Solution to Gravity Divergence, Gravity Renormalization, and Physical Origin of Planck-Scale Cut-off

Hyoyoung Choi * [1]

∗ *Independent Researcher, Department of Physics, Seoul, Republic of Korea.*

Abstract

By incorporating binding energy into an effective mass M_{eff}, we derive the RG flow and gravitational coupling $G(k)$. $G(k) = (1 - \frac{3G_N}{5R_m c^3}k)G_N = (1 - \frac{R_{gp}}{R_m})G_N$, where R_m is the radius of the mass or energy distribution and $R_{gp} = \frac{3}{5}\frac{GM_{fr}}{c^2}$ is the radius at which the negative gravitational self-energy (binding energy) balances the mass-energy. At $R_m = R_{gp}$, where $G(k) = 0$ and the gravitational coupling vanishes, $G(k)$ resolves gravitational divergences without quantum corrections and provides effective renormalization. This study proves that the QFT cut-off $\Lambda \sim M_P c^2$ serves as a physical boundary across all energy scales in quantum gravity. Quantum fluctuations ($\Delta E \sim M_P c^2$) with $\Delta t \sim t_P$ yield an energy distribution radius $R_m \sim l_P$, where negative gravitational self-energy balances mass-energy, yielding $E_T \approx 0$, thus eliminating divergences via $G(k) = 0$ and preventing negative energy states. In contrast, for proton or electron masses, $R_m \gg R_{gp}$, leading to $E_T \approx Mc^2$, rendering gravitational effects negligible and unsuitable for a cut-off. Thereby affirming the Planck scale's unique role in quantum gravity. If $R_m > R_{gp}$, $G(k) > 0$, yielding an attractive force. If $R_m = R_{gp}$, $G(k) = 0$, and the gravitational coupling vanishes. If $R_m < R_{gp}$, $G(k) < 0$, yielding a repulsive force or antigravity. This repulsive force prevents singularity formation in black holes. Therefore, the singularity problem is also solved.

1. Introduction

Gravity is basically given by the Einstein-Hilbert action, where G is Newton's constant and R is the scalar curvature derived from the Riemann curvature tensor.

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \tag{1}$$

However, it is known that several problems arise when trying to quantize this gravity theory.

1) The coupling constant has a dimension

Newton's constant G, which is the coupling constant of gravity, has the dimension of $mass^{-2}$. [1] On the other hand, the gauge theory covered in the Standard Model has a dimensionless coupling constant (scale-invariant), so it can control the flow in an appropriate way at high energies.

However, in the case of gravity, since the coupling constant has a dimension, the divergence is not controlled at high energies, and as it becomes more and more severe, it cannot be renormalized with only a finite number of terms.

2) Non-renormalizability

In two-loop and above, non-renormalizable divergence inevitably appears.

Therefore, it is known that quantum gravity based on general relativity is fundamentally not renormalized, and a new concept is needed. There are several methods to solve the divergence problem of gravity, but among them, there is a method called Asympotic Safety proposed by Weinberg. [2] [3] [4] The concept of Asympotic

[1] E-mail:7icarus7@gmail.com

Safety is a hypothesis that if the coupling constant of gravity converges to a specific fixed point at high energy, the theory can be maintained finitely. [4]

In this paper, I will present a solution to the renormalization problem of gravity using a method similar to this asympotic safety. To do so, I will look at the solution to the singularity problem inside a black hole, where the idea started, and then approach the renormalization problem of gravity.

2. Solution of the singularity problem of a black hole [2]

2.1. Mass defect effect due to gravitational binding energy or gravitational potential energy
When two masses m are separated by r, the total energy of the system is

$$E_T = 2mc^2 - \frac{Gmm}{r} \tag{2}$$

If we introduce the negative equivalent mass $-m_{gp}$ for the gravitational potential energy,

$$-\frac{Gmm}{r} = -m_{gp}c^2 \tag{3}$$

$$E_T = 2mc^2 - \frac{Gmm}{r} = 2mc^2 - m_{gp}c^2 = (2m - m_{gp})c^2 = m^*c^2 \tag{4}$$

The gravity of a composite particle composed of two objects acting on a mass m_3 that is relatively far away is

$$F = -\frac{Gm^*m_3}{R^2} = -\frac{G(2m - m_{gp})m_3}{R^2} = -\frac{G(2m)m_3}{R^2} - \frac{G(-m_{gp})m_3}{R^2} \tag{5}$$

That is, **when considering the gravitational action of a bound system, not only the mass in its free state but also the binding energy term ($-m_{gp}$) should be considered.** The total mass or equivalent mass m^* of the system is less than the mass of 2m when the two objects were in a free state. The bound objects experience a mass loss (defect) due to the gravitational binding energy. This is equivalent to having a negative equivalent mass in the system.

In general, the binding energy is very small compared to the mass energy. However, as the mass increases, the ratio of binding energy to mass energy increases. Therefore, in the case of a black hole, there arises a situation where this gravitational binding energy must be considered.

2.2. In a gravitationally bound system such as the Sun-Earth system, when the orbit changes, stable orbit and the change in total energy
In a gravitationally bound system like the Sun-Earth system, as the orbit changes, the gravitational potential energy must change, and with it the total energy.

1) Initial state (r_0)
The Sun and Earth are gravitationally bound at a distance r_0.
The total mechanical energy is $E_{me} = K_0 + U_0$
In this case, the equivalent mass is $M_{eff,0} = M_{free} - M_{binding,0}$

2) Orbital change ($r_1 < r_0$)
When Earth moves to a lower orbit r_1, the gravitational potential energy decreases ($U_1 < U_0$).
To stabilize the system, the excess energy must be radiated away. As a result, the total energy of the system decreases, and so does the effective mass. That is, $M_{eff,1} < M_{eff,0}$

[2] Chapter 2 is almost the same as the contents of the previous paper. [5] And, some research contents have been added. It is cited to understand the mass defect effect due to gravitational binding energy or gravitational potential energy, and the negative equivalent mass effect.

In the intermediate stage of the orbit change, according to the law of conservation of mechanical energy, the negative gravitational potential energy decreases, and the positive kinetic energy also increases, so the total energy is conserved.

