Proof that the Legendre conjecture is unprovable

T.Nakashima

Abstract

In this paper, we attempt to prove the unprovability of the Legendre Conjecture, a known open problem. The method is similar to the author's "Guessing that the Riemann Hypothesis is unprovable using nonstandard analysis."

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Assume that the following proposition is unprovable.

proposition 1.
$$\pi(x) = li(x) + O(x^{\frac{1}{2}+\epsilon})$$

For more details, please refer to the author's paper [2]. This unprovability is mainly due to the existence of the following "distorted function". We will proceed with the discussion using nonstandard analysis by imitating the following method. For a certain prime number P, set $\pi(P)$ to $M >> 1$ instead of 1. Since the use of infinity is allowed in nonstandard analysis, we will set P to infinity and create a consistent negation model.

If
$$\pi(x) < li(x) + Kx^{\frac{1}{2}+\epsilon}$$
is true in the normal axiom system, and
$$\pi(x) > li(x) + Kx^{\frac{1}{2}+\epsilon}$$
is true by taking the "distorted function" in nonstandard analysis, then this proposition is unprovable. (It can be disproved.)

Theorem 1. *(Independence of the Legendre Conjecture) It is unprovable (independent proposition) that there is at least one prime number between* n^2 *and* $(n+1)^2$.

Consider the Legendre conjecture. Consider a number N and the number $N+1$. Then, in nonstandard analysis, let N be infinite, as nonstandard analysis allows. Then, consider an arbitrarily wide interval between the prime number after $N + 1$ and the prime number before N by counting N many times. This leads to a consistent model, providing a counterexample to the Legendre conjecture in nonstandard analysis.

If nothing is done. In other words, in normal mathematics, the Legendre Conjecture is "probably" correct. However, in a model based on nonstandard analysis, the Legendre Conjecture is false. In other words, the Legendre Conjecture is an unprovable proposition, that is, an independent proposition.

If the model in nonstandard analysis includes a positive proof of the Legendre Conjecture, which has common ground with normal mathematics, there is no way that a negative model can be created in nonstandard analysis. The negative proposition of the Legendre Conjecture is provable, but the positive proposition is unprovable since the negation can be created by one of the extensions.

Note: About Bertrand's postulate. It is known that there are prime numbers between N and $2N$. In this case, by counting the same number N many times, when increasing the interval, the method of taking an interval that hits N is nonsense because it is like counting N just N times, I consider the number 1 and interval N, (remember N and $N + 1$ is the main parts of the model,) $N + 1$ is too small number, and Bertrand's postulate can be proven without any problem.

Note 2: Regarding the transfer principle in nonstandard analysis. Infinity is included in the external set. The prime number theorem cannot coexist with the negation model of the Legendre conjecture, but since the transfer principle cannot be applied at infinity, the author's construction of the negation model of the Legendre conjecture at infinity can be constructed without any problems. ([1])

References

[1] "An Introduction to Nonstandard Real Anaiysis", A. E. Hard, P. A. Loeb. Pure and Applied Mathematics, A Series of Monographs and Textbooks, Columbia University, Ney York.

[2] "'Guessing that the Riemann Hypothesis is unprovable using Non-Standard Analysis" T. Nakashima, Vixra. org. Number theory [2503: 0188]