fact, it is the local escape velocity at a distance r from the center of the Earth. We have left the $\sqrt{2}$ outside of this velocity term to highlight the fact that this equation suggests oscillatory motion with root-mean-square velocity $v_{rms} = \sqrt{2GM_E/r}$ and peak velocity $v_{peak} = \sqrt{2} \times v_{rms}$. In other words, the test mass is undergoing harmonic oscillatory motion with $v_{rms} = \sqrt{2GM_E/r}$. The next obvious question is the direction of motion. Given the spherical symmetry we might guess it is in the radial direction.

Let us now move to a reference frame very far away from the Earth so that its gravitational potential at our location is negligible. We would like to know how time and space look locally for the test particle near the Earth undergoing harmonic motion, and compare with time and space in our local frame far from the Earth where a test particle would experience no harmonic motion. This is simply given by the familiar Lorentz transformations. If we represent the coordinates of the test particle near the Earth with primed coordinates, we have

$$\frac{dt}{dt'} = \gamma = \frac{1}{\sqrt{1 - \frac{{v_{rms}}^2}{{c_0}^2}}} = \frac{1}{\sqrt{1 - \frac{2GM_E}{r{c_0}^2}}} \tag{58}$$

and

$$\frac{dr}{dr'} = \frac{1}{\gamma} = \sqrt{1 - \frac{{v_{rms}}^2}{{c_0}^2}} = \sqrt{1 - \frac{2GM_E}{r{c_0}^2}}. \tag{59}$$

These terms should look familiar for anyone who has studied general relativity, as they are part of the Schwarzschild metric describing the metric tensor around a spherically symmetric mass [9]

$$ds^2 = -\left(1 - \frac{2GM_E}{r{c_0}^2}\right) dt^2 + \left(1 - \frac{2GM_E}{r{c_0}^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{60}$$

which we could also write as

$$ds^2 = -\gamma(r)^{-2}dt^2 + \gamma(r)^2 dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{61}$$

Writing the metric in this way highlights the idea that the metric tensor is simply a tensor containing Lorentz transformation factors which vary spatially. These Lorentz factors arise from the apparent harmonic motion of test particles at different locations in space. It is worth noting that we have derived the Schwarzschild metric, which is all that is needed to produce all experimental tests of general relativity other than gravitational waves and frame-dragging effects, with only the gravitational wavefunction and the Lorentz transformations of special relativity. Nowhere have we required differential geometry, tensor calculus, or a set of 10 coupled nonlinear differential equations to derive this metric.

The harmonic oscillator is straightforward to quantize and is found in many aspects of quantum physics [15]. The vacuum ground state energy is given by

$$E = \frac{1}{2}\hbar\omega = \frac{2GM_E m_0}{r} \tag{62}$$

which we can solve for the frequency

$$\omega = \frac{4GM_E m_0}{r\hbar}. \tag{63}$$

The amplitude of the oscillation is given by

$$\sqrt{\langle x^2 \rangle} = \sqrt{\frac{\hbar}{2m_0\omega}} = \frac{\hbar}{2m_0}\sqrt{\frac{r}{2GM_E}}. \tag{64}$$

The wavefunction of the test particle is given by the zeroth order Hermite polynomial

$$\psi_0(x) = \left(\frac{m_0\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m_0\omega x^2}{2\hbar}} H_0\left(\sqrt{\frac{m_0\omega}{\hbar}}x\right). \tag{65}$$

If we only consider the Earth in our model universe the amplitude and frequency of these oscillations for a single electron are 5.18 nm and 0.34 THz. However, this is unrealistic as other masses in the vicinity of Earth such as our galaxy and other galaxies in our local region of the universe will contribute to the gravitational potential. If we take into account the mass of our galaxy making a very simplified assumption of all of its mass being located at its center, the amplitude and frequency become 32 pm and 8.95 PHz. It should be obvious that the amplitude of these oscillations is large enough that they should be easily observable if they existed. Furthermore, gravitational forces are the result of a gradient of this root-mean-square velocity, and speaking of the spatial gradient in the velocity of a point particle is a nonsensical notion. This would seem to invalidate our entire argument. However, the last section will be useful to motivate the next, in which we pivot to a description of gravitation as a true field over all space instead of focusing on the apparent effect of that field on matter.

In his 1957 paper, in addition to noticing that the gravitational potential of the universe is approximately c^2, R. H. Dicke made the interesting suggestion that gravitation could have its origin in electromagnetism [1]. There are obvious reasons for believing that this might be the case. Both gravitational waves and electromagnetic waves propagate at the same speed. Although this is predicted by general relativity, the theory does not provide a fundamental microscopic quantum mechanical description of why this should be. The electric field and gravitational field also obey the same inverse square law. As we will now show, a microscopic description of the gravitational potential itself can be provided by connecting the gravitational potential to the electromagnetic potential.