However, in order for the system to become stable in a low orbit, the kinetic energy exceeding the kinetic energy required to move through the low orbit must be released outside the system. And, due to this energy release, the total mass or equivalent mass of the system decreases.

This process is also observed in the process of celestial bodies forming black holes through gravitational collapse. [6]

2.3. Gravitational self-energy or total gravitational potential energy of an object

The concept of gravitational self-energy is the total of gravitational potential energy (U_{gp}) possessed by a certain object M itself. Since a certain object M itself is a binding state of infinitesimal mass dMs, it involves the existence of gravitational potential energy among these dMs and is the value of adding up these. $M = \sum dM$. The gravitational self-energy is equal to the minus sign of the gravitational binding energy. Only the sign is different because it defines the gravitational binding energy as the energy that must be supplied to the system to bring the bound object into a free state.

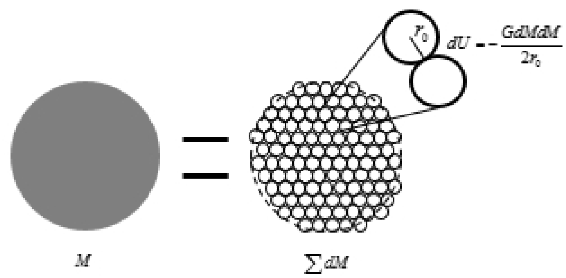

Figure 1: Since all mass M is a set of infinitesimal mass dMs and each dM is gravitational source, too, there exists gravitational potential energy among each of dMs. Generally, mass of an object measured from its outside corresponds to the value of dividing the total of all energy into c^2.

In the case of a spherical uniform distribution, total gravitational potential energy or gravitational binding energy ($-U_{gp}$) is

$$U_{gp} = -\frac{3}{5}\frac{G{M_{fr}}^2}{R} \tag{6}$$

$$U_{gp-Black-hole}(R = R_S) = -\frac{3}{5}\frac{G{M_{fr}}^2}{R} \approx -\frac{3}{5}\frac{G{M_{fr}}^2}{(\frac{2GM_{fr}}{c^2})} = -0.3M_{fr}c^2 \tag{7}$$

Strictly speaking, the mass M of a black hole is not the mass M_{fr} in the free state, but the equivalent mass (or effective mass) including the binding energy. Here, M_{fr} is used for simple estimation.

2.4. In the case of black hole, the gravitational potential energy or gravitational binding energy must be taken into account

In the general case, the value of gravitational potential energy is small enough to be negligible, compared to mass energy Mc^2. So generally, there was no need to consider gravitational potential energy. However the smaller R becomes, the higher the absolute value of U_{gp}. For this reason, we can see that U_{gp} is likely to offset the mass energy in a certain radius.

Thus, **looking for the size in which gravitational potential energy becomes equal to mass energy by comparing both,**

$$U_{gp} = |-\frac{3}{5}\frac{G{M_{fr}}^2}{R_{gp}}| = M_{fr}c^2 \tag{8}$$

$$R_{gp} = \frac{3}{5}\frac{GM_{fr}}{c^2} \tag{9}$$

This equation means that if mass M_{fr} is uniformly distributed within the radius R_{gp}, negative gravitational potential energy for such an object equals positive mass energy in size. So, in case of such an object, positive mass energy and negative gravitational potential energy can be completely offset while total energy is zero. Since total energy of such an object is 0, gravity exercised on another object outside is also 0.

Comparing R_{gp} with R_S, the radius of Schwarzschild black hole,

In the rough estimate above, since the gravitational potential energy at the event horizon is $U_{gp} = -0.3M_{fr}c^2$, the mass energy of the black hole will be approximately $E_{BH} = 0.7M_{fr}c^2$.

$$R_S = \frac{2GM}{c^2} \approx \frac{2G(\frac{7}{10}M_{fr})}{c^2} = \frac{7}{5}\frac{GM_{fr}}{c^2} \tag{10}$$

$$R_{gp} = \frac{3}{5}\frac{GM_{fr}}{c^2} = \frac{3}{7}(\frac{7GM_{fr}}{5c^2}) \approx \frac{3}{7}R_S \approx 0.43R_S \tag{11}$$

This means that there exists the point where negative gravitational potential energy becomes equal to positive mass energy within the radius of black hole, and that, supposing a uniform distribution, the value exists approximately at the point $0.43R_S$.

Even if we apply the kinetic energy and virial theorem, the radius only decreases as negative energy cancels out positive energy, but the core claim that "there is a region that cannot be compressed any further due to negative gravitational potential energy" remains unchanged. Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system.

Considering the virial theorem ($K = -U/2$),

$$R_{gp-vir} = \frac{1}{2}R_{gp} \tag{12}$$

2.5. There is no singularity at the center of a black hole

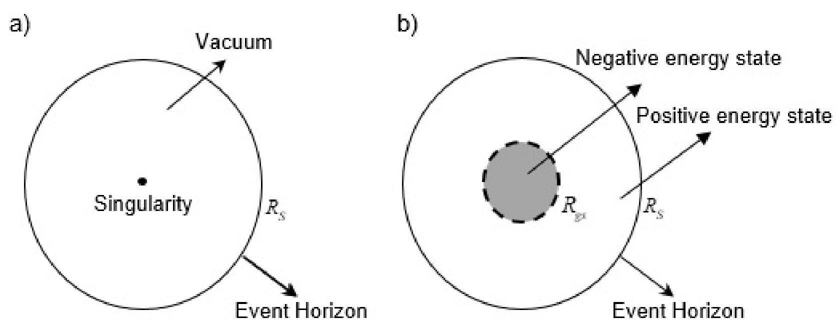

Figure 2: The internal structure of a black hole based on the radius of the mass (or energy) distribution. a) Existing Model. b) New Model : Internal state of a black hole according to the range of mass distribution. The area of within $R_{gp}(or R_{gp-vir})$ has gravitational potential energy (gravitational self-energy) of negative value, which is larger than mass energy of positive value. If r is less than $R_{gp}(or R_{gp-vir})$, this area becomes negative energy(mass) state. There is a repulsive gravitational effect between the negative masses, which causes it to expand again. This area (within $R_{gp}(or R_{gp-vir})$) exercises anti-gravity on all particles entering this area, and accordingly prevents all masses from gathering to $r = 0$. Therefore the distribution of mass (energy) can't be reduced to at least radius $R_{gp}(or R_{gp-vir})$.

