A SOLUTION TO WAVE PARTICLE DUALITY CONUNDRUM ?

Jayesh Suresh

Abstract

This article attempts to model spin of fermions and to explore possible connection to wave function. Zitterbewegung, originally introduced to describe the rapid oscillatory motion of relativistic particles, is not commonly used in modern physics. Here, zitterbewegung has been employed to develop a model that accounts for spin of fermions. It can explain wave particle duality

Key words- zitterbewegung, quantum, spin, fermion,relativistic

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In this article, an approach using zitterbewegung (ZB) as a model for quantum behavior has been explored. Originally predicted by Schrödinger for the Dirac electron [1], zitterbewegung describes a rapid oscillatory motion of relativistic particles, an idea largely neglected in modern quantum mechanics. Here, ZB in the context of a free fermion has been examined, modeling its motion using geometric considerations.

Relativistic Energy and Formulation of Equation for Zitterbewegung

The relativistic energy equation is given by:

$$E^2 = (pc)^2 + (m_0c^2)^2$$

p is the relativistic momentum. For a free relativistic particle undergoing zitterbewegung, we assume total energy incorporate both translational and rotational motion [2]:

$E^2= [1/2(p/c)v^2 + 1/2 (p/c)r^2\omega^2]^2+ (m_0c^2)^2$,where m_0c^2 is the constant rest energy. Momentum for a relativistic particle may be written as mc where m is the relativistic mass. Here the particle's velocity is taken as $v\approx c$,falls within the relativistic range.

So total energy may be represented as

$$E^2= [1/2\ (p/c)v^2 + 1/2\ (p/c)r^2\omega^2]^2+ (m_0c^2\ )^2.$$

Since for relativistic particle $v\approx c$, the translational and tangential velocities are in the relativistic range and $v=r\omega\approx c$. The rotational energy and the translational energy will be equal. Let x be the displacement of the particle along the x axis and r be the radius of its rotational motion in xy plane over time period t. Then, the translational velocity can be expressed as:

Therefore ,$v=r\omega= 2\pi r/t=x/t$

Substituting $v=r\omega$ into the equation gives:

$$E^2= [1/2\ (p/c)v^2 + 1/2\ (p/c)vr\omega]^2+ (m_0c^2\ )^2$$

where the term $\frac{1}{2}(p/c)vr\omega$ is introduced to simplify the equation for modelling of coupled translational and rotational motion. Expanding further,

$$E^2= [\tfrac{1}{2}\ (p/c)x^2/\ t^2+ \tfrac{1}{2}\ (p/c)2\pi rx/\ t^2]^2+(m_0c^2\ )^2$$

Which simplifies to:

$$E^2= [\tfrac{1}{2}(p/c)\ /t^2(x^2+ 2\pi rx]]^2+(m_0c^2\ )^2$$

Since E=constant. Differentiating with respect to x using the chain rule, dE/dt=(dE/dx).(dx/dt)=0. Therefore dE/dx=0 (dx/dt is a non zero term for a particle in motion).

Taking the derivative,

$$2E\,(dE/dx)= 2[\tfrac{1}{2}\,(p/c)/t^2(x^2+2\pi rx)]d/dx[\tfrac{1}{2}\,(p/c)/t2(x^2+2\pi rx)]+0$$

Which simplifies to

$$0=(1/2(p/c))\,t^2\,d/dx(x^2+2\pi rx)$$

Since $[\tfrac{1}{2}\,(p/c)/t^2(x^2+2\pi rx)]$ cannot be zero, the derivative simplifies to,

$$2x + 2\pi r + 2\pi x\,dr/dx= 0$$

$$xr+dr/dx=-x/\pi$$

Rearranging, $r=-x(1/\pi+dr/dx)$

Setting boundary conditions $x=0,\lambda;r=0$ and forced symmetry allows a trivial solution. The trivial solution can give three points on the XY plane, with which a curve can be traced.

On X-Y coordinates, x is the transverse displacement in X axis. It ranges from 0 to λ. r is the changing radius with respect to the displacement in Y axis. A graph can be plotted in Cartesian co-ordinates by taking Y=+/- r. The graph has been drawn using boundary condition

$x=0, \lambda; r=0$ and inflection point of graph at $x= \lambda/2$ where $dr/dx=0$. The amplitude of oscillation has to increase and decrease symmetrically if the particle has to travel as wave. A trivial solution is possible and it gives three points to draw the curve.

At inflection point of the slope, $dr/dx=0$

Where $r= R= \lambda /2(1/\pi+0)$, R is the radius of zitterbewegung

Therefore, $\lambda= 2\pi R$ and $R=\lambda/2\pi$ This is equal to the predicted radius for zitterwebegung.[3] .

The graph has to be drawn with $\lambda =\pi$ unit, R=1/2 unit. This is done for approximation to half Sine curve for modeling of fermion.

When $x=0$, $r= 0(1/\pi+dr/dx)=0$
Therefore when $x =\lambda$, $r =0$ and $dr/dx= -2R/\lambda=-1/\pi=-0.3$, which is the minimum slope of the graph.

The three points are (0,0), $(\lambda/2,R)$ and $(\lambda,0)$.The equation can be written as, $|r|= x(1/\pi+dr/dx)$.

Here $|r|= x/\pi+xdr/dx$. The graph can be drawn as the following.

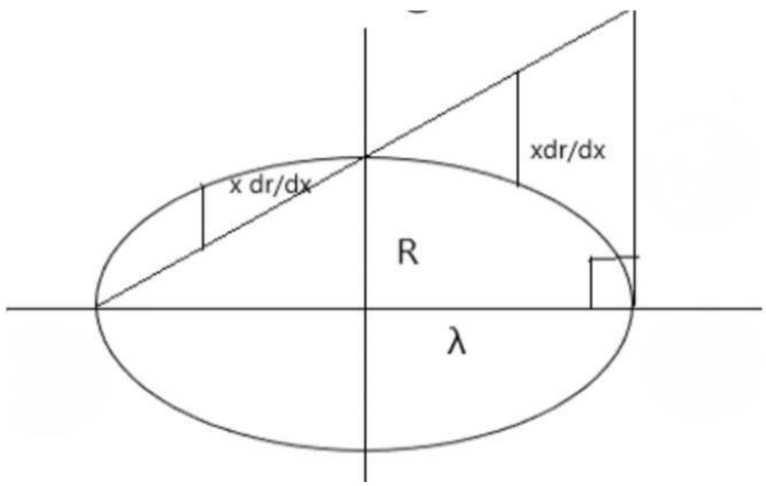


Fig 1. Outline formed by the particle in one cycle of zitterbewegung

By integrating the equation, $xr+dr/dx=-x/\pi$

$xr=-x2/2\pi+C$

Setting boundary conditions, $x=0,r=0$; $C=0$ is obtained

Therefore $x=-2\pi r$

The equation has multiple solutions,

if $x= \lambda$, $r=\lambda/2\pi$

If $x=\lambda/2$, $r= \lambda/4\pi$

If $x= \lambda/4$ $r= \lambda/8\pi$

Here r is the radius of oscillation. In the next sections, single rotation solution is only considered as ½ spin fermion is considered in the article.Considering the equation $\lambda = 2\pi R$, if both translational and rotational circumference to be equal, the rotational motion cannot be perfectly circular.

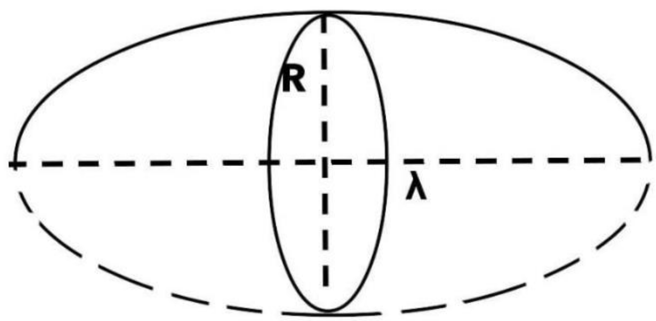


Fig 2. Translational and rotational motion in one cycle of zitterbewegung

Physical Implications:

The graph may be approximated to a standing wave pattern and can be represented as standing Sine waves travelling in opposite direction. If one wave length of standing wave is de Broglie wave length, then $\lambda^1 = h/p$ and the expected angular frequency of zitterbewegung,

$$\omega_{zb} = 2\omega = 2\ ck = 2c(2\pi/\lambda 1)$$

$\omega_{zb}=2pc/\hbar= 2mc^2/\hbar$ which aligns with existing predictions[4].

Here $m = \gamma m_0$, the relativistic mass of the particle.

The angular momentum of zitterbewegung of relativistic particle,

$L=pr= pR=p\ \lambda^1/4\pi=ph/4\pi p=\hbar/2$.This matches spin value for fermions and may explain the need for 720 degree rotation for getting back to initial configuration.

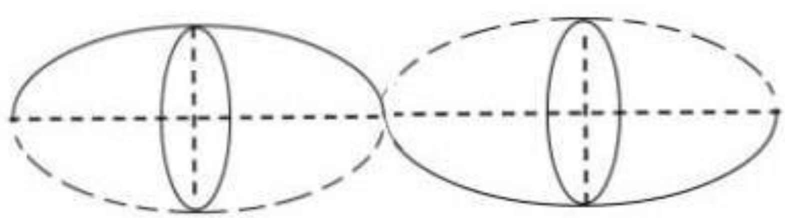


Fig 3. Zitterbewegung of fermion in one de Broglie wave length

The standing wave wave can be represented as

$y= Re^{ikx- \pi/2}+ Re^{-ikx-\pi/2}$

$y= R[Cos\ (kx-\pi/2)+ iSin(kx-\pi/2)]+ R[Cos\ (kx-\pi/2)- iSin(kx-\pi/2)]$

$y=2RCos(kx-\pi/2)$. Here, k is the wavenumber, which is equal to $k=2\pi/\lambda^1$ and phase difference of $-\pi/2$ as the original wave is a Sine wave.

Integration with quantum mechanics

$E^2 = [1/2\ (p/c)v^2 + 1/2\ (p/c)vr\omega]^2 + (m_0c^2\ )^2$

Since v=rw, the total energy can be rewritten as

$E^2 = [1/2\ (p/c)v^2 + 1/2\ (p/c)v^2]^2 + (m_0c^2\ )^2$

Which equals to

$E^2 = [(p/c)v^2]^2 + (m_0c^2\ )^2$ which

can be simplified to

$E^2 = [pc]^2 + (m_0c^2)^2$ as v=c.

The structure described above is two dimensional representation of a three-dimensional helical motion.

If P is the pitch of helix, $P\tan\theta = 2\pi R_1$, where R_1 is radius of helix and θ is the helix angle.

The equation is very similar to equation, $\lambda = 2\pi R$.

If helix angle is 45^o, then $\tan\theta = 1$ and $P = \lambda = 2\pi R_1$.

The helix diameter is the diameter of the cylinder on which the helix is wound and in the model, it is

$R_1 = R/\sqrt{2}R_1$

The length difference will be,

$k = 2\pi(R - R/\sqrt{2}) = 1.84\ R$

Therefore, 1.84 R can be extended to the pitch for our modeling.

The minimum slope in the two dimensional graph derived earlier was $-1/\pi$ at $x = \lambda$. If we do horizontal shear transformation of the graph in – X direction, slope becomes

$dy_1/dx_1 = dy/d(x-my)$, where m is the shear factor

The two dimensional graph is an approximation of half Sine wave ($\lambda = \pi$ unit, R=1/2 unit). So at $x=\pi$, slope is equal to -0.5.

So dy/dx=0.5.

dy/d(x-my) can be written as (dy/dx)/[d(x-my)/dx]

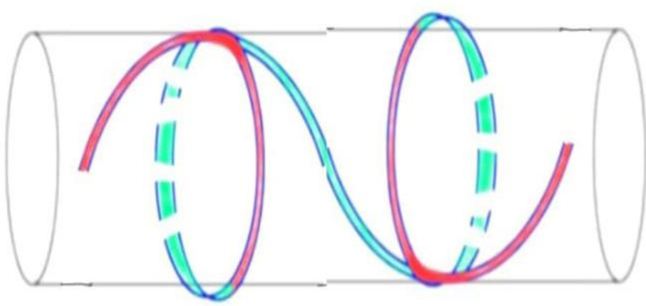
Therefore 0.318=0.5/[1-0.5m]

m=1.14

Shear angle $= \text{Cot}^{-1}(1.14) = 41.2^{o}$

This is less than the expected value 45^{o}. [8.4% error percentage].

There is a gap between two limbs of helix in XY plane. The corresponding angle in XY plane is responsible for the disparity. So a lesser shear angle is required to obtain a helix angle of 45^o.Therefore, 2D graph is proven to be a reasonable approximation of 3D helical structure. The picture may be drawn as below.



4. 720^o Rotation of Fermion around an imaginary cylinder

The three dimensional helix can explain spin chirality. In quantum mechanics, eigenspinors of x axis and y axis requires normalisation with 1/√2. This may be due to the angle created by helical motion,equals to 45^o , with the momentum vector of the particle in x and y axis ($\mathbf{p}_x$,$\mathbf{p}_y$). The translational motion may be in X axis and and helical motion will be in YZ plane so that eigenspinors make a 45 degree angle with $\mathbf{p}_y$ and donot make an angle with $\mathbf{p}_z$.

Since $\sin(45^{o})=1/\sqrt{2}$, the normalisation factor is equal for both x and y axes. So naturally 'p' represent three dimensional momentum. From the four momentum of Dirac particle, hamiltonian can be derived.

$H=c\alpha\cdot p+m_0c^2\beta$ [5]

Conclusion

This analysis demonstrates that the zitterbewegung trajectory can be approximated by a wave function, offering a more tangible physical interpretation of wave-particle duality than the traditional probabilistic approach. However, this formulation remains a mathematical approximation, requiring further rigorous analysis for a complete theoretical framework. This interpretation provides a foundational step for deeper investigations into quantum determinism.

References

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Conflict of Interest

The authors declare no conflicts of interest.

Ethical Approval

This article does not involve studies with human or animal participants.

Availabilty of data and material

There is no data and material available for the study