The Time Symmetry in Special Relativity Is Broken?

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Abstract

This paper presents a fundamental revision of the theoretical framework of special relativity by abandoning both the Lorentz transformations and the concept of time dilation. By critically analyzing symmetry issues in relativistic effects, this study argues that time is a universally invariant quantity, remaining unaffected by the relative motion between reference frames. By re-evaluating the core principles of relativity, the study rederives the mass-energy equivalence formula without invoking time dilation or Lorentz transformations. This novel approach resolves long-standing paradoxes in relativity, such as the twin paradox, and opens new possibilities for theoretical and experimental research in fundamental physics.

Introduction

Since its inception in 1905, Einstein's special relativity has served as a cornerstone of modern physics. At its core lies the Lorentz transformations, which describe the relationship between spatial and temporal measurements across different inertial reference frames. These transformations led to key predictions, such as time dilation, length contraction, and the invariance of the speed of light, all of which have been extensively validated by experiments.

Upon closer examination, significant issues with the traditional framework become evident. The concept of time dilation, which implies that time passes at different rates in different reference frames, conflicts with the inherent symmetry of relativistic effects and leads to paradoxes, such as the twin paradox. Such inconsistencies indicate that time dilation might not represent a fundamental aspect of nature.

This paper proposes a radical departure from traditional special relativity by abandoning time dilation and the Lorentz transformations altogether. Instead, a new framework is introduced in which time remains an invariant quantity across all reference frames, and spacetime transformations are redefined while preserving the constancy of the speed of light. Using this refined approach, this paper derives the mass-energy equivalence formula by indirectly introducing time dilation, providing a more consistent explanation of relativistic phenomena. The proposed model not only resolves conceptual paradoxes but also lays a solid foundation for future research into the dynamics of spacetime and the fundamental nature of time and energy.

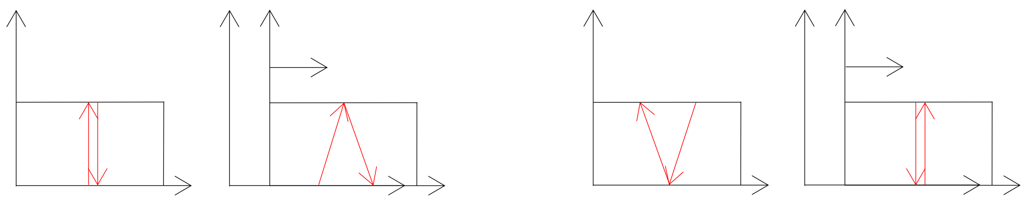
The Problems of Time Dilation

The theory of relativity, introduced by Albert Einstein in 1905, has been a cornerstone of modern physics. Central to this theory are the Lorentz transformations, which mathematically describe how measurements of time and space change for observers in different inertial frames of reference. These transformations have led to the well-known phenomena of time dilation, both of which have been extensively studied and experimentally validated.

However, the formulas of time dilation and derived from the original theory of relativity are not satisfactory to some extent. Here are two illustrative examples are provided to highlight these aspects.

Consider a train traveling at relativistic speed, with a carriage of height h and a mirror mounted on its ceiling. Consider a scenario where a beam of light is emitted vertically within the carriage along the mirror, and the time it takes for the light to reach the floor of the train after the first reflection is denoted as $\Delta\tau$. Now with the subgrade as the reference frame (K' reference frame), the path of the train is Δx, and the time it takes for the light to reach the floor of the train is denoted as Δt. Since the speed of light does not change, the Pythagorean theorem leads to $\Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}}$, where $\beta = v/c$. This is how Einstein derived his formula for the time dilation. He interpreted the phenomenon that the path of light relative to the roadbed becomes longer than that of light relative to the car to mean that the time passing by the roadbed is not the same as the time passing by the train, and the time relationship is the above expression.

A Train Story



Lights on Trains: Simultaneity is Relative?

Alternatively, if the path of light is considered vertical relative to the roadbed, then the path of light is longer relative to the train. When we use the Pythagorean theorem to link this relationship, we get a different expression from the original formula for the time dilation. The derivation process is as follows:

The height of the train carriage can be represented as

$$h = \frac{c\Delta t}{2}.$$

In the K reference frame, the path of the photon can be represented as

$$L = c\Delta\tau\,.$$

The distance traveled by the train can be represented as

$$\Delta x = v\Delta t\,.$$

According to the Pythagorean theorem:

$$h^2 + \left(\frac{\Delta x}{2}\right)^2 = \left(\frac{L}{2}\right)^2$$

Expanding:

$$\left(\frac{c\Delta t}{2}\right)^2 + \left(\frac{v\Delta t}{2}\right)^2 = \left(\frac{c\Delta \tau}{2}\right)^2$$

$$\Delta t^2(c^2 + v^2) = c^2\Delta\tau^2$$

Divide both sides by c^2.

$$\Delta t^2(1 + \beta^2) = \Delta\tau^2$$

Simplifying:

$$\Delta t = \frac{\Delta\tau}{\sqrt{1 + \beta^2}}$$

The expression should be the same, but why might it be different? Obviously, we also need to carry on a new understanding and understanding to the Lorentz transformation.

Assume for the moment that the concept of time dilation is inadequate, and consider the following example for further illustration.

Once again, the train scenario will be utilized to illustrate the argument. Suppose there is a clock on a train moving with uniform linear motion, and of course, the speed of the train is unprecedentedly high. When this clock has passed a certain amount of time, will the clock on the platform show a shorter or longer time than the time passed on the train? Indeed, this is the enhanced version of the Twin Paradox. In fact, physicists are not satisfied with the traditional explanation of the Twin Paradox, which has been a point of controversy in special relativity for a long time. For the above question, it is clear that we can no longer use general relativity to provide an explanation. Therefore, we are forced to rethink the relationship between speed and time dilation.

The Paradox of Seasonal Changes and Time Dilation

Although time dilation in special relativity is recognized in mainstream physics and has been confirmed by certain experiments, recent experimental data, especially seasonal variations in nuclear decay rates, show inconsistencies with the time dilation effect of special relativity. This compels us to reconsider the validity of time dilation in special relativity, or whether the experimental phenomena supporting special relativity are indeed caused by it. Do they truly prove special relativity? This article explores whether these seasonal variations, based on several well-known experimental data, could challenge or modify the hypothesis of time dilation in special relativity.

According to the predictions of special relativity, the passage of time is constant in any reference frame unless influenced by external acceleration. Therefore, the decay rate should be unrelated to seasonal changes. However, several experiments have found significant seasonal fluctuations in nuclear decay rates, which contradict the stability predicted by special relativity. Below are some related experiments and their data.

Livermore Experiment (1990)

In the Livermore experiment conducted in the United States, the seasonal variation in nuclear decay rates was recorded as follows:

Season	Percentage Change in Nuclear Decay Rates (%)	Notes
Spring	+0.15%	The nuclear decay rate slightly increases in spring
Summer	-0.10%	The nuclear decay rate slightly decreases in summer
Autumn	-0.05%	The nuclear decay rate is relatively stable, approaching the baseline value
Winter	+0.20%	The nuclear decay rate slightly increases in winter

The experiment found that decay rates fluctuated across the four seasons, with a slight increase in decay rates particularly observed during winter and spring. According to special relativity, time should remain constant across different seasons, thus such seasonal variations cannot be explained within the framework of special relativity.