The total energy of the system, including the gravitational potential energy or binding energy, is

$$E_T(R) = \sum_i m_i c^2 + \sum_{i<j} -\frac{Gm_i m_j}{r_{ij}} = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \tag{13}$$

Let's gradually reduce R from when R is infinite.

This is assuming that it is stationary after the orbital transition. If there is kinetic energy due to rotation in the orbit, we can reflect only half of the negative gravitational potential energy term by using the virial theorem. $K = -\frac{1}{2}U$

$$E_T(R=\infty) = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{R} = M_{fr}c^2 \tag{14}$$

$$E_T(R=R_S) = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{R} \approx M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{(\frac{2GM_{fr}}{c^2})} = M_{fr}c^2 - \frac{3}{10}M_{fr}c^2 = 0.7M_{fr}c^2 \tag{15}$$

$$E_T(R=R_{gp}) = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{R} = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{(\frac{3}{5}\frac{GM_{fr}}{c^2})} = M_{fr}c^2 - M_{fr}c^2 = 0 \tag{16}$$

$$E_T(R=\frac{1}{10}R_{gp}) = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{R} = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{(\frac{3}{50}\frac{GM_{fr}}{c^2})} = M_{fr}c^2 - 10M_{fr}c^2 = -9M_{fr}c^2 \tag{17}$$

From the equation above, even if some particle comes into the radius of black hole, it is not a fact that it contracts itself infinitely to the point $R=0$. From the point R_{gp}(or R_{gp-vir}), gravity is 0, and when it enters into the area of R_{gp}(or R_{gp-vir}), total energy within R_{gp}(or R_{gp-vir}) region corresponds to negative values enabling anti-gravity to exist. This R_{gp}(or R_{gp-vir}) region comes to exert repulsive effects of gravity on the particles outside of it, therefore it interrupting the formation of singularity at the near the area $R=0$.

However, it still can perform the function as black hole because the emitted energy will exist in a region larger than $r > R_{gp}(or R_{gp-vir})$. Since the emitted energy cannot escape the black hole, it is distributed in the region $R_{gp}(or R_{gp-vir}) < r < R_S$. Since the total energy of the entire range ($0 \leq r < R_S$) inside the black hole is positive, it functions as a black hole.

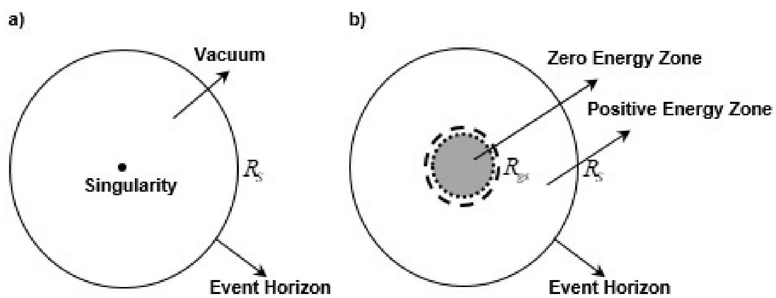

Figure 3: The internal structure of the black hole when the energy distribution inside the black hole is stabilized over time. a)Existing model b)New model. If, over time, the black hole stabilizes, the black hole does not have a singularity in the center, but it has a zero (total) energy zone. Since there is a repulsive gravitational effect between negative energies (masses), the mass distribution expands, and when the mass distribution expands, the magnitude of the negative gravitational potential energy decreases, so it enters the positive energy state again. When the system (mass distribution) becomes a positive energy state, gravitational contraction will exist again. In this way, gravitational contraction and expansion will be repeated until the total energy of the system becomes 0, and finally it will stabilize at a state where the total energy is 0. The maximum size of the Zero Energy Zone is R_{gp}.

If you have only the concept of positive energy, please refer to the following explanation.

The total energy of the system, including the gravitational potential energy, is

$$E_T = \sum_i m_i c^2 + \sum_{i<j} -\frac{Gm_i m_j}{r_{ij}} = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \tag{18}$$

If, $R = R_{gp}$

$$E_T(R = R_{gp}) = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{R} = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{(\frac{3}{5}\frac{GM_{fr}}{c^2})} = M_{fr}c^2 - M_{fr}c^2 = 0 \tag{19}$$

From the point of view of mass defect, $r = R_{gp}(orR_{gp-vir})$ is the point where the total energy of the system is zero. For the system to compress more than this point, there must be an positive energy release from the system. However, since the total energy of the system is zero, there is no positive energy that the system can release. Therefore, the system cannot be more compressed than $r = R_{gp}(orR_{gp-vir})$. So black hole doesn't have singularity.

2.6. The gravitational singularity can be solved by gravity, not by quantum mechanics

Think about a black hole with the size 10 billion times bigger than the solar mass. [7] [8] Schwarzschild radius of this black hole is $R_S = 3 \times 10^{10}km$ and R_{gp} of this black hole $1.29 \times 10^{10}km$. Average density of this black hole is about $0.18kg/m^3$. And average density of the Earth is about $5,200kg/m^3$. The average density of air is roughly $1.2kg/m^3$, which is lower than air.

Is it a size that requires quantum mechanics? Black hole of this size is Newtonian mechanics' object and therefore, gravitational potential energy must be considered.

Let's reduce the mass of this black hole gradually and approach three times the solar mass, the smallest size of black hole where stars can be formed!

In case of the smallest black hole with three times the solar mass, [9] $R_S = 9km$. R_{gp} of this object is as far as $3.87km$. In other words, **even in a black hole with smallest size that is made by the contraction of a star, the distribution of internal mass can't be reduced to at least radius 3.87km($R_{gs-vir} = 1.94km$).**

Before reaching quantum mechanical scales, the singularity problem is solved by gravity itself.

2.7. The minimal size of existence

[Existence = the sum of infinitesimal existences composing an existence]

A single mass M for some object means that it can be expressed as $M = \sum dM$ and, for energy, $E = \sum dE$. The same goes for elementary particles, which can be considered a set of dMs, the infinitesimal mass.

R_{gp} equation means that if masses are uniformly distributed within the radius R_{gp}, the size of negative binding energy becomes equal to that of mass energy. This can be the same that the rest mass, which used to be free for the mass defect effect caused by binding energy, has all disappeared. This means the total energy value representing "some existence" coming to 0 and "extinction of the existence". Therefore, R_{gp} is considered to act as "the minimal radius" or "a bottom line" of existence with some positive energy.

Gravitational self-energy can provide the concept of minimal length or minimal radius, one of the reasons for introducing string theory.

$$l_{\min} \approx R_{\min} \geq R_{gp} = \frac{3}{5}\frac{GM}{c^2} \tag{20}$$

The important point here is that the minimum length or minimum radius is proportional to the fundamental physical quantity of existence, mass M, or energy E. In other words, there is a limit to compressing large energy into a small space.

3. Extension of general relativity and new solution [3]

In all existing solutions, the mass term M must be replaced by $(M_{fr} - M_{gp})$

[3]Chapter 3 is almost the same as the contents of the previous paper. [5] It is cited to understand the mass defect effect due to gravitational binding energy or gravitational potential energy, and the negative equivalent mass effect.

Let's think about the simplest case, the solution of the Schwarzschild black hole. When we find the Schwarzschild solution, we find the solution by making the Schwarzschild metric consistent with Newtonian mechanics in the Newton limit.

However, in this comparison with Newtonian mechanics, we use the following relationship:

$$\Phi_N = -\frac{GM}{r} \tag{21}$$

$$1 - \frac{C}{r} \approx 1 + \frac{2GM}{c^2 r} \tag{22}$$

$$C = 1 - \frac{2GM}{c^2} \tag{23}$$

Here, we should have asked one question. Is the mass M the mass in the free state, M_{free}? Or is it the equivalent mass or the total mass M^* because it is bound?

The mass of an object or the Earth in Newtonian mechanics is the equivalent mass or the total mass M^*. That is,

$M = M_{free} + M_{binding-energy}$

In a weak gravitational field, $M_{binding-energy}$ can be ignored because it is very small compared to M_{free}. For example, in the case of the Earth, the gravitational binding energy is about 4.17×10^{-10} times the mass energy of the Earth. Therefore, in a weak gravitational field, $M \approx M_{free}$. However, in a strong gravitational field, the mass defect effect due to binding energy must be considered. In general relativity, the gravitational potential energy is a localized physical quantity included in the energy-momentum tensor, so it conforms to the classical Einstein field equation. It is also different from nonlocal pseudo-tensors such as the Landau-Lifshitz pseudo-tensor.

$M = M_{fr} - M_{gb} = M_{fr} - M_{gs} = M_{fr} - M_{gp}$

M_{fr}: Total mass when all components of the object are in a free state

M_{gb}: Equivalent mass of gravitational binding energy

$-M_{gs}$: Equivalent mass of gravitational self-energy

$-M_{gp}$: Equivalent mass of total gravitational potential energy of the object

In Newtonian mechanics, $-M_{gb}$, $-M_{gs}$, and $-M_{gp}$ have the same form and value in general situations. In addition, the energy of the gravitational field is also in the form of $U_{gf} = -k\frac{GM^2}{R}$, and depending on the integration interval, it can be the same as or different from the gravitational potential energy.

We can solve the problem of singularity by separating the term($-M_{gp}$) of gravitational potential energy (gravitational self-energy) from mass and including it in the solutions of field equation.

$M \rightarrow (M_{fr}) + (-M_{gp})$, In all existing solutions(Schwarzschild, Kerr, Reissner-Nordström, ...), the mass term M must be replaced by $(M_{fr} - M_{gp})$.

For example, Schwarzschild solution is,

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2 dt^2 + \frac{1}{(1 - \frac{2GM}{c^2 r})}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{24}$$

Schwarzschild-Choi solution is

$$ds^2 = -(1 - \frac{2G(M_{fr} - M_{gp})}{c^2 r})c^2 dt^2 + \frac{1}{(1 - \frac{2G(M_{fr} - M_{gp})}{c^2 r})}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{25}$$

In the case of a spherical uniform distribution,

$$-M_{gp} = -\frac{3}{5}\frac{G{M_{fr}}^2}{Rc^2} \tag{26}$$

1) If $M_{fr} \gg |-M_{gp}|$, in other words if $r \gg R_S$, we get the Schwarzschild solution.

2) If $M_{fr} = |-M_{gp}|$

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{27}$$

It has a flat space-time.
3) If $M_{fr} \ll |-M_{gp}|$, in other words if $0 \le r \ll R_{gp}$,

$$ds^2 \simeq -(1+\frac{2GM_{gp}}{c^2r})c^2dt^2 + \frac{1}{(1+\frac{2GM_{gp}}{c^2r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{28}$$

In the domain of $0 \le r \ll R_{gp}$,

The area of within R_{gp} has gravitational potential energy of negative value, which is larger than mass energy of positive value. Negative mass has gravitational effect which is repulsive to each other. [10] So, we can assume that $-M_{gp}$ is almost evenly distributed. Therefore $-\rho_{gp}$ is constant. And we must consider the Shell Theorem.

$$-M_{gp} = \frac{4\pi r^3}{3}(-\rho_{gp}) \tag{29}$$

$$(1+\frac{2GM_{gp}}{c^2r}) = 1 + \frac{2G(\frac{4\pi}{3}r^3\rho_{gp})}{c^2r} = 1 + \frac{8\pi G\rho_{gp}r^2}{3c^2} \tag{30}$$

$$ds^2 \simeq -(1+\frac{8\pi G\rho_{gp}r^2}{3c^2})c^2dt^2 + \frac{1}{(1+\frac{8\pi G\rho_{gp}r^2}{3c^2})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{31}$$

If $r \to 0$,

$$ds^2 \simeq -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{32}$$

There is no singularity.

4. Effective renormalization of gravity

4.1. Asymptotic Safety Method

Since Newton's constant G_N has a negative mass dimension ($[G_N] = -2$ in 4 dimensions), it is difficult to renormalize because high-order infinities appear during perturbation expansion. However, the Asymptotic Safety method is the idea that even in theories such as quantum gravity, which are difficult to renormalize using traditional perturbation methods, a theory that can be predicted at UV (ultra-high energy) can be constructed using a nonperturbative method. [2] [3] [4]

Generally, the RG (Renormalization Group) flow for coupling g_i is expressed as follows:
Beta function equation

$$\beta_i(g) = \frac{dg_i}{d\ln k} \tag{33}$$

The conventional beta function form of $G(k)$ in nonperturbative RG flow

$$\frac{dG(k)}{d\ln k} = \beta(G) = (d-2)G - cG^2 \tag{34}$$

d: spacetime dimension (usually d = 4 is assumed)
c: quantum correction factor, which varies depending on the details of the theory.
When solving the RG flow equation, the general solution of $G(k)$ is expressed as follows.

$$G(k) = \frac{G_0}{1 + cG_0\ln(k/k_0)} \tag{35}$$

G_0: Initial Newton constant (value at low energy, usually known as G_N)
k_0 : Initial energy scale

4.2. Find $G(k)$ or $M_{eff}(k)$

Usually, when applying RG flow, $G(k)$ is used as follows.

$$F = -\frac{G(k)Mm}{r^2} \tag{36}$$

$G(k)$ is a function that varies with distance or energy, and k basically means energy scale (or momentum scale). $k \sim p \sim \frac{E}{c}$

Existing researchers are having difficulties while focusing on $G(k)$, but let's think a little differently,

$$F = -\frac{G(k)Mm}{r^2} = -\frac{G_N(\frac{G(k)}{G_N}M)m}{r^2} = -\frac{G_N M_{eff} m}{r^2} \tag{37}$$

In other words, **instead of changing the gravitational coupling constant $G(k)$, we can change it to changing the mass M**. G_N is Newton's gravitational constant

$M \to M_{eff} = \frac{G(k)}{G_N} M$

Previously, when solving the singularity problem of black holes, we were able to know that the mass M changes by including binding energy or gravitational potential energy. This is a method that utilizes that.

For a simple calculation, assuming a spherical uniform distribution,

$$M_{eff} = M_{fr} - M_{gp} = M_{fr} - \frac{3}{5}\frac{G_N {M_{fr}}^2}{Rc^2} \tag{38}$$

$$M_{eff}(k) = (1 - \frac{3}{5}\frac{G_N M_{fr}}{Rc^2})M_{fr} = (1 - \frac{3}{5}\frac{G_N \frac{E}{c^2}}{Rc^2})M_{fr} = (1 - \frac{3G_N}{5Rc^3}k)M_{fr} \tag{39}$$

This can be reorganized and expressed in the form of $G(k)$. [4]

$$F = -\frac{G(k)Mm}{r^2} = -\frac{G_N M_{eff} m}{r^2} = -\frac{G_N(1 - \frac{3G_N}{5Rc^3}k)M_{fr} m}{r^2} = -(1 - \frac{3G_N}{5Rc^3}k)G_N \frac{M_{fr} m}{r^2} \tag{40}$$

$$G(k) = (1 - \frac{3G_N}{5Rc^3}k)G_N \tag{41}$$

If $B \equiv \frac{3G_N}{5Rc^3}$ is defined,

$$G(k) = (1 - \frac{3G_N}{5Rc^3}k)G_N = (1 - Bk)G_N \tag{42}$$

If, $k^* = \frac{1}{B} = \frac{5Rc^3}{3G_N}$ or $R = R_{gp}$, $G(k^*) = 0$

$G(k^*) = 0$ means that at that particular energy scale (for example, in the UV regime) the effective gravitational coupling vanishes. In other words, rather than diverging to infinity at high energies, the gravitational interaction actually disappears at that scale.

We want $\lim_{r \to 0} \frac{M_{eff}}{r^2} = 0$ to eliminate divergence. That is, M_{eff} must decrease faster than r^2.

In the previous analysis, $R_{gp} = \frac{3}{5}\frac{GM_{fr}}{c^2} \approx \frac{3}{7}R_S$

At $R_{gs} = \frac{3}{5}\frac{GM_{fr}}{c^2}$ before r reaches 0, M_{eff} goes to 0. Therefore, we can solve the gravitational divergence problem.

Also, in the low energy limit, $G(k) \to G_N$. And, in the M_{eff} equation, when $r \gg R_S$, we can see that it is consistent with the Newton equation.

[4] Here I am using the gravitational potential energy value from Newtonian mechanics. This may not be a completely accurate value. However, we use approximations in many fields. If you can find a better binding energy function or gravitational potential energy function, you can use that.

4.3. New beta function

$$G(k) = (1 - \frac{3G_N}{5Rc^3}k)G_N = (1 - Bk)G_N \tag{43}$$

Differentiating both sides with respect to $\ln k$:

$$\frac{dG(k)}{d\ln k} = \frac{d}{d\ln k}(1 - Bk)G_N = G_N(-B)\frac{dk}{d\ln k} \tag{44}$$

$$\frac{dk}{d\ln k} = k \tag{45}$$

$$\beta(G) = -BG_N k \tag{46}$$

At a specific $k^* = \frac{1}{B} = \frac{5Rc^3}{3G_N}$, $G(k^*) = 0$, the value of the beta function is

$$\beta(G)|_{k=\frac{1}{B}} = -BG_N k = -G_N \tag{47}$$

Therefore, the new $\beta(G)$ is, if we adjust the existing equation,

$$\beta(G) = (d-2)G(k) - cG(k)^2(1 - \frac{G(k)}{G_N}) - G_N \tag{48}$$

Looking at this equation, if $k = k^* = \frac{1}{B} = \frac{5Rc^3}{3G_N}$ or $R = R_{gp}$, $G(k) = 0$, and we get $\beta(0) = -G_N$, which is consistent with the previous result.

In the Asymptotic Safety method, when the energy goes to infinity ($k \to \infty$), we find a Non-Gaussian Fixed Point (NGFP) where the coupling constants have a specific finite value. However, in this model, G(k) does not simply converge to a finite value, but there is a point where $G(k) \to 0$ at a specific scale $R = R_{gp}$. This solves the divergence problem of gravity in a new way.

Also, when $k > \frac{1}{B} = \frac{5Rc^3}{3G_N}$ or $R < R_{gp}$, we get $G(k) < 0$, a repulsive force occurs. This repulsive force prevents gravitational collapse, so that a singularity is not formed at the center of the black hole.

And, in the existing model, a quantum correction term was added to produce the Non-Gaussian Fixed Point (NGFP) and repulsive effects. However, in this model, if $k > \frac{1}{B}$, antigravity is generated, solving the singularity problem. Therefore, there is no need to introduce a quantum correction term.

Therefore, If the quantum correction term is deleted, the beta function becomes a simpler form.

$$\beta(G) = (d-2)G(k) - G_N \tag{49}$$

To find a fixed point, if $d = 4$, $\beta(G) = 0$

$$\beta(G) = 2G(k) - G_N = 2(1 - \frac{R_{gp}}{R})G_N - G_N = (1 - \frac{2R_{gp}}{R})G_N = 0 \tag{50}$$

Fixed point is

$$R = 2R_{gp} = \frac{6}{5}\frac{GM_{fr}}{c^2}$$

4.4. Solving the problem of gravitational divergence at high energy

At low energy scales ($E \ll M_P c^2, \Delta t \gg t_P$), the divergence problem in gravity is addressed through effective field theory (EFT). [11] [12] However, at high energy scales ($E \sim M_P c^2, \Delta t \sim t_P$), EFT breaks down due to non-renormalizable divergences, leaving the divergence problem unresolved. [13]

Since the mass M is an equivalent mass including the binding energy, this study proposes the running coupling constant $G(k) = (1 - \frac{R_{gp}}{R})G_N$ that reflects the gravitational binding energy. **At the Planck scale ($R \sim R_{gp} = \frac{3G_N M_P}{5c^2} = \frac{3}{5}l_P$), $G(k) = 0$ eliminates divergences, and on higher energy scales than**

Planck's ($R < R_{gp}$), a repulsion occurs as $G(k) < 0$, solving the divergence problem in the entire energy range.

$$G(k) = (1 - \frac{3G_N}{5R_m c^3}k)G_N = (1 - \frac{3G_N M_{fr}}{5R_m c^2})G_N = (1 - \frac{R_{gp}}{R_m})G_N \tag{51}$$

R_m is the radius of the mass distribution or energy distribution. To avoid confusion with the rich scalar R or the usual R notation, the previously used R is changed to R_m.

R_{gp}: The radius where the gravitational potential energy (or binding energy) with a negative value is equal to the positive energy.

If $R_m > R_{gp}$, $G(k) > 0$, yielding an attractive force.
If $R_m = R_{gp}$, $G(k) = 0$, the gravitational coupling vanishes. Gravity is also zero.
If $R_m < R_{gp}$, $G(k) < 0$, yielding a repulsive force or antigravity.

This repulsive force prevents gravitational collapse and prevents the formation of a singularity at the center of the black hole. Since the point where $R_m < R_{gp}$ exists inside the event horizon of the black hole, it solves the singularity problem without colliding with observations.

4.4.1. At Planck scale

If, $M \sim M_P = \sqrt{\frac{\hbar c}{G_N}}$

$$R_{gp} = \frac{3}{5}\frac{G_N}{c^2}\sqrt{\frac{\hbar c}{G_N}} = \frac{3}{5}\sqrt{\frac{\hbar G_N}{c^3}} = \frac{3}{5}l_P \tag{52}$$

This means that R_{gp}, where $G(k) = 0$, i.e. gravity is zero, is the same size as the Planck scale.
At $R_m = R_{gp}$,
$G(k) = 0 \Rightarrow \Pi^{div} \sim \frac{G(k)}{\varepsilon}R^2 = 0$
This means that divergence is eliminated at the Planck scale.

4.4.2. At high energy scales larger than the Planck scale

If $R_m < R_{gp}$ or $k > \frac{5Rc^3}{3G_N}$ (That is, roughly $E > M_P c^2$)

$$G(k) = (1 - \frac{R_{gp}}{R_m})G_N < 0 \tag{53}$$

At high energies beyond the Planck scale, the gravitational coupling constant becomes negative, causing a repulsive (antigravity) effect. When $R_m < R_{gp}$, the region becomes a negative mass state, and since there is a repulsive gravitational effect between negative masses, it has a uniform distribution characteristic. This uniform density avoids singularity and keeps the curvature terms finite, thus solving the problem of divergence at high energy scales.

If antigravity effect exists, the mass (or energy) distribution expands, and antigravity exists until $R_m = R_{gp}$. Since $R_m = R_{gp}$ is the point where $G(k)$ becomes 0, the gravitational divergence problem is solved even at high energies above the Planck scale.

Therefore, the gravitational divergence problem is solved at all energy scales.

Einstein-Hilbert action is

$$S = \int dx^4 \frac{\sqrt{-g}}{16\pi G(k)}R \tag{54}$$

$$S = \int dx^4 \frac{\sqrt{-g}}{16\pi(1 - \frac{R_{gp}}{R_m})}R \tag{55}$$

In the case of a spherical uniform distribution, $R_{gp} = \frac{3}{5}\frac{GM_{fr}}{c^2} \approx \frac{3}{7}R_S$. The point where $G(k) = 0$ suggests an inflection point where the force changes from attractive to repulsive.

4.5. The physical origin of the cut-off energy at the Planck scale

In quantum field theory (QFT), the cut-off energy Λ or cut-off momentum is introduced to address the infinite divergence problem inherent in loop integrals, a cornerstone of the renormalization process [14]. However, this cut-off has traditionally been viewed as a mathematical convenience, with its physical origin or justification remaining poorly understood [15].

This work proposes that Λ represents a physical boundary determined by the scale where the sum of positive mass-energy and negative gravitational self-energy equals zero, preventing negative energy states at the Planck scale. This mechanism, rooted in the negative gravitational self-energy of positive mass or energy, provides a physical explanation for the Planck-scale cut-off.

4.5.1. $G(k) = 0$ and Planck scale

The running coupling constant $G(k)$, where $k \sim p \sim E/c$ denotes the momentum scale, is defined as:

$$G(k) = (1 - \frac{R_{gp}}{R_m})G_N \tag{56}$$

$$R_{gp} = \frac{3GM_{fr}}{5c^2} \tag{57}$$

For a mass $M \sim M_P = \sqrt{\frac{\hbar c}{G_N}}$, the characteristic radius is:

$$R_{gp} = \frac{3}{5}\frac{G_N}{c^2}\sqrt{\frac{\hbar c}{G_N}} = \frac{3}{5}\sqrt{\frac{\hbar G_N}{c^3}} = \frac{3}{5}l_P \tag{58}$$

At $R_m = R_{gp}$, $G(k) = 0$, marking the Planck scale where divergences vanish.

If $R_m < R_{gp}$, then $G(k) < 0$, which means that the system is in a negative mass state. Therefore, the Planck scale acts as a boundary energy where an object is converted to a negative energy state by the gravitational self-energy of the object. Although the system can temporarily enter a negative mass state, the mass distribution expands again because there is a repulsive gravitational effect between the negative masses. Thus, the Planck scale (l_P) serves as a boundary preventing negative energy states driven by gravitational self-energy.

4.5.2. Uncertainty principle and total energy with gravitational self-energy

To elucidate the interplay between quantum fluctuations and gravitational effects, we apply the energy-time uncertainty principle ($\Delta E \Delta t \geq \frac{\hbar}{2}$) to the total energy of a system, incorporating gravitational self-energy.

The energy-time uncertainty principle provides:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{59}$$

At the Planck time, $\Delta t = t_P$, energy fluctuation is:

$$\Delta E \geq \frac{\hbar}{2t_P} = \frac{1}{2}m_P c^2 \tag{60}$$

During Planck time, let's suppose that quantum fluctuations of $\frac{5}{6}m_P$ mass have occurred.

Since all mass or energy is combinations of infinitesimal masses or energies, positive mass or positive energy has a negative gravitational self-energy.

The total energy of the system, including the gravitational self-energy, is

$$E_T = \sum_i m_i c^2 + \sum_{i<j} -\frac{Gm_i m_j}{r_{ij}} = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \tag{61}$$

Here, the factor $\frac{3}{5}$ arises from the gravitational self-energy of a uniform mass distribution. Substituting $\frac{5}{6}m_P$ and $R = \frac{ct_P}{2}$ (where $c\Delta t$ represents the diameter of the energy distribution, constrained by the speed of light (or the speed of gravitational transfer). Thus, $\Delta x = 2R = c\Delta t$.

$$E_T = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \simeq \frac{5}{6}m_P c^2 - \frac{3}{5}\frac{G(\frac{5}{6}m_P)^2}{\frac{ct_P}{2}} = \frac{5}{6}m_P c^2 - \frac{5}{6}m_P c^2 = 0 \tag{62}$$

This demonstrates that at the Planck scale, the negative gravitational self-energy balances the positive mass-energy, defining a cut-off energy $\Lambda \sim m_P c^2$. For energies $E > \Lambda$, the system enters a negative energy state ($E_T < 0$), which is generally prohibited due to the repulsive gravitational effects of negative mass states.

Quantum Fluctuations at Different Mass Scales

We evaluate Δt, R, and E_T for three representative masses: the Planck mass ($M_P = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-8} kg$), the proton mass ($M_{proton} \approx 1.67 \times 10^{-27} kg$), and the electron mass ($M_{electron} \approx 9.10 \times 10^{-31} kg$).

1) Planck Mass

$$R = \frac{c\Delta t}{2} \geq \frac{c}{2}\frac{\hbar}{2\Delta E} = \frac{1}{4}\frac{c\hbar}{M_P c^2} = \frac{1}{4} l_P \tag{63}$$

$$E_T \approx M_P c^2 - \frac{3}{5}\frac{GM_P^2}{R} = M_P c^2 - \frac{3}{5}\frac{GM_P^2}{\frac{1}{4} l_P} \approx -1.4 M_P c^2 \tag{64}$$

This negative E_T indicates that $R(=\frac{1}{4} l_P) < R_{gp}(=\frac{3}{5} l_P)$, where $R_{gp} = \frac{3}{5}\frac{GM}{c^2} \approx 9.698 \times 10^{-36} m \sim \frac{3}{5} l_p$ is the critical radius at which $E_T = 0$. Increasing $\Delta t \sim t_p$, $R \to R_{gp}$, and $E_T \to 0$, suggesting that the Planck scale is where gravitational self-energy can balance the mass-energy, supporting a physical cut-off at $\Lambda \sim M_P c^2$.

2) Proton Mass

For $M = M_{proton}$, $\Delta E \approx 938 MeV$, and $\Delta t \approx 3.506 \times 10^{-25} s \sim 10^{19} t_p$, $R \approx 5.254 \times 10^{-17} m \sim 10^{18} l_p$

The total energy is

$$E_T \approx M_{proton} c^2 - \frac{3}{5}\frac{GM_{proton}^2}{R} \approx M_{proton} c^2 \tag{65}$$

Here, $R \gg R_{gp} \approx 7.450 \times 10^{-55} m$, and the gravitational self-energy ($\sim 10^{-48} J$) is negligible compared to $M_{proton} c^2 \approx 1.504 \times 10^{-10} J$.

3) Electron Mass

For $M = M_{electron}$, $\Delta E \approx 0.511 MeV$, and $\Delta t \approx 6.439 \times 10^{-22} s \sim 10^{22} t_p$, $R \approx 9.652 \times 10^{-14} m \sim 10^{21} l_p$

$$E_T \approx M_{electron} c^2 - \frac{3}{5}\frac{GM_{electron}^2}{R} \approx M_{electron} c^2 \tag{66}$$

Here, $R \gg R_{gp} \approx 4.058 \times 10^{-58} m$, and the gravitational self-energy ($\sim 10^{-58} J$)) is negligible compared to $M_{electron} c^2 \approx 8.187 \times 10^{-14} J$.

Physical Implications

The Planck scale is unique: only for $M \sim M_P$, $\Delta t \sim t_p$, and $R \sim l_p$ does the gravitational self-energy ($U_{gs} = U_{gp} = -\frac{3}{5}\frac{GM^2}{R}$) approach the mass-energy, enabling $E_T \approx 0$. This balance, absent in non-gravitational theories like QED or ϕ^4 (where $U_{elec-self-energy} > 0$), suggests that the QFT cut-off $\Lambda \sim M_P c^2$ is a physical boundary where quantum and gravitational effects converge. For proton or electron masses, $R \gg R_{gp}$, and gravitational effects are negligible, aligning with QED/QCD cut-offs ($\Lambda \sim GeV$).

4.5.3. Generalization and Exceptions

This mechanism applies to systems dominated by negative gravitational self-energy, such as gravitational effective field theories [15] or quantum gravity scenarios. In non-gravitational theories like QED or ϕ^4, positive binding energies (e.g., electrostatic self-energy, $U_{es} > 0$) yield cut-offs unrelated to the Planck scale. A Planck scale cut-off appears only when quantum gravitational effects are included.

While negative energy states are generally avoided, exceptions exist. The observable universe is estimated to have a negative total energy, likely due to mechanisms such as cosmic inflation or dark energy [16] [17]. However, this negative energy state is estimated to have been achieved by a slightly different mechanism. [16]

4.5.4. In gravitational problems, the physical meaning of cut-off energy

The cut-off energy $\Lambda \sim M_P c^2$ is not a mathematical artifact but a physical boundary driven by the balance of positive mass-energy (or positive energy) and negative gravitational self-energy. This mechanism provides a novel explanation for the Planck scale as the natural cut-off in gravitational systems, resolving the long-standing issue of the physical origin of QFT cut-offs.

5. Conclusion

In the case of a combined object, we must consider the binding energy. Thus, it is possible that the mass M of the combined object varies with the binding energy. By using the fact that the mass M changes to M_{eff}, which reflects the binding energy, we can obtain the RG flow and gravitational coupling constant $G(k)$.

When $k^* = \frac{5Rc^3}{3G_N}$, $G(k^*) = 0$, which means that gravity is zero. Therefore, we can solve the problem of gravitational divergence at high energy. In addition, when $R < R_{gp}$, $G(k) < 0$, so antigravity occurs, and it prevents substances from gravitational collapse and forming a singularity. Therefore, the singularity problem is also solved.

[**The important problems in physics and cosmology related to gravity**]
1) Black hole singularity problem
2) Dark energy problem
3) Problem with the cause and mechanism of inflation
4) Gravitational divergence problem and gravity renormalization problem

The mainstream recognizes all four problems as different problems, and therefore presents as-hoc hypotheses for each of them. But these four problems may actually be different aspects of one problem. That is, problems that can be explained by the existence of repulsion or antigravity in the gravitational problem.

The singularity problem, inflation problem, divergence problem, and dark energy seem to be on different scales, right? So it seems like multiple sources are needed?

The only thing we need is a mechanism that creates repulsion or anti-gravity in the problem of gravity.

For a simple analysis, let's assume a spherical uniform distribution, and look at the gravitational potential energy or gravitational self-energy.

$$U_{gp} = -\frac{3}{5}\frac{GM^2}{R} \tag{67}$$

The total energy, including the gravitational potential energy or gravitational self-energy, is

$$E_T = \sum_i m_i c^2 + \sum_{i<j} -\frac{Gm_i m_j}{r_{ij}} = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \tag{68}$$

When the energy distribution radius R is very small and the mass M is large, the negative gravitational potential energy term can be larger than the positive mass energy. This applies to the singularity problem [5], inflation problem [17] [18], and divergence problem.

The negative gravitational potential energy term can be larger than the positive mass energy when M is very large. It applies to the dark energy problem, which accelerates the expansion of the universe. [16]

References

[1] Shomer, A. A Pedagogical Explanation for the Non-Renormalizability of Gravity. (2007). [arXiv:0709.3555].

[2] Weinberg, S. Ultraviolet Divergences in Quantum Theories of Gravitation. In General Relativity: An Einstein Centenary Survey (pp. 790-831). (1979).

[3] Reuter, M. Nonperturbative Evolution Equation for Quantum Gravity. Physical Review D, 57(2), 971. (1998).

[4] Percacci, R. Asymptotic Safety. (2007). [arXiv:0709.3851].

[5] Hyoyoung, Choi, Solution of the Singularity Problem of Black Hole, (2016). [https://www.researchgate.net/publication/313314666].

[6] Roger Penrose, Gravitational Collapse and Space-Time Singularities, Phys. Rev. Lett. 14, 57. (1965)

[7] Valtonen, M. J., Lehto, H.J., nilsson, K. et al., A massive binary black-hole system in OJ 287 and a test of general relativity, Nature, **452**, 851 (2008).

[8] N. McConnell, C.-P. Ma, K. Gebhardt et al., Two ten-billion-solar-mass black holes at the centres of giant elliptical galaxies, Nature, **480**, 215 (2011).

[9] Oppenheimer, J. R. and Volkoff, G. M., On massive neutron cores, Phys. Rev. **55** (4) 374 (1939).

[10] H. Bondi, Negative Mass in General Relativity, Rev. Mod. Phys. **29** 423 (1957).

[11] J. F. Donoghue, General relativity as an effective field theory: The leading quantum corrections, Gen. Relativ. Gravit. 26, 1913 (1994).

[12] C. P. Burgess, Quantum gravity in everyday life: General relativity as an effective field theory, Living Rev. Relativ. 7, 5 (2004).

[13] M. H. Goroff and A. Sagnotti, The ultraviolet behavior of Einstein gravity, Nucl. Phys. B 266, 709 (1986).

[14] K. G. Wilson, The renormalization group: Critical phenomena and the Kondo problem, Rev. Mod. Phys. 47, 773 (1975).

[15] J. F. Donoghue, General relativity as an effective field theory: The leading quantum corrections, Gen. Relativ. Gravit. 26, 1913 (1994).

[16] Hyoyoung, Choi, Dark Energy is Gravitational Potential Energy or Energy of the Gravitational Field, (2022). [https://www.researchgate.net/publication/360096238].

[17] Hyoyoung, The Birth Mechanism of the Universe from Nothing and New Inflation Mechanism, (2023). [https://www.researchgate.net/publication/371951438].

[18] Hyoyoung, Choi, Problems and Solutions of Black Hole Cosmology, (2022). [https://www.researchgate.net/publication/359192496].