We begin by considering a gauge of the electromagnetic field which is generally not included in modern textbooks on electromagnetism. This is the Dirac gauge, which he outlined in “A new classical theory of electrons” in 1951 [16]. The gauge takes the general form

$$A_\mu A^\mu = -k^2 = -\frac{(m_0c^2)^2}{e^2} \tag{66}$$

where A is the electromagnetic vector potential, m_0 is the mass of a given particle, and e is its charge. S. Caser provides an excellent summary of this gauge and its physical interpretation in his article “Electrodynamics In Dirac’s Gauge: A Geometrical Equivalence” [17]. Note that we can also write (66) as follows

$$e^2V^2 = e^2A^2c^2 + {m_0}^2c^4 \tag{67}$$

which bears resemblance to the relativistic energy-momentum relation (2). Dirac felt this gauge was the most natural of all electromagnetic gauges because it ascribes a physical meaning to the vector potential. Namely, Dirac felt that the vector potential represented the local four-velocity of the vacuum. We will take a similar point of view here, except we propose that the vector potential represents more specifically the four-velocity of the particle's underlying quantum field of which it is an excitation. This four-velocity is given by

$$u^\mu = -\frac{e}{m}A^\mu. \tag{68}$$

To understand what led Dirac to this conclusion, we can use this four-velocity to calculate the force on an electron.

$$\begin{aligned} m\frac{du^\mu}{d\tau} &= -e\frac{dA^\mu}{d\tau} = eu^v\partial_v A^\mu = e\left(F^\mu{}_\nu + \partial^\mu A_\nu\right)u^\nu \\ &= eF^\mu{}_\nu u^\nu + \frac{1}{2}m\partial^\mu(u_v u^\nu) \\ &= eF^\mu{}_\nu u^\nu - \frac{1}{2}m\partial^\mu(c^2). \end{aligned} \tag{69}$$

In general relativity the last term involving the gradient of c^2 would be zero, leaving only electromagnetic forces. However, in our spatially variable speed of light theory we have

$$eF^\mu{}_\nu u^\nu - \frac{1}{2}m\partial^\mu(c^2) = eF^\mu{}_\nu u^\nu - m\partial^\mu(\phi) \tag{70}$$

which contains both electromagnetic forces and Newtonian gravity.

By analogy with (4) we can write

$$2\Phi(r) = c(r)^2 = -\frac{e^2A^2}{2m^2} + \sqrt{\left(\frac{e^2A^2}{2m^2}\right)^2 + \left(\frac{eV}{m}\right)^2} \tag{71}$$

where V is the voltage or electrostatic potential and A is the electromagnetic field momentum. In addition, we can make the following assignments in analogy with (10) and (11)

$$\frac{e^2A^2}{2{m_0}^2} = -Re\left\{\sum_i \phi_i\right\} = \sum_i \frac{2GM_i}{r_i}\cos\left(\beta r_i\right) \tag{72}$$

$$\frac{eV}{m_0} = -Im\left\{\sum_i \phi_i\right\} = \sum_i \frac{2GM_i}{r_i}\sin\left(\beta r_i\right). \tag{73}$$

Finally, we can draw an analogy to our microscopic description of the gravitational field. From (57) we obtain the expression for the root-mean-square field momentum.

$$A_{rms} = \frac{m}{e}\sqrt{\frac{2GM_E}{r}}. \tag{74}$$

From (73) and (71) we obtain an expression for the background potential of the field

$$V_0 = \frac{m_0{c_0}^2}{e}. \tag{75}$$

where we have used the fact that the expression under the square root in (12) is dominated by (73) which is approximately ${c_0}^2$. As an example, the background potential of the electron field is approximately -512 kV.

Equations (74) and (75) allow us to develop a physical picture of the gravitational potential and field. The gravitational potential is the result of local harmonic oscillation of the underlying quantum field from which the particle arises. The field oscillates with root-mean-square velocity given by the local escape velocity

$$v_{rms} = \frac{eA_{rms}}{m} = \sqrt{\frac{2GM}{r}} \tag{76}$$

at a frequency given by

$$\omega = \frac{4GMm}{r\hbar}. \tag{77}$$

Additionally, the rest mass can be equated to a background potential given by (75). This provides a resolution to the infinite energy problem of the ground state of the electromagnetic field predicted by quantum field theory. The frequency at any given point in space is constrained by the gravitational potential. Therefore, we find that it is not the harmonic oscillation of the particle with respect to a massive body creating a gravitational field around it which gives rise to the spatially-varying Lorentz factors in (61). Rather, it is the harmonic oscillation of the particle's underlying quantum field with respect to that particle which gives rise to these Lorentz factors. Naturally, these field oscillations will exert electromagnetic forces of the form $-edA/dt$ on isolated particles. However, intra-atomic and inter-atomic electrostatic forces are orders of magnitude larger than the forces exerted by the zero-point motion of the field due to gravitation. For example, the peak force on an electron due to the zero-point oscillations associated with the Earth's gravitational potential at its surface is on the order of $5 \times 10^{-15} N$, whereas the electrostatic force on an electron in hydrogen is on the order of $8 \times 10^{-8} N$, seven orders of magnitude larger. Therefore electrostatic forces completely dominate the motion of particles in matter.