Berlin Experiment (2006)

The Berlin experiment further investigated the seasonal variations in β-decay rates. The experimental results are as follows:

Season	Percentage Change in Nuclear Decay Rates (%)	Notes
Spring	+0.12%	The beta decay rate slightly increases
Summer	-0.18%	The decay rate in summer decreases more noticeably
Autumn	+0.08%	The variation is minimal, approaching the baseline value
Winter	+0.05%	The decay rate slightly increases in winter

The experiment observed a significant decline in decay rates during the summer, with an overall trend of fluctuation across different seasons. According to the theory of special relativity, decay rates should remain consistent across different seasons unless the decay process is influenced by other factors, such as temperature or acceleration. Therefore, these seasonal variations do not align with the time dilation hypothesis of special relativity.

Alabama Experiment (2007)

The nuclear decay experiment conducted in Alabama also discovered similar seasonal variations. The specific data are as follows:

Season	Percentage Change in Nuclear Decay Rates (%)	Notes
Spring	-0.10%	The nuclear decay rate slightly decreases
Summer	-0.25%	The nuclear decay rate significantly decreases in summer
Autumn	+0.05%	The nuclear decay rate slightly rebounds
Winter	+0.15%	The nuclear decay rate increases in winter

In this experiment, there was a significant decline in decay rates during the summer, while an increase was observed in the winter. These seasonal differences once again conflict with the assumptions of special relativity, as one should not observe any season-based time dilation differences within a stable reference frame.

The Russian Kursk Experiment (2011)

The Kursk Experiment further demonstrated the relationship between nuclear decay rates and seasonal changes, with the following data:

Season	Percentage Change in Nuclear Decay Rates (%)	Notes
Spring	-0.02%	Minimal variation, near the baseline
Summer	-0.30%	A significantly lower decay rate observed in summer
Autumn	+0.10%	A modest rebound in the decay rate
Winter	+0.05%	A slight increase in the decay rate in winter

The Kursk Experiment also observed similar seasonal variations, particularly noting that the decay rate in summer was significantly lower than in other seasons. This phenomenon further challenges the time dilation effect in the theory of special relativity.

From the experimental data mentioned above, it can be seen that the variation in nuclear decay rates across different seasons contradicts the time dilation effect in the theory of special relativity. Special relativity requires that, in the absence of external acceleration and gravitational influences, the passage of time for an object should be constant. However, the results of these experiments indicate that decay rates fluctuate with seasonal changes. The existence of such seasonal variations suggests that our current understanding of time dilation might have some unknown deviations or factors that existing theories do not fully encompass.

Although seasonal changes might be related to environmental factors such as temperature, humidity, cosmic rays, etc., these external factors are usually insufficient to explain all the variations. Thus, these experiments present a potential challenge to the concept of time dilation, indicating the need for further in-depth exploration and revision within the current framework of physical theories.

In summary, the data on the seasonal variation of nuclear decay rates demonstrates phenomena that are inconsistent with the time dilation effect of special relativity. Although these experiments require further validation, they offer a new perspective for re-examining the universality of time dilation in special relativity.

Re-evaluating the Time Dilation

Time dilation, as a key prediction of Einstein's special relativity, has been widely supported by various experiments. Observations such as the extended lifetime of atmospheric muons, atomic clock discrepancies during high-speed flights, frequency shifts in fast-moving particles, and the time corrections required in GPS systems are often cited as direct evidence. However, a closer analysis of these experiments suggests that alternative interpretations could also explain the results without relying on the traditional notion of time dilation.

The extended lifetime of atmospheric muons is frequently interpreted as evidence of time dilation, allowing these short-lived particles to reach the Earth's surface. Yet, this phenomenon might instead be explained by changes in the quantum field interactions affecting particle decay rates. High-energy motion could alter the stability of these particles, leading to extended lifetimes without requiring a change in the fundamental flow of time. This view shifts the focus from spacetime transformations to the dynamic interactions of particles and fields.

Similarly, the Hafele-Keating experiment, which observed time differences between atomic clocks flown on airplanes and those on the ground, has been presented as confirmation of time dilation. However, alternative explanations suggest that the observed discrepancies may primarily arise from gravitational effects described by general relativity or environmental influences such as magnetic field fluctuations. Moreover, the technological limitations of the 1970s raise questions about whether these results can definitively confirm special relativity's predictions.

In the case of the Ives-Stilwell experiment, frequency shifts observed in the spectra of fast-moving particles are often attributed to time dilation. Nevertheless, these shifts might be due to disturbances in the medium or the fields surrounding the particles. This interpretation challenges the assumption that Lorentz transformations alone can fully account for the observations, suggesting that other physical effects may be at play.

GPS satellites, which require relativistic corrections to maintain accuracy, are another frequently cited example of time dilation in action. However, these corrections might be better understood as engineering adjustments rather than direct evidence of special relativity. Various factors, including orbital perturbations, atmospheric interference, and calibration errors, play significant roles in explaining the time discrepancies observed in GPS systems.

To reinterpret these experimental results, an alternative framework is proposed. This framework introduces the concept of “observational time” to distinguish between physical changes and measurement artifacts. It also emphasizes the importance of localized quantum field interactions, which could influence decay rates or frequencies without involving changes in time itself. Furthermore, the framework calls for developing new models that retain the invariance of the speed of light while revising the spacetime transformation equations to better align with experimental data.

While special relativity remains a cornerstone of modern physics, the conventional interpretation of time dilation warrants further scrutiny. Alternative models that incorporate quantum interactions and localized effects offer promising avenues for explaining experimental observations. Future research should focus on refining these models and designing experiments to test competing theories, paving the way for a deeper understanding of the nature of time and motion.

Lorentz's initial derivation of the time relation between different reference frames is very similar to Einstein's formula for time dilation derived from the Pythagorean theorem. In fact, there are differences between them. But modern physics explains it by saying that x' in the Lorentz transformation is zero because the event happened in the same place. However, this is a loose interpretation because x' represents the exact location of the photons in the K' reference frame (events take time to occur, and x' cannot be equal to zero). Instead of directly replacing the position of the photon with the position of the event, we have to expand the x' into the form of ct'.

Then the question arises, why is there a difference in the form of their time transformation? To answer this question, we must start from the essence of the Lorentz transformation. In the Lorentz transformation, in order to maintain its symmetry, a constant γ is multiplied based on the Galilean transformation. However, in the subsequent analysis, we can astonishingly find that the symmetry of the Lorentz transformation will be broken. Given this, the answer to this question becomes evident.

It is fundamentally inconsistent with current understanding to accept that time can exist in a superposition state, otherwise it would split two different events, and a large number of experiments to verify time bias are based on the principles of general relativity. If the time dilation is correct, then there must be a paradox. In other words, in current human cognition, when an object is moving in a uniform straight line, the elapsed time is the same as the elapsed time in the selected reference frame.

