Classification of Composite numbers

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In this research different and distinct subsets of composite numbers are identified together with their Logical formulae. The aim of the research is to come up with an efficient method of sieving out primes from a set of positive integers.

Keywords Logical formula; set and subsets of composite numbers; Goldbach partition semiprimes

Justification for a proper classification of composite numbers

A proper classification of composite numbers helps in creating an effective sieve for separating prime numbers from composite numbers. Set-builder notation specifies a set as being the set of all elements that satisfy some logical formula, see reference [1]. In this research logical formulae will be identified for different subsets of the set of composite numbers.

Representing a composite number

$$N = p_i n_j \mid n_j = \prod p_j \mid p_j \ge p_i \tag{1}$$

Here n_j represents a composite number with prime factors greater than or equal to p_i . Thus the set of composite even numbers are represented as

$$N = p_1 n_j = 2n_j \mid n_j = \prod p_j \mid p_j \ge p_1 = 2$$
(2)

and has elements

$$(4, 6, 8, 10, 12, \dots, 2n_i)$$

. The set made using logical formula (3) below

$$N = p_2 n_j = 3n_j \mid n_j = \prod p_j \mid p_j \ge p_1 = 3$$
(3)

is made of the infinite set of elements

$$(9, 15, 21, 27, 33, 39, 51, 57, 63, 69, 75, 81, 87, 93...)$$

Goldbach partition semiprimes with prime factors of 3 are in this set. Semiprimes for Goldbach partion of even numbers 6 and above are found in this set. The element 3^n to the above set. The set based on logical formula (4) below

$$N = p_3 n_j = 5n_j \mid n_j = \prod p_j \mid p_j \ge p_3 = 5$$
(4)

is made of the infinite set of elements

$$(25, 35, 55, 65, 85, 95, 115....)$$

and contains Goldbach partition semiprimes with prime factor 5. Semiprimes for Goldbach partition of even numbers 10 and

greater are found in this set. The element 5^n belongs to the above set. In the above set based on logical formula (4) for example:

$$65(\frac{1}{5} + \frac{1}{13}) = 18$$
$$95(\frac{1}{5} + \frac{1}{19}) = 24$$

The set based on logical formula (5) below

$$N = p_4 n_j = 7n_j \mid n_j = \prod p_j \mid p_j \ge p_4 = 7$$
 (5)

is made of elements

and contains Goldbach partition semiprimes with prime factor of 7. Semiprimes for Goldbach partition of composite even numbers 14 and above are found in this set. The element 7^n belong to the above set. In the above set based on logical formula 7 for example:

$$203(\frac{1}{7} + \frac{1}{29}) = 36$$
$$91(\frac{1}{7} + \frac{1}{13}) = 20$$

References

[1]John F. Lucas (1990). Introduction to Abstract Mathematics. Rowman & Littlefield. p. 108. ISBN 978-0-912675-73-2.