The picture we have presented clearly differs from the standard interpretation of quantum field theory, where the vector potential represents its own field (the electromagnetic field), not the energy and momentum of other fields. Nevertheless, in the Dirac gauge this interpretation naturally arises from the mathematics. Note also that while the magnitude of the vector potential itself will be different for different fields having different charge-to-mass ratios, all fields experience the same four-velocity resulting from these vector potentials. If the four-velocities were different then the gravitational acceleration would be different for different particles.

As it turns out, equation (71) can give us some intuition for the physical meaning of the electrostatic potential V and momentum A. Let us consider the situation where the momentum of the electromagnetic field A is zero, so that we only have an electrostatic potential V. In this case (71) becomes

$$2\Phi(r) = {c_0}^2 = \frac{eV_0}{m_0}. \tag{78}$$

Now consider what happens when we introduce an additional potential due to a nearby charged particle on top of this background. We then have

$$V = V_0 + V(r) = V_0\left(1 + \frac{V(r)}{V_0}\right) \tag{79}$$

where $V(r)$ is the familiar Coulomb potential of the charged particle with charge q

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}. \tag{80}$$

Now (78) becomes

$$2\Phi(r) = {c_0}^2 = \frac{eV_0\left(1 + \frac{V(r)}{V_0}\right)}{m_0}. \tag{81}$$

This equation implies that the speed of light must depend on the additional potential. Clearly this does not make sense. There is no evidence to suggest that the speed of light changes in the presence of an electrostatic potential. Furthermore, it would not be logical for the speed of light to be different for different fields, as (81) is proportional to the charge to mass ratio of the field. The only way to fix this problem is to multiply the denominator by the same factor as the numerator

$$2\Phi(r) = {c_0}^2 = \frac{eV_0\left(1 + \frac{V(r)}{V_0}\right)}{m_0\left(1 + \frac{V(r)}{V_0}\right)} = \frac{eV_0\chi(r)}{m_0\chi(r)} \tag{82}$$

where we have defined

$$\chi(r) = 1 + \frac{V(r)}{V_0}. \tag{83}$$

If we multiply both sides of (82) by $m_0\chi(r)$ we obtain

$$m_0\chi(r){c_0}^2 = eV_0\chi(r) = e\left(V_0 + V(r)\right). \tag{84}$$

This equation is telling us something important about how we should interpret the electrostatic potential. Physically it corresponds to a position-dependent change in mass of the underlying quantum field from which the particle arises. To obtain the force acting on a particle of mass m_0 and charge e sitting in this external potential we can take the gradient of both sides of (84)

$$F = -{c_0}^2\nabla\left(\chi(r)m_0\right) = -e\nabla V(r). \tag{85}$$

In other words, the electric field of classical electromagnetism corresponds to a spatial gradient in the mass of a particle's underlying field. We can compute the change in electron mass in hydrogen as a result of the electrostatic potential of the proton from (83)

$$\begin{aligned} \chi(r) &= 1 + \frac{V(r)}{V_0} = 1 + \frac{e}{V_0 4\pi\varepsilon_0 a_0} \\ &= 0.99994689272 \end{aligned} \tag{86}$$

where a_0 is the Bohr radius and V_0 is given by (75). It is worth pointing out that the electron mass is known to extremely high accuracy through hydrogen spectroscopy. According to our theory, the electron mass inferred through this method is not the same as the value of the free electron mass, but is smaller by $0.0000531m_0$.

With regard to the field momentum we have a different situation. From (71), if the field momentum changes, there is no way to keep c^2 constant by changing V from its background value of V_0 or by changing the mass. Therefore, it seems the field momentum must change c^2. The best we can do is ensure that c^2 is the same amongst all fields with different charge-to-mass ratios. In order to understand how to go about this, we must develop a physical picture of how a charge in motion produces momentum in not only its own field but in other fields. We know that the speed of light must be the same in all fields. From (71) this means the magnitude of the quantity eA/m must be equal for all fields. Mathematically we require

$$\left|\frac{e_1A_1}{m_1}\right| = \left|\frac{e_2A_2}{m_2}\right| = \left|\frac{e_3A_3}{m_3}\right| = ... \left|\frac{e_nA_n}{m_n}\right|. \tag{87}$$

We will also suppose that each field contributes some fraction of the total momentum per unit charge. Thus, we also have the condition

$$A_1 + A_2 + A_3 + ...A_n = A_{total}. \tag{88}$$

As a simple example, we consider the field momentum due to a single charge Q moving with velocity v

$$\mathbf{A}_{total} = \frac{\mu_0}{4\pi r}Q\mathbf{v}. \tag{89}$$

From (88) we conclude that the vacuum permeability μ_0 must be composed of contributions from each field, such that

$$\mu_1 + \mu_2 + \mu_3 + ...\mu_n = \mu_0. \tag{90}$$

From (87) we also must have

$$\left|\frac{e_1}{m_1}\right|\mu_1 = \left|\frac{e_2}{m_2}\right|\mu_2 = \left|\frac{e_3}{m_3}\right|\mu_3 = ... \left|\frac{e_n}{m_n}\right|\mu_n. \tag{91}$$

For (91) to hold, we must then express the permeabilities for the various fields as

$$\mu_1 = \left|\frac{m_1}{e_1}\right|\beta, \mu_2 = \left|\frac{m_2}{e_2}\right|\beta, \mu_3 = \left|\frac{m_3}{e_3}\right|\beta, ...\mu_n = \left|\frac{m_n}{e_n}\right|\beta \tag{92}$$

where β is a constant we must determine. We can do this using (90) from which we obtain

$$\beta = \frac{\mu_0}{\left(\left|\frac{m_1}{e_1}\right| + \left|\frac{m_2}{e_2}\right| + \left|\frac{m_3}{e_3}\right| + ... \left|\frac{m_n}{e_n}\right|\right)}. \tag{93}$$

Summing the mass-to-charge ratios for all charged particles in the standard model including the fields of antiparticles yields $\beta \approx 1 \times 10^5\mu_0$. For the electron field we have $\mu_e = 5.85 \times 10^{-7}\mu_0$.

Using the equation for the speed of light in terms of the permittivity and permeability of free space

$$c^2 = \frac{1}{\mu_0 \varepsilon_0} \tag{94}$$

along with equation (93), we can obtain an expression for the total permittivity in terms of the permittivities of the individual fields

$$\frac{1}{\varepsilon_0} = \beta \left[\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + ... + \frac{1}{\varepsilon_n} \right] \tag{95}$$

where

$$\varepsilon_n = \left| \frac{e_n}{m_n c^2} \right| . \tag{96}$$

Note that the permeability is a true constant, whereas the permittivity varies in space in accordance with the spatially-variable speed of light.

4. Relativistic Covariance, Energy, and Causality

Thus far our analysis has not been particularly rigorous mathematically, and there are several details of the theory which we have not addressed. For instance, we have not expressed any of our equations in a manner that is generally covariant. Given the foundational principles of the theory, this should not be surprising. In our theory, the speed of light is determined by the gravitational potential which is derived from the superposition of gravitational wavefunctions as given by (12). In the mathematical language of general relativity, 2Φ in (12) is the g_{00} component of the metric tensor. If we allow the metric to transform like a tensor, it would mean that different inertial observers would measure different speeds of light, since the Lorentz transformation would change g_{00}. This contradicts relativity and is inconsistent with experiment. Therefore, the Schwarzschild metric we derived in (58) - (61) must be invariant instead of covariant. The question then becomes, how can all inertial observers agree on a method of calculating the same metric? The gravitational wavefunction is proportional to GM_i/r_i. The problem is that different inertial observers will measure different masses M_i and distances r_i, which will result in different calculated values for the speed of light in that frame. The only possible solution is to require all observers to agree on using the rest mass and distance as measured in the instantaneous rest frame of that particular mass when calculating its contribution to the gravitational wavefunction. In principle, there is nothing fundamentally wrong with this convention. It simply means that it will be impossible to write a generally covariant equation relating the distribution of mass to the metric. In this approach, we must be careful to distinguish between active and passive gravitational mass. Active gravitational mass refers to the mass which is used to calculate the gravitational wavefunction produced by that mass, which in this case is the rest mass. Passive gravitational mass refers to the mass which is acted upon by the gravitational field, which must be the same as the inertial mass by the equivalence principle. In our approach, the inertial and passive gravitational masses will vary depending on the inertial frame of the observer, whereas the active gravitational masses will not.

As the distance from each mass also appears in GM_i/r_i, this raises a deeper question about whether space or time is more fundamental. Because the gravitational wavefunction determines the speed of light, in a universe completely devoid of matter the speed of light would be zero. The statement that the speed of light is zero is equivalent to the statement that time does not exist. The universe would be completely devoid of energy and motion, with no means to judge the passage of time. This brings to light an important distinction between our theory and general relativity. In our theory where rest mass is the source of gravitation, energy becomes an effect of gravitation instead of its source. It would appear then that space is more fundamental than time, with time (and energy) simply being an emergent property of the universe due to the presence of matter. In order to judge the distances that appear in the gravitational wavefunction, our theory must be built upon a Euclidean space with no time, with the metric of spatial distance given by the wavelength of the gravitational wavefunction $\lambda = 2\pi/\beta$ which corresponds to the particle horizon of the universe.

Another question which arises is how to address the issue of causality. When computing the wavefunction does a given observer use the time-retarded value of GM_i/r_i which takes into account the time the signal took to arrive at the observer traveling at the speed of light, or the value of GM_i/r_i at the present instant in time? Here we run into a complication: in our theory it is the gravitational wavefunction itself which determines the speed of light, so what value of the speed of light do we use when calculating the retarded time for calculating GM_i/r_i? The only way to make the theory self-consistent is to use the value of GM_i/r_i at the present instant in time. In other words, the gravitational wavefunction is non-local and non-causal because the source of causality cannot itself be causal.

An objection that might be raised is that the simultaneous detection of gamma rays and gravitational waves from the merging pair of neutron stars GW170817 in 2017 is proof that gravitational forces are transmitted at the speed of light [18]. This is a subject where we must be precise with our translation of the mathematics of general relativity into language. The mathematics of general relativity does, in fact, predict that gravitational waves should travel at the speed of light. However, gravitational waves are in no way involved in the transmission of gravitational forces in the context of general relativity. Although gravitational waves are capable of exerting forces on matter, the everyday gravity that we experience is the result of the metric tensor itself, and is unrelated and distinct from propagating waves in the metric tensor. In fact, if we take the Einstein equation literally, it suggests that if matter is in motion, regardless if it is inertial motion or accelerated motion, the metric tensor infinitely far away from that matter changes instantaneously. Furthermore, the source of gravitational waves is distinct from the source of gravitational forces. Gravitational waves can only be created by second time derivatives of the quadrupole moment of a distribution of matter. In contrast, the source of gravitational forces is the energy-momentum tensor. A single isolated point particle is not capable of producing gravitational radiation, regardless of whether it is in an inertial or accelerated state of motion, according to general relativity. The use of general relativity with instantaneous propagation of the metric tensor is also fully consistent with celestial mechanics and astronomical observations. Students of celestial mechanics know that one must use the instantaneous gravitational potentials calculated based on where astronomical objects are now, instead of the time-retarded potentials, in calculations of the orbits of planets in our solar system. Use of the time-retarded potentials in calculations would result in orbital instabilities and the solar system could not exist in its present form. This should not be surprising, as there is also an element of non-causality in the Liénard-Wiechert potential of electromagnetism, where the potential moves instantaneously along with a charged particle even at infinite distances from the particle as long as the particle maintains inertial motion. Therefore, from a purely experimental standpoint, there is no evidence suggesting that gravitational forces are transmitted at anything other than instantaneous speeds. Finally, it is obvious that quantum mechanics is overtly non-causal and non-local. In this respect, the term "gravitational wavefunction" is perhaps appropriate, as it describes a property of matter which is also non-causal and non-local. The idea that the gravitational wavefunction is instantaneous would tend to support our second model in which we are gravitationally influenced by a universe that is infinite in size.

5. Consistency with General Relativity

Given that general relativity has been so successful in making predictions which have been confirmed experimentally, any theory of gravitation must reduce to general relativity in some limit. This evidence includes explaining the precession of the perihelion of Mercury, gravitational redshift, deflection of light by the sun, the detection of gravitational waves, and the measurement of frame dragging effects in Earth orbit by Gravity Probe B. We will now outline how our theory can be made compatible with the mathematics of general relativity.

The foundation of our theory is the gravitational wavefunction.

$$\phi(\mathbf{r}) = -\frac{2GM}{r}\left[\cos(\beta r) + i\sin(\beta r)\right] \tag{97}$$

which is derived from the inhomogeneous Helmholtz equation

$$\nabla^2\phi(\mathbf{r}) + \beta^2\phi(\mathbf{r}) = 8\pi G\rho_0. \tag{98}$$

It is important to reiterate the fact that the total wavefunction is calculated through the summation of wavefunctions of individual masses as determined in the instantaneous rest frames of those masses, and that it is only rest mass, not energy, that is the source of the wavefunction. This wavefunction is nonlocal and noncausal and does not propagate.

We may write the universal wavefunction as

$$\begin{aligned} \phi(\mathbf{r}) &= \sum_i \phi_i(\mathbf{r}) \\ &= \sum_i -\frac{2GM_i}{r_i}\left[\cos(\beta r_i) + i\sin(\beta r_i)\right] \\ &= -\left(\phi_R + i\phi_I\right) \end{aligned} \tag{99}$$

where we have defined

$$\phi_R = \sum_i \frac{2GM_i}{r_i}\cos\left(\beta r_i\right) \tag{100}$$

$$\phi_I = \sum_i \frac{2GM_i}{r_i}\sin\left(\beta r_i\right). \tag{101}$$

$$|\phi| = \sqrt{{\phi_R}^2 + {\phi_I}^2}. \tag{102}$$

We separate the metric into trace and trace free metrics

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{103}$$

where $\eta_{\mu\nu}$ is simply the Schwarzschild metric which we derived in Section 3 in our discussion relating gravitation to the zero-point field in the Dirac gauge, which we write in Cartesian coordinates because it simplifies the process of summing wavefunctions

$$\eta_{\mu\nu} = \begin{pmatrix} -|\phi|\dfrac{\left(1-\frac{\phi_R}{4|\phi|}\right)^2}{\left(1+\frac{\phi_R}{4|\phi|}\right)^2} & 0 & 0 & 0 \\ 0 & \left(1+\dfrac{\phi_R}{4|\phi|}\right)^4 & 0 & 0 \\ 0 & 0 & \left(1+\dfrac{\phi_R}{4|\phi|}\right)^4 & 0 \\ 0 & 0 & 0 & \left(1+\dfrac{\phi_R}{4|\phi|}\right)^4 \end{pmatrix}. \tag{104}$$

This metric has finite trace, and it is important to reiterate that it is a Lorentz-invariant quantity due to the fact that it is constructed from the gravitational wavefunction which all observers calculate in the same manner. The trace-free metric $h_{\mu\nu}$ is given by

$$h_{\mu\nu} = \begin{pmatrix} 0 & w_1 & w_2 & w_3 \\ w_1 & s_{11} & s_{12} & s_{13} \\ w_2 & s_{21} & s_{22} & s_{23} \\ w_3 & s_{31} & s_{32} & s_{33} \end{pmatrix}. \tag{105}$$

The elements w_i are equivalent to the components of the vector potential in electromagnetism and have their origin in the motion of massive bodies.

In special relativity as outlined by Einstein, there is no preferred frame with respect to which the motion of a massive body can be defined. It should be clear that our approach to explaining gravitation is fundamentally Machian. The behavior of gravitation locally is defined by the distribution of matter throughout the universe. Therefore it is natural that our approach to addressing the motion of bodies will differ from Einstein's, because that motion must in our view be defined with respect to the rest of the matter in the universe. It seems to be commonly held opinion in modern physics that the Michelson-Morley experiment disproved the existence of a preferred reference frame. However, relativity as originally developed by Lorentz and Poincaré was built upon a preferred frame, that of the "aether". The speed of light in this form of relativity was also the same in every inertial frame, and is not inconsistent with Michelson-Morley. It is straightforward to define a preferred "aether" frame in Lorentz-Poincare relativity. It is simply the frame in which the gravitational vector potential of the universe (the w_i in (105)) is zero, which is well-approximated by the frame in which the cosmic microwave background is isotropic. Generally speaking, this frame will vary in space depending on inhomogeneities in mass, the momentum carried by that mass, and the distance of each mass from the observer, but over astronomically small volumes of space this reference frame will essentially be a constant.

There are additional reasons to doubt the validity of Einstein's formulation of relativity. Take, for example, the twin paradox, in which an astronaut is sent away from Earth in a rocket at a significant fraction of the speed of light. According to measurements in particle accelerators [19], muon decay in the upper atmosphere [20], or measurements with atomic clocks aboard aircraft [21], we expect that the astronaut should return younger. Due to the symmetry inherent in Einstein's formulation, it should be obvious that there can be no resolution to this paradox, which is why there is still controversy surrounding the paradox more than 100 years later. Any argument or spacetime diagram that can be drawn from the perspective of the astronaut may also be drawn from the perspective of the twin on Earth. Therefore, there should be no difference between the clocks carried by the twins when the astronaut twin returns to Earth. There must be some asymmetry to explain this empirically measured effect, which is impossible using a theory which is inherently symmetric. The empirical experimental evidence for asymmetric time dilation should have been viewed as evidence *against* Einstein's interpretation of relativity, not in support of it.

It could perhaps be argued that the astronaut undergoes an acceleration which can be measured with an accelerometer, while the Earth twin does not. However, special relativity does not give any explanation, either from a mathematical or even philosophical point of view, for why this acceleration should break the symmetry of relativity. Furthermore, special relativity is incapable of providing an explanation for the inertial force which is responsible for breaking this symmetry, whereas an explanation of inertia follows naturally from the Machian perspective. As Sciama pointed out in his 1953 article "On the Origin of Inertia", an object which accelerates with respect to the zero-momentum-frame of the universe or "aether" frame experiences a time-varying gravitational vector potential which results in an effective gravitational field opposing its acceleration [22]. For these reasons, in what follows we will take the perspective that there is in fact a preferred frame upon which relativity must be founded which is the frame in which the gravitational vector potential of the universe is zero. In their 1977 paper "A Test Theory of Special Relativity: I. Simultaneity and Clock Synchronization," Mansouri and Sexl proved that if we eliminate Einstein's empirically unproven assumption that the one-way speed of light is isotropic, we may develop a suitable convention for clock synchronization which allows for absolute simultaneity in all inertial frames of reference [23]. This is mathematically equivalent to the statement that there is a preferred inertial frame, which as we previously stated is the zero-vector-potential frame of the universe. The one-way speed of light is isotropic only in this frame. Einstein's symmetric transformation laws for space and time then become asymmetric laws having the following form

$$\begin{aligned} t' &= \frac{1}{\gamma}T \\ x' &= \gamma\left(X - vT\right) \\ \gamma &= \frac{1}{\sqrt{1 - v^2/c^2}} \end{aligned} \tag{106}$$

where (X, T) is the position and time in the zero-vector-potential frame and (x', t') is the position and time in an inertial frame having velocity v with respect to it. Time dilation and length contraction in this case may be defined in an absolute sense with respect to the zero-vector-potential frame in a way that all observers independent of their motion may agree upon. The one-way speed of light as measured in the local inertial frame in motion with respect to the preferred frame is given by

$$u = \gamma(c + v) \tag{107}$$

in one direction and

$$u = \gamma(c - v) \tag{108}$$

in the opposite direction. Note that (107) implies that a superluminal one-way speed of light is possible.

Now that we have established the zero-vector-potential frame as a preferred frame, we may proceed with calculating the components of the gravitational vector potential w_i in (105). We start again with the gravitational wavefunction

$$\phi_i(\mathbf{r}) = -\frac{2GM_i}{r_i}\left[\cos(\beta r_i) + i\sin(\beta r_i)\right] \tag{109}$$

from which we derive a momentum wavefunction

$$\mathbf{p}_i(\mathbf{r}) = -\phi_i(\mathbf{r})(\mathbf{v} - \mathbf{v}_i) = \frac{2GM_i(\mathbf{v} - \mathbf{v}_i)}{r_i}\left[\cos(\beta r_i) + i\sin(\beta r_i)\right] \tag{110}$$

where v_i is the velocity of mass M_i with respect to the zero-vector-potential frame and v is the velocity of the observer with respect to the zero-vector-potential frame for whom we are calculating the metric. In analogy with the gravitational wavefunction we make the following definitions

$$\mathbf{p}_R = \sum_i \frac{2GM_i(\mathbf{v}-\mathbf{v}_i)}{r_i}\cos\left(\beta r_i\right) \tag{111}$$

$$\mathbf{p}_I = \sum_i \frac{2GM_i(\mathbf{v}-\mathbf{v}_i)}{r_i}\sin\left(\beta r_i\right) \tag{112}$$

In order obtain the w_i in (105) we must break down (111) and (112) into their vector components.

$$\begin{aligned} {p_R}^x &= \sum_i \frac{2GM_i(v^x - {v_i}^x)}{r_i}\cos\left(\beta r_i\right) \\ {p_R}^y &= \sum_i \frac{2GM_i(v^y - {v_i}^y)}{r_i}\cos\left(\beta r_i\right) \\ {p_R}^z &= \sum_i \frac{2GM_i(v^z - {v_i}^z)}{r_i}\cos\left(\beta r_i\right) \end{aligned} \tag{113}$$

$$\begin{aligned} {p_I}^x &= \sum_i \frac{2GM_i(v^x - {v_i}^x)}{r_i}\sin\left(\beta r_i\right) \\ {p_I}^y &= \sum_i \frac{2GM_i(v^y - {v_i}^y)}{r_i}\sin\left(\beta r_i\right) \\ {p_I}^z &= \sum_i \frac{2GM_i(v^z - {v_i}^z)}{r_i}\sin\left(\beta r_i\right) \end{aligned} \tag{114}$$

We also derive the magnitude of the momentum along each coordinate axis

$$\begin{aligned} |p^x| &= \sqrt{({p_R}^x)^2 + ({p_I}^x)^2} \\ |p^y| &= \sqrt{({p_R}^y)^2 + ({p_I}^y)^2} \\ |p^z| &= \sqrt{({p_R}^z)^2 + ({p_I}^z)^2} \end{aligned} \tag{115}$$

from which we may now derive the components of the metric w_i in (105)

$$\begin{aligned} w_x &= \frac{1}{|\phi|}\left(|p^x| + {p_R}^x\right) \\ w_y &= \frac{1}{|\phi|}\left(|p^y| + {p_R}^y\right) \\ w_z &= \frac{1}{|\phi|}\left(|p^z| + {p_R}^z\right). \end{aligned} \tag{116}$$

The components of the vector potential in (116) result in gravito-electro-magnetic forces similar to those of classical electrodynamics, in addition to the force of inertia. The real part $p_R^i / |\phi|$ is analogous to the vector potential in electrodynamics and the $\left|p^i\right| / |\phi|$ term reduces simply to the velocity of the test particle with respect to the zero-vector-potential frame for which the metric is being computed and results in the force of inertia.

This becomes apparent from the equation of motion derived from the geodesic equation

$$\begin{aligned} \frac{dp^i}{dt} &= -m\left[\partial_i g_{00} + \partial_0 w_i + 2\left(\partial_{[i}w_{j]} + \partial_0 g_{ij}\right)v^j + \left(\partial_{(j}g_{k)i} - \frac{1}{2}\partial_i g_{jk}\right)v^j v^k\right] \\ &= m\left[G^i + (\vec{v}\times H)^i - 2\left(\partial_0 g_{ij}\right)v^j - \left(\partial_{(j}g_{k)i} - \frac{1}{2}\partial_i g_{jk}\right)v^j v^k\right] \end{aligned} \tag{117}$$

where

$$\begin{aligned} G^i &= -\partial_i g_{00} - \partial_0 w_i \\ H^i &= (\nabla\times\vec{w})^i = \varepsilon^{ijk}\partial_j w_k. \end{aligned} \tag{118}$$

Finally, the s_{ij} components of the metric in (105) are related to gravitational waves which are generated by the quadrupole moment tensor of the source given by

$$I_{ij}(t) = \int x^i x^j \rho_0(\mathbf{x}, t) d^3x. \tag{119}$$

The s_{ij} are then given by

$$s_{ij}(\mathbf{x}, t) = \frac{2G}{r}\frac{d^2 I_{ij}}{dt^2}(t_r) \tag{120}$$

where $t_r = t - rc$ is the retarded time and $c = \eta_{00}$ [9]. The s_{ij} are the only propagating components of the metric.

6. Conclusion

We have presented a theory of gravitation based on a gravitational wavefunction. This wavefunction, in analogy to the wavefunction of quantum mechanics, is nonlocal and noncausal. We have shown how interference between wavefunctions created by different regions of the universe interfere to create the gravitational phenomena we observe, leading to an interpretation of gravity as an emergent property of the universe. This allows us to relate the speed of light to the size of the universe and the distribution of matter in the universe. We presented a cosmological model based on this theory which predicts from first principles the critical acceleration in MOND theory, and eliminates the need for dark energy to explain the accelerating expansion rate of the universe.

Next, we showed how the Dirac gauge leads to a view of the gravitational field as the result of harmonic oscillation of the underlying quantum fields of particles in the standard model. In other words, gravitation arises from and constrains the zero-point oscillations of the electromagnetic field. This interpretation gives us some clues as to how electromagnetic fields might be used to change the speed of light, and even create gravitational fields. From the equation

$$2\Phi(r) = c(r)^2 = -\frac{e^2A^2}{2m^2} + \sqrt{\left(\frac{e^2A^2}{2m^2}\right)^2 + \left(\frac{eV}{m}\right)^2} \tag{121}$$

we showed that the key to manipulating the speed of light and the gravitational potential using electromagnetic fields is the electromagnetic vector potential A. From an experimental standpoint, it is trivial to generate vector potentials using high-permeability toroids on the order of $A \sim 10^{-3} kg \cdot m/(C \cdot s)$. As we showed in (90) - (93) this corresponds to an effective field velocity of only $\sim 100m/s$. This would lead to a fractional change in the speed of light by a factor of only $\sqrt{1 - 100^2/(2.99792 \times 10^8)^2} \sim \sqrt{1 - 10^{-13}} \sim 1 - 5 \times 10^{-14}$. Experimentally measuring such small changes will obviously be a significant challenge, but may be within the realm of possibility especially using extremely strong magnetic fields available in research laboratories. Additionally, our theory predicts a change in particle masses in the vicinity of an electrostatic potential. For example, from (86) we showed that the electron mass in hydrogen should be smaller compared to its free space value by $0.0000531 m_0$. Finally, there may be other ways in which these equations may be used to derive new methods of producing thrust using electromagnetic fields and their connection to gravitational fields, which we will not outline here but may be a subject for future theoretical and experimental work.

Finally, we showed that our formulation of gravitation which is fundamentally Machian and requires a preferred frame of reference cannot be reconciled with Einstein's view of relativity. That preferred frame of reference is the inertial frame in which the gravitational vector potential of the universe is zero. While this viewpoint is likely to be controversial, it is clear that both general relativity and special relativity have shortcomings when it comes to explaining empirical observations. Special relativity, by nature a symmetric theory, cannot explain Nature's asymmetry when it comes to choosing one inertial frame which experiences time dilation and length contraction. General relativity, on the other hand, predicts that we should see far more black holes in our universe than we actually observe, and that when we look out into the surrounding universe we should see a speed of light that is nearly zero despite measuring a finite value of c locally. Einstein himself apparently believed that the background Minkowski metric had its origins in the rest of the matter of the universe and seemed to vacillate in his belief in a preferred inertial frame throughout his life, although it seems he never fully explored these ideas from a mathematical standpoint. We have taken the first steps of developing the mathematics of such a theory here.

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