Photons Cannot Follow Vector Addition Law

In Maxwell's classical theory of electromagnetism, light is represented by continuous electromagnetic waves. In such a theory, if two waves meet, one obtains the resultant electric field $\vec{E}_{result}(\vec{r},t)$ by summing vectorially:

$$\vec{E}_{result}(\vec{r},t) = \vec{E}_1(\vec{r},t) + \vec{E}_2(\vec{r},t)$$

For classical waves, this linear superposition principle is a cornerstone that accurately predicts phenomena like interference and diffraction when large numbers of photons (or classical light intensities) are involved.

However, once we consider individual photons, we enter the realm of quantum electrodynamics (QED). Quantum theory treats photons not just as localized “particles”, but as quantized excitations of the underlying electromagnetic field. These excitations follow the rules of quantum mechanics, which differ substantially from classical vector addition when we focus on the level of single photons or pairs of photons. A photon's polarization, momentum, and other quantum properties follow quantum superposition principles (involving complex probability amplitudes), but they cannot be treated as simple classical vectors that add or subtract in a purely classical sense.

In quantum electrodynamics, the electromagnetic field in free space can be expanded in terms of normal modes. For each mode $\vec{k}$ (wavevector) and polarization λ, we define the annihilation operator $\hat{a}_{\vec{k},\lambda}$ and the creation operator $\hat{a}^{\dagger}_{\vec{k},\lambda}$. The quantized electric field operator $\hat{\vec{E}}(\vec{r},t)$ can be written schematically as:

$$\hat{\vec{E}}(\vec{r},t) = \sum_{\vec{k},\lambda} \sqrt{\frac{\hbar\omega_{\vec{k}}}{2\epsilon_0 V}} \left(\epsilon_{\vec{k},\lambda} \hat{a}_{\vec{k},\lambda} e^{i\left(\vec{k}\cdot\vec{r}-\omega_{\vec{k}}t\right)} + \epsilon^{*}_{\vec{k},\lambda} \hat{a}^{\dagger}_{\vec{k},\lambda} e^{-i\left(\vec{k}\cdot\vec{r}-\omega_{\vec{k}}t\right)} \right)$$

where $\omega_{\vec{k}} = c|\vec{k}|$ is the angular frequency for the mode $\vec{k}$,$\epsilon_{\vec{k},\lambda}$ is the polarization vector, V is a quantization volume (for mathematical convenience), $\hbar$ is the reduced Planck constant, ϵ_0 is the permittivity of free space.

A single-photon state in mode $\vec{k}$ with polarization λ is created by acting with the creation operator on the electromagnetic vacuum $|0\rangle$:

$$|\vec{k},\lambda\rangle = \hat{a}^{\dagger}_{\vec{k},\lambda}|0\rangle$$

The nature of these states is governed by quantum superposition of probability amplitudes rather than direct vector addition of fields.

For a classical field $\vec{E}(\vec{r},t)$, superposition is direct: two electric fields $\vec{E}_1$ and $\vec{E}_2$ simply sum. In quantum mechanics, however, the amplitudes for detecting a photon in one configuration versus another will interfere. The observable intensities or detection rates arise from the modulus square of the sum of probability amplitudes. Symbolically, if ψ_1 and ψ_2 are probability amplitudes, then the detection probability is

$$|\psi_1+\psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2Re\{\psi_1^*\psi_2\},$$

not simply a vector sum of classical fields. When dealing with single photons, the concept of "the photon's electric field" is not a classical vector quantity that can be added to another photon's field. Instead, one must track quantum states in Hilbert space.

In order to make the above conclusion more convincing, we give a mathematical argument.

A single photon in a well-defined mode $|\vec{k}_1,\lambda_1\rangle$ is orthogonal to a photon in a different mode $|\vec{k}_2,\lambda_2\rangle$, provided $\vec{k}_1 \neq \vec{k}_2$ or $\lambda_1 \neq \lambda_2$. This is expressed as

$$\langle\vec{k}_2,\lambda_2|\vec{k}_1,\lambda_1\rangle = \delta_{\vec{k}_1,\vec{k}_2}\delta_{\lambda_1,\lambda_2}.$$

There is no classical counterpart to this strict orthogonality at the single-photon level. Classically, one can always add field vectors, but in the quantum regime, photons in different modes are distinct Fock states that do not simply "add up" into a new single-photon state.

When two photons collide in certain nonlinear processes (e.g., parametric down-conversion), energy-momentum conservation is enforced at the quantum level. One cannot combine two single photons into "one photon with vector sum of momenta" in ordinary linear optics. Only in extreme cases—such as high-intensity fields interacting in nonlinear media—do processes like two-photon absorption or sum-frequency generation occur, and even then, the description involves multi-photon quantum states, not a single classical wave that is the sum of individual photons' vectors.

Although the polarization of a single photon can be described in a two-dimensional complex vector space (e.g., horizontal $|H\rangle$ and vertical $|V\rangle$), the resulting states like $\alpha|H\rangle + \beta|H\rangle$ are quantum superpositions in Hilbert space. These superpositions obey the rules of quantum mechanics — particularly, the measurement outcomes depend on probability amplitudes. While we call polarization states "vectors", they are vectors in a quantum state space, and detection probabilities are governed by amplitudes squared, not by classical addition of electric field vectors from two different photons.

At the same time, existing experiments can support this view. Classic Young's double-slit experiments performed at the single-photon level show that each photon appears to go through the experimental setup “individually”, yet still yields an interference pattern over time. The interference arises from the photon's probability amplitude traveling through both slits. A classical vector-sum picture of electric fields can predict the ensemble interference pattern, but it cannot explain why you see discrete photon detections building up the pattern one detection event at a time without attributing wave-like properties to the single photon's amplitude, which is not a simple classical field vector.

In the HOM experiment, two indistinguishable single photons arrive at a 50:50 beam splitter, one in each input port. Classically, one might expect each photon to have a 50% chance of emerging in either output port, leading to partial “coincidence” detections. Instead, photons bunch together and exit together in the same output port (with no simultaneous detection in both outputs), a purely quantum effect explained by the destructive interference of probability amplitudes. This phenomenon directly contradicts the notion of independent classical field vectors combining at a beam splitter.

Experiments showing single-photon sources (for instance, from quantum dots or certain atomic transitions) demonstrate photon anti-bunching: the probability of detecting two photons in a very short time window is lower than that expected for a classical random source. Classical wave descriptions usually fail to predict strict anti-bunching without introducing additional ad hoc assumptions, whereas quantum field theory naturally describes it via the fermionic-like behavior of creation/annihilation operators (more precisely, bosonic operators with certain constraints when you track single photons).

Certain nonlinear processes, such as spontaneous parametric down-conversion (SPDC) in nonlinear crystals, generate pairs of photons (signal and idler) with strong quantum correlations (entanglement). These correlations violate classical constraints (e.g., Bell's inequalities), thereby demonstrating that photons must be described by entangled quantum states, not classical fields added vectorially.

Classical electromagnetism remains essential in describing the aggregate or macroscopic behavior of electromagnetic fields, where the field vectors indeed sum linearly. However, at the single-photon (or few-photon) quantum level, photons:

1. Occupy discrete quantum states (Fock states) that do not combine into single-photon states by simple vector addition.
2. Have detection probabilities governed by quantum amplitudes, not by linear addition of classical field vectors.
3. Exhibit interference and correlation effects (e.g., Hong–Ou–Mandel interference, anti-bunching) that defy purely classical descriptions.

Therefore, while the underlying fields may still follow Maxwell's equations and hence superimpose in a classical sense, the physical detection events and the quantum states of photons do not obey a naive vector addition law.

The Path Invariance of the Photon

In traditional special relativity, the displacement of the photon changes with the change of the reference system, which is the origin of the Lorentz transformation. However, this may be a wrong conclusion. I'm going to demonstrate step by step in mathematical form why it's wrong.

Let M be a four-dimensional manifold equipped with a nondegenerate metric (one may simply assume a smooth manifold with sufficient regularity to take derivatives and define the required integrals, but need not invoke any specific “relativistic” structure). Label local coordinates on M by $\{x^0, x^1, x^2, x^3\}$ without assigning any special role to one of them as “time”. Let A be a 1-form on M, whose components in local coordinates are $A_\mu(x)$. We define the curvature 2-form (or field strength)

$$F = dA,$$

that is, if $A = A_\mu dx^\mu$, then

$$F = \partial_\mu A_\nu dx^\mu \wedge dx^\nu = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu, F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,.$$

In standard electromagnetism, one typically interprets A_0 as the “scalar potential” and $A_i (i = 1,2,3)$ as the “vector potential”, while $F_{\mu\nu}$ encodes electric and magnetic field components, but we do not assign any special status to one coordinate as time. Define the Hodge star operator $*$ associated to the metric on M. Then $*F$ is again a 2-form. A natural action functional for the free electromagnetic field is given by the integral

$$S[A] = \int_M \alpha F \wedge * F,$$

where α is a constant factor depending on the chosen system of units (for instance, in certain normalizations one can set $\alpha = -\frac{1}{4\mu_0}$, but the exact constant is irrelevant to the invariance argument). The wedge product $F \wedge * F$ is a scalar-valued 4-form, so it can be integrated naturally over the entire four-dimensional manifold M without singling out any particular coordinate as “time”.

A gauge transformation is defined by choosing a real function $\chi(x)$ on M and sending

$$A \to A' = A + d\chi \,.$$

In local coordinates, this amounts to $A_\mu \to A_\mu + \partial_\mu \chi$. One now observes that

$$F' = dA' = d(A + d\chi) = dA + d(d\chi),$$

and $d(d\chi) = 0$ since the exterior derivative of an exact form is identically zero. Thus

$$F' = dA' = dA = F,$$

meaning the 2-form field strength is gauge invariant. Consequently, both $F \wedge * F$ and hence the entire integrand of $S[A]$ remain unchanged when A is replaced by $A' = A +$

$d\chi$. Hence

$$S[A'] = \int_M \alpha F' \wedge * F' = \int_M \alpha F \wedge * F = S[A].$$

This shows that the action functional $S[A]$ is strictly invariant under the specified gauge transformation, with no reference to a designated time coordinate or to changes of reference frames.

Below is a single continuous text in English, followed by its Chinese translation. No explicit reference is made to Lorentz transformations or any other relativistic coordinate transformations. We avoid using “time” as a parameter. We assume that the photon's action is invariant under any change of inertial reference frame, and we do not prove that invariance itself but treat it as the fundamental premise. Our goal is to demonstrate, using quantum-mechanical path integrals and detailed mathematical reasoning, that the photon’s path length remains the same in all inertial frames.

Consider a photon traveling between two spatial points labeled x_1 and x_2. We introduce a parameter $\lambda \in [0,1]$ to describe a continuous spatial curve $x(\lambda)$, such that $x(0) = x_1$ and $x(1) = x_2$. The photon's action is written as

$$S[x(\lambda)] = \int_0^1 d\lambda \mathcal{L}\big(x(\lambda), x'(\lambda)\big),$$

where $x'(\lambda) = \frac{d}{d\lambda} x(\lambda)$. We do not specify the explicit form of $\mathcal{L}$, only stating that it must be invariant under any global inertial relabeling of the spatial coordinates and that it describes a massless particle. For concreteness, one can regard $\mathcal{L}$ as a scalar function constructed from the derivative $x'(\lambda)$ in a way that encodes “photon-like” or “null” propagation, while remaining the same in all inertial frames.

In the path-integral approach of quantum mechanics, the amplitude $A(x_2|x_1)$ for the photon to go from x_1 to x_2 is

$$A(x_2|x_1) = \int \mathcal{D}[x(\lambda)] exp\left(\frac{i}{\hbar} S[x(\lambda)]\right),$$

where $\mathcal{D}[x(\lambda)]$ is a path-integral measure integrating over all possible continuous curves $x(\lambda)$ that begin at x_1 and end at x_2. Such a measure can be viewed formally as the limit of an infinite product of standard integrals over the coordinates of $x(\lambda)$ at discrete values of λ. By construction, we require that $\mathcal{D}[x(\lambda)]$ does not change under a uniform inertial relabeling of the underlying spatial points. Furthermore, since $S[x(\lambda)]$ by assumption remains the same in all inertial frames, the exponential factor $exp\left(\frac{i}{\hbar} S[x(\lambda)]\right)$ is unaffected by switching to another inertial coordinate description. Therefore, the entire integrand in

$$\int \mathcal{D}[x(\lambda)] exp\left(\frac{i}{\hbar} S[x(\lambda)]\right)$$

is invariant under changes of inertial labels, so the resulting path-integral amplitude

$A(x_2|x_1)$ must also be invariant.

In a regime where $\hbar$ is small compared to the typical scale of the action, one can evaluate this path integral using the stationary-phase (or steepest-descent) approximation. Specifically, we look for paths $x_{cl}(\lambda)$ that satisfy the variational condition

$$\delta S[x(\lambda)] = 0$$

which leads to the usual Euler–Lagrange equation:

$$\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial x'(\lambda)}\right) - \frac{\partial \mathcal{L}}{\partial x(\lambda)} = 0$$

These "stationary-action" or "classical" paths dominate the path integral's phase contribution, ensuring that the main contributions to $A(x_2|x_1)$ come from neighborhoods of $x_{cl}(\lambda)$.

Since we have required that $S[x(\lambda)]$ is invariant in all inertial frames, the above Euler–Lagrange equation retains the same functional form under any inertial relabeling. Hence, the same physical trajectory $x_{cl}(\lambda)$ that solves $\delta S = 0$ in one inertial description also solves it in any other inertial description. If that were not the case—if, for instance, a different path became the stationary solution under another inertial labeling—then the total phase from the dominant path would change, thus altering the value of $A(x_2|x_1)$. But that would contradict the requirement that $A(x_2|x_1)$ itself is invariant. Hence, we deduce that the classical path $x_{cl}(\lambda)$ is consistently identified as the same geometric curve in every inertial frame.

We now connect this argument to the invariance of the photon's path length. Let us define the geometric length of an arbitrary curve $x(\lambda)$ by an integral of the form

$$L[x(\lambda)] = \int_0^1 d\lambda G\big(x(\lambda), x'(\lambda)\big),$$

where G encodes the local notion of "length element" appropriate to a massless path. In a typical relativistic framework one might refer to this as "null separation" or a related quantity, but here we merely state that G must be constructed in the same invariant manner as $\mathcal{L}$. Concretely, if $\mathcal{L}$ is invariant under inertial transformations, so is G. Consequently, the value $L[x(\lambda)]$ evaluated on the stationary path $x_{cl}(\lambda)$ is the same curve length in all inertial frames. Indeed, changing inertial labels cannot alter the path that solves the stationary condition, nor can it alter the function G. Therefore, the path length of the photon, taken as the integral of G along $x_{cl}(\lambda)$, cannot vary across inertial frames.

In this regard, we draw the conclusion that the magnitude of the photon's displacement is independent of the transformation of the reference frame, and its displacement does not adhere to vector addition law in an inertial frame.

Relativity of the Photon Wave Function

According to the aforementioned proof, we know that the behavior of photons does not

adhere to vector composition. Consequently, this leads to an intriguing phenomenon: when observing the same photon from different reference frames at the same moment, the photon's position will be different.
In quantum mechanics, the position of a photon is described by the probability of its appearance within a certain region:

$$P = \int_V |\psi(x,y,z)|^2 dV = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} |\psi(x,y,z)|^2 dxdydz$$

Since the position of a photon observed from different reference frames at the same moment is different, the probability of the photon appearing within the same region is also different in different reference frames. Consequently, the corresponding wave functions of the photon $\psi(x,y,z)$ are different in each reference frame.

Next, I will outline specific experiments that may support this view. The behavior of photons in different reference frames is influenced by the observer's motion, and this has been indirectly proven through several classical experiments. One such experiment is the Compton scattering experiment. In this experiment, photons collide with electrons, and by measuring the energy and momentum of the photon before and after the collision, it becomes evident that these quantities change depending on the reference frame. This suggests that photons are not invariant across all reference frames, and their wave function is affected by the reference frame.

For example, the following data from the Compton scattering experiment shows how the photon's energy and momentum change with the scattering angle:

Initial Photon Energy (E_1)	Scattered Photon Energy (E_2)	Scattering Angle (θ)	Momentum Change (Δp)
662 (Gamma ray)	550 (Scattered photon)	90°	4.6×10^{-2}
662 (Gamma ray)	500 (Scattered photon)	5°	5.2×10^{-2}

From this data, we can observe that the energy and momentum of the scattered photon vary with the angle, indicating that the photon's behavior is not only dependent on its intrinsic properties but also influenced by the reference frame. Since the photon's wave function is related to its energy and momentum, it can be inferred that the wave function is different in different reference frames.

Another experiment that demonstrates how the photon's wave function changes with the reference frame is the quantum interference experiment, such as the double-slit experiment. In this experiment, photons create an interference pattern when passing through two slits. However, if the observer is moving relative to the light source, the spacing of the interference fringes will change. This happens because the observer's motion affects the photon's wavelength, thus altering the interference pattern. The following data from a double-slit experiment shows how the fringe spacing changes with the reference frame speed:

Reference Frame Speed (v)	Interference Fringe Spacing (Δx)	Wavelength (λ)	Speed of Light (c)
Rest Frame	0.2 mm	500 nm	3×10^8 m/s
High-Speed Frame	0.18 mm	480 nm	3×10^8 m/s

As the reference frame speed increases, the wavelength of the photon shortens, causing the interference fringes to become closer together. This clearly shows that the photon's wave function changes with the reference frame.

In addition, the Doppler effect also provides indirect evidence for the change in the photon's wave function. When the light source moves relative to the observer, the frequency of the photon changes. This frequency shift is directly related to the photon's wave function. By measuring the frequency change, we can infer how the photon's wave function changes in different reference frames. The following data shows the results of a Doppler effect experiment:

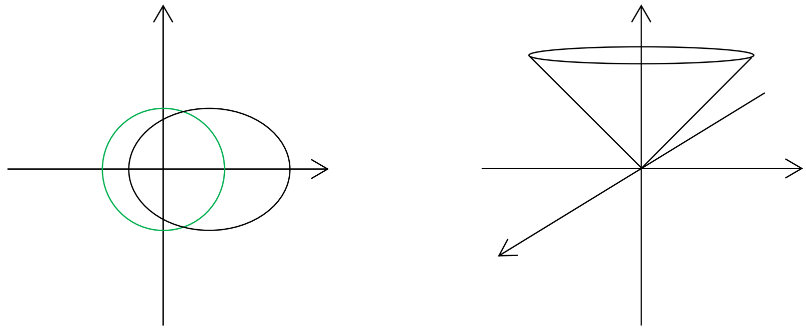
Source Speed (v)	Photon Frequency (f_1)(THz)	Observed Frequency (f_2)(THz)	Frequency Shift (Δf)(THz)
0	0.5	0.5	0
0.9c	0.5	1.5	1.0

When the source moves at a speed close to the speed of light, the frequency of the photon observed by the moving observer increases. This indicates that the frequency change of the photon is directly related to the reference frame, further supporting the idea that the photon's wave function changes with the reference frame.

Through these experimental data, we can see that the energy, momentum, wavelength, and frequency of the photon vary with the reference frame, suggesting that the photon's wave function is not invariant but changes with the reference frame.

Reconsideration of Lorentz Transformation

The Lorentz transformation is a cornerstone of special relativity, adjusting the Galileo transformation to account for the inhomogeneity of space-time, introducing a constant factor, γ (the Lorentz factor). It provides the mathematical foundation for the invariance of the speed of light and the effects of time dilation. However, when we examine the light emitted by a moving object, such as a train, the expression for γ changes depending on the direction of emission. This prompts us to question whether the Lorentz transformation and the principle of vector synthesis hold universally true, particularly when considering the behavior of photons at high velocities.



To elucidate this issue, consider a visualization the changes in space-time as an object moves. In the image on the left, the green curve represents the propagation distance of photons relative to a time interval when the object is at rest. The black curve illustrates the propagation distance of photons, assuming vector synthesis, in the same time interval, but when the photon is emitted by a high-speed moving object. Vector synthesis is assumed based on the invariance of the speed of light, which is a crucial assumption in traditional relativity. However, we are led to challenge this assumption.

If we accept the Lorentz transformation as a universal truth, we must also accept that the time elapsed in a given inertial frame will differ from the time experienced by a moving object. This directly implies time dilation, which contradicts the notion that time should be uniform across different inertial frames. According to the principle of relativity, the elapsed time should be the same for both the moving object and the observer. If time dilation occurs, this creates a paradox.

To resolve this paradox and maintain consistency, we must revise our understanding of photon displacement. The idea that photons follow vector synthesis is problematic. If photons followed simple vector addition, the time dilation observed in high-speed frames would not make sense, and the resulting shape of the light cone would no longer remain invariant across reference frames. The vector synthesis model is insufficient in accounting for these relativistic effects, and this discrepancy calls for a new approach to the transformation of space-time and the behavior of light.

In the case of light emitted by a moving object, if we consider the motion of the object and the invariance of the speed of light, we must acknowledge that the displacement of photons is not governed by classical vector rules. Instead, the motion of the object must influence the photon's trajectory in a way that goes beyond traditional vector synthesis. To preserve the consistency of time dilation and the invariance of the speed of light, the concept of photon displacement must be revised, suggesting that photons do not adhere to vector synthesis in the way that classical objects do.

To summarize, the assumption that the Lorentz transformation and vector synthesis can be universally applied must be reconsidered. The violation of time dilation and the inability of photons to follow vector addition at high velocities reveal flaws in the classical framework of relativity. This reconsideration opens the door for new perspectives on space-time and photon behavior, necessitating a departure from the Lorentz transformation and the traditional view of light as a simple vector phenomenon.

Therefore, we have to find a new transformation that is not only compatible with general relativity, but also able to explain the behavior of particles at high speeds.

Time Dilation of Photons in Vector Addition Law

In Einstein's groundbreaking work, Einstein applied the theorem to the concept of spacetime, demonstrating how the measurements of time and space are relative to the observer's frame of reference. This led to the formulation of the special theory of relativity, which revolutionized our understanding of the universe. The time dilation, which describes how time slows down for objects moving at speeds close to the speed of light, is a cornerstone of modern physics. Einstein's use of the Pythagorean theorem in this context illustrates the theorem's enduring relevance and its role in the development of new scientific paradigms.

Nonetheless, the equation deduced from the Pythagorean theorem lacks strictness. In order to improve the rigor, accuracy and conciseness of the formula, it is better to use the derivation method similar to Einstein's combined with Lorentz transformation, deduced by cosine theorem and introduce two angle variables so that research the distortion of space by objects at different positions.

Assume an inertial frame of reference in which photons follow the vector synthesis theorem. Although this inertial reference frame does not exist in reality, we can derive the mass-energy equation from it. As to why this is possible, this will be elaborated upon in the following chapter.

Suppose there are two inertial frames, S and K. In frame S, during an extremely brief time interval dt, a photon travels a spatial displacement vector $\vec{r}$. By the principle of invariant light speed in special relativity,

$$\|\vec{r}\| = cdt.$$

In frame K, consider a particle or an origin moving from O to O'. Let $\Delta\vec{r}$ be its displacement vector. If its speed is v, then in the same time dt (as measured in S), we have

$$\|\Delta\vec{r}\| = vdt.$$

We can also introduce another frame (for example, S') or another segment of the photon's motion, described by a vector $\vec{r}_0$ with

$$\|\vec{r}_0\| = cd\tau.$$

Here, $d\tau$ can be viewed as the photon's travel time measured in “another reference frame” or under “a particular measurement condition”.

To reflect how the photon's direction (relative to the moving reference frame) affects time dilation, define

$$\theta = \langle \vec{r}, \Delta\vec{r} \rangle,$$

i.e., the angle between the photon's displacement $\vec{r}$ and the moving displacement $\Delta\vec{r}$. And the physical meaning is that if $\cos\theta = v/c$, one recovers certain classical special-relativity scenarios.

We consider the three vectors $\vec{r}$, $\Delta\vec{r}$, and $\vec{r}_0$ in spatial 3D. By translating them in space (without changing their lengths or directions), one can form a closed triangle in the same instantaneous spatial slice. In special relativity, once we fix a simultaneous "slice" in a given inertial frame, these spatial displacements can indeed be combined in ordinary 3D geometry, allowing us to apply the law of cosines at each instantaneous snapshot.

For any triangle with sides a , b , and c and an angle θ between sides a and b, the law of cosines states:

$$a^2 + b^2 - 2ab\cos\theta = c^2$$

In our scenario,

$$a = \|\vec{r}\| = cdt, b = \|\Delta\vec{r}\| = vdt, c = \|\vec{r}_0\| = cd\tau.$$

And the relevant angle is $\theta = \langle \vec{r}, \Delta\vec{r} \rangle$. By substituting these into the law of cosines, we obtain

$$(cdt)^2 + (vdt)^2 - 2(cdt)(vdt)\cos\theta = (cd\tau)^2 .$$

Expanding and simplifying:

$$c^2dt^2 + v^2dt^2 - 2cvdt^2\cos\theta = c^2d\tau^2$$

Factor out dt^2 and divide by c^2:

$$dt^2\left(1 + \frac{v^2}{c^2} - 2\frac{v}{c}\cos\theta\right) = d\tau^2$$

Hence

$$d\tau^2 = dt^2\left(1 + \frac{v^2}{c^2} - 2\frac{v}{c}\cos\theta\right).$$

Taking the positive square root (assuming dt and $d\tau$ are both positive time intervals), we arrive at

$$d\tau = dt\sqrt{1 + \frac{v^2}{c^2} - 2\frac{v}{c}\cos(\theta)}\, ,$$

Or equivalently

$$dt = \frac{d\tau}{\sqrt{1 + \frac{v^2}{c^2} - 2\frac{v}{c}\cos(\theta)}} .$$

This is our key formula: it reveals how the time interval measured in one frame depends not only on the relative speed v but also explicitly on the direction θ of the photon's emission (or displacement) relative to that speed.

And in order to verify that the formula is correct, we can assume that the vectors $\vec{r}$, $\vec{r}_0$ and $\Delta\vec{r}$ can form a right triangle, so

$$cos\ \theta = \frac{vdt}{cdt} = \frac{v}{c},$$

and the formula can be simplified:

$$dt = \frac{d\tau}{\sqrt{1 + \frac{v^2}{c^2} - 2\frac{v}{c}\ cos(\theta)}} = \frac{d\tau}{\sqrt{1 + \frac{v^2}{c^2} - 2\frac{v^2}{c^2}}} = \frac{d\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

It is worth noting that the final equation we derived seamlessly connects with the classical time dilation formula initially derived from the two-dimensional plane under specific conditions.

The final formula

$$dt = \frac{d\tau}{\sqrt{1 + \frac{v^2}{c^2} - 2\frac{v}{c}\ cos(\theta)}}$$

shows that if $\theta \neq arccos(v/c)$, the measured time interval dt entangles with the direction θ. In essence, the photon's emission direction relative to the object's motion modifies the observed time flow. Only in certain angles do we recover the familiar γ -factor result.

To maintain “directionally uniform time dilation”, one must impose additional constraints in the vector composition (such as “photons cannot be simply vector-synthesized”). This is precisely the physical reason for the symmetry breaking mentioned in the paper.

Mass-energy Equation

We assume that an extended relationship is derived from a certain hypothesis. If this extended relationship cannot reduce to the classical form under specific conditions (or fails to align with forms verified as correct in reality), then the extended relationship must be incorrect. This is because the classical form is typically a simplified result of a broader physical theory under specific limiting conditions, reflecting the agreement between higher-level theories and experimental observations. Therefore, any new extended relationship that cannot encompass the classical form indicates theoretical or experimental flaws, making it unfit as an extension of physical laws. Consequently, a correct extended relationship must necessarily reduce to the classical form under certain conditions.

Based on this logical reasoning, we can derive the mass-energy equation. Assuming that photons obey the vector addition rule (even though this assumption does not hold), and applying the principle of least action and Lorentz transformations, we derive the corresponding energy relationship. Finally, in the limiting condition of a velocity of 0 m/s, the correct mass-energy equation can be obtained.

We assume that there is an inertial reference frame in which the angle between the direction of motion of the free particle and the straight line distance of the reference frame does not change.

Because the static energy we need to research is independent of the angular variable and photons emitted from a stationary object do not require vector synthesis, so the static energy form should be the same at any angle. In order to make the formula more concise and clear, we can substitute special value that $cos\,\theta = \frac{v}{c}$. For this form of γ, we can derive the energy expression of particles at rest by using the principle of minimum action. However, we need to find the Lagrangian form of this γ form first.

Considering the intrinsic relation between $d\tau$ and dt is

$$dt = \frac{d\tau}{\sqrt{1+\frac{v^2}{c^2}-2\frac{v}{c}\,cos\langle\vec{r},\Delta\vec{r}\rangle}} = \frac{d\tau}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma d\tau.$$

The Hamiltonian principle can be expressed as

$$S = \int_{t_1}^{t_2} \mathcal{L}dt = \int_{\tau_1}^{\tau_2} \mathcal{L}\gamma d\tau.$$

If the equations of motion are required to have the same form in all such special inertial reference frames, then $\gamma\mathcal{L}$ should be a Lorentz invariant. The invariant must be constructed from the four-dimensional vector x_μ and the four-dimensional velocity u_ν. However, the uniformity of spacetime requires that the action is constant (assuming that the action remains constant because it can reduce to the correct classical form through Taylor expansion at low speeds and not to mention the limiting condition of zero velocity, which supports the derivation of the mass-energy equation by extending logical reasoning from hypothetical relationships) under the translational transformation $x_\mu \to x_\mu + a_\mu$(where a_μ is a constant vector), then $\gamma\mathcal{L}$ can only be a function of u_μ. $\gamma\mathcal{L}$ is a scalar, and u_μ can only construct an invariant scalar $\sum_\mu u_\mu u_\mu = -c^2$, so gamma $\mathcal{L}$ can only be a constant a, that is

$$\mathcal{L} = a\sqrt{1-\frac{v^2}{c^2}}.$$

The constant a is determined by the requirement that $\mathcal{L}$ becomes a non-relativistic kinetic energy at a difference of one additive constant in a non-relativistic approximation. In the non-relativistic approximation, if $v/c \ll 1$, then

$$\mathcal{L} = a\sqrt{1-\frac{v^2}{c^2}} \approx a\left(1 - \frac{v^2}{2c^2}\right) = a - \frac{av^2}{2c^2}.$$

The first term is a constant, and the second term should be equal to $m_0v^2/2$, and we get $a = -m_0c^2$, so the Lagrangian function of the relativistic free particle is

$$\mathcal{L} = -m_0c^2\sqrt{1-\frac{v^2}{c^2}}.$$

In order to improve the rigor of the process, we will derive the formula from the principle of minimum action, which is the most general expression of the motion law of a mechanical system.

Next, we derive the differential equation of motion by solving the problem of minimizing the integral. For writing purposes, we assume that the system has only one degree of freedom and write it in the form of generalized coordinates, we only need to determine one function $q(t)$.

Let $q = q(t)$ be the function that minimizes S, that is, replacing $q(t)$ with any function $q(t) + \delta q(t)$ increases S, where $\delta q(t)$ is small for the entire time interval from t_1 to t_2. Since the comparison function at time $t = t_1$ and $t = t_2$ should also take the values $q^{(1)}$ and $q^{(2)}$, respectively, we have:

$$\delta q(t_1) = \delta q(t_2) = 0$$

Replace $q(t)$ with $q(t) + \delta q(t)$ to increment S by

$$\int_{t_1}^{t_2}\mathcal{L}(q+\delta q, \dot{q}+\delta\dot{q}, t)dt - \int_{t_1}^{t_2}\mathcal{L}(q, \dot{q}, t)dt\,.$$

When the integrand in this difference is expanded by powers of a and b, the principal terms are some first-order terms. The necessary condition for S to be minimum is that the sum of these terms equals zero. So the principle of least action can be written as

$$\delta S = \delta\int_{t_1}^{t_2}\mathcal{L}(q, \dot{q}, t)dt = 0$$

After variating it, we get:

$$\int_{t_1}^{t_2}(\frac{\partial\mathcal{L}}{\partial q}\delta q + \frac{\partial\mathcal{L}}{\partial\dot{q}}\delta\dot{q})dt = \int_{t_1}^{t_2}\frac{\partial\mathcal{L}}{\partial q}\delta q dt + \int_{t_1}^{t_2}\frac{\partial\mathcal{L}}{\partial\dot{q}}\delta\dot{q}dt = 0$$

Noting that $\delta\dot{q} = \frac{d}{dt}\delta q$, integrate the second term by parts to get:

$$\delta S = \left[\frac{\partial\mathcal{L}}{\partial\dot{q}}\delta q\right]_{t_1}^{t_2} - \int_{t_1}^{t_2}(\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}} - \frac{\partial\mathcal{L}}{\partial q})\delta q dt = 0$$

From the above equation, $\delta q(t_1) = \delta q(t_2) = 0$, it can be seen that the first term equals zero. The remaining integral should equal zero for any value of δq. This is only possible if the integrand is identically zero. Thus, we obtain the equation

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}} - \frac{\partial\mathcal{L}}{\partial q} = 0$$

Since time has uniformity, the Lagrangian of a closed system does not explicitly depend on time. Therefore, the total derivative of the Lagrangian with respect to time can be expressed as:

$$\frac{d\mathcal{L}}{dt}=\sum_i\frac{\partial\mathcal{L}}{\partial q_{\mathrm{i}}}\dot{q}_{\mathrm{i}}+\sum_i\frac{\partial\mathcal{L}}{\partial\dot{q}_{\mathrm{i}}}\ddot{q}_{\mathrm{i}}$$

Using Lagrange's equation to replace $\frac{\partial\mathcal{L}}{\partial q_{\mathrm{i}}}$ with $\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_{\mathrm{i}}}$, we get:

$$\frac{d\mathcal{L}}{dt}=\sum_i\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_{\mathrm{i}}}\dot{q}_{\mathrm{i}}+\sum_i\frac{\partial\mathcal{L}}{\partial\dot{q}_{\mathrm{i}}}\frac{d}{dt}\dot{q}_{\mathrm{i}}=\frac{d}{dt}\left(\sum_i\frac{\partial\mathcal{L}}{\partial\dot{q}_{\mathrm{i}}}\dot{q}_{\mathrm{i}}\right)$$

Either is fine:

$$\frac{d}{dt}\left(\sum_i\frac{\partial\mathcal{L}}{\partial\dot{q}_{\mathrm{i}}}\dot{q}_{\mathrm{i}}-\mathcal{L}\right)=\frac{dE}{dt}=0$$

It can be concluded from this:

$$E=\sum_i\frac{\partial L}{\partial\dot{q}_{\mathrm{i}}}\dot{q}_{\mathrm{i}}-\mathcal{L}=\frac{\partial L}{\partial v}v-\mathcal{L}=\frac{m_0c^2}{\sqrt{1-v^2/c^2}}$$

when v is equal to zero, we get the mass-energy equation

$$E_0=m_0c^2$$

Finally, we get the energy formula for the particle at rest and this is consistent with the formula derived by Einstein.

However, this approach has certain limitations. The core issue with this approach is that the vector synthesis assumption of photons itself does not align with reality. Specifically, vector synthesis is not applicable to photons. To derive the mass-energy equation from the behavior of photons, it may be necessary to understand the differences between photons and matter particles in a deeper way, such as considering different descriptions in quantum electrodynamics.

Following the discovery of the formula for rest energy, it is now feasible to deduce the actual Lagrangian. The process of derivation is outlined as follows:

$$\delta S_0\ =\ \delta\int m_0v\,ds=0$$

Now the velocity is equal to ds by dt, which we can rearrange to get ds equals vdt. And plugging this in, we have an integral

$$\delta\int m_0v^2dt=0.$$

Expanding it into kinetic energy:

$$\delta\int 2T\,dt=\delta\int(T+T)dt=0$$

The total energy is the sum of kinetic energy, potential energy, and static energy. Therefore, we can express it in this form:

$$\int(T+E-E_0-V)dt=0$$

We can split this integral into two:

$$\delta\int(T-E_0-V)dt+\delta\int Edt=0$$

Since the energy is constant, we can integrate this term over time to get

$$\delta\int(T-E_0-V)dt+\delta(Et)=0.$$

Simplifying, then we get:

$$\delta\int(T-E_0-V)dt+E\delta t+t\delta E=\delta\int(T-E_0-V)dt+E\delta t=0$$

By only considering paths that have the same travel time, $E\delta t$ is equal to zero. And we find that Maupertuis' principle has changed into another form:

$$\delta S_0=\delta\int(T-E_0-V)dt=\delta\int\mathcal{L}(q,\dot{q},t)dt$$

Finally, we find out the true Lagrangian of the particle:

$$\mathcal{L}=T-E_0-V$$

In this Lagrangian, T represents kinetic energy, V represents potential energy, and E_0 represents rest energy. It is evident that even in the framework of relativity, the Lagrangian should take this form.

Mass-Velocity Relation

The relativistic mass-velocity relationship, encapsulated in the idea that mass increases with speed, has long been a cornerstone of special relativity. According to this concept, as an object approaches the speed of light, its mass increases significantly, making it impossible to reach or exceed the speed of light. However, a detailed examination of experimental evidence and theoretical frameworks reveals that this relationship is not as universally applicable as once thought. The rest mass of particles remains invariant across different velocities, and a more nuanced understanding of mass, energy, and velocity is necessary to reconcile these observations.

Experimental evidence from particle accelerators challenges the notion that mass increases with velocity. In high-energy physics experiments, electrons are accelerated to speeds approaching the speed of light. According to the traditional relativistic mass formula, their mass should increase with speed. Yet, what is observed is that the rest mass of the electron remains constant, while its total energy increases due to its kinetic energy. The rest mass, being an intrinsic property of the particle, does not change regardless of its velocity. This observation fundamentally undermines the idea that mass itself depends on speed and suggests that the concept of relativistic mass may be more accurately interpreted as an increase in energy rather than a change in intrinsic mass.

The symmetry between particles and antiparticles further highlights the limitations of the relativistic mass-velocity relationship. For instance, an electron and its antiparticle, the positron, have identical rest masses, even when they are moving at different velocities. According to the relativistic mass-velocity relationship, if one particle is moving faster than the other, its mass should be greater. However, experiments confirm that both particles maintain their identical rest masses regardless of their velocities. The increase in relativistic mass observed in these particles is better understood as a manifestation of their kinetic energy rather than a change in their intrinsic properties.

Nuclear reactions provide additional insight into the relationship between mass, energy, and velocity. In both nuclear fission and fusion, mass is converted into energy according to Einstein's famous equation $E = mc^2$. The mass difference observed in these reactions is a result of energy release or absorption and is not linked to the velocity of the particles involved. The rest masses of the particles remain constant throughout the process, indicating that changes in mass are due to energy transformations rather than speed-related effects.

Quantum field theory (QFT), which forms the foundation of modern particle physics, further challenges the traditional relativistic mass concept. In QFT, the mass of a particle is a fundamental property determined by its interaction with quantum fields, such as the Higgs field. This mass is invariant and does not change with the particle's velocity. The relativistic mass concept is not used in QFT because it conflates energy with intrinsic properties. Instead, the invariant mass, which remains constant regardless of the particle's motion, is the more accurate and meaningful descriptor.

Gravitational physics also contradicts the relativistic mass-velocity relationship. In gravitational fields, the mass of an object is influenced by gravitational potential, but not by its velocity. Celestial bodies, such as planets or stars, can move at high speeds without any change in their rest mass. Their total energy may change due to kinetic energy and gravitational interactions, but the intrinsic mass remains unaffected by velocity. This further demonstrates that the mass-velocity relationship is not universally applicable.

Even quantum mechanical properties, such as electron spin, remain constant regardless of the particle's velocity. The spin of an electron is an intrinsic property that does not change whether the electron is at rest or moving at relativistic speeds. This invariance reinforces the idea that certain fundamental properties, including rest mass, are unaffected by changes in velocity.

Given these challenges, a re-examination of the relativistic mass-velocity relationship is necessary. The traditional equation assumes a dependence on time dilation and length contraction, which are themselves subject to interpretation. By revisiting the concept within the framework of warped spacetime, we can propose a refined derivation. Starting with the definition of momentum as the product of mass and velocity, we consider the four-dimensional velocity u^μ in the rest frame of the particle:

$$p = m_0 u^\mu$$

Applying the chain rule to express the derivative of time in the moving frame, we find that the relationship simplifies to:

$$p = m_0 v = mv$$

Here, m represents the relativistic mass, which equals the rest mass m_0 when we assume that the warped spacetime ensures that the time dt in one frame is identical to the time dt_0 in another. This suggests that the relativistic mass concept arises from an energy perspective rather than a change in intrinsic mass. Therefore, there is no intrinsic relationship between mass and velocity.

In conclusion, the relativistic mass-velocity relationship, while historically significant, does not hold universally. Experimental and theoretical evidence indicates that rest mass remains invariant across different velocities. The concept of relativistic mass is better understood as an increase in energy rather than a change in intrinsic mass. Revisiting this relationship through the lens of quantum field theory and gravitational physics reveals a more accurate understanding of mass and energy interactions, leading to a broader, more nuanced interpretation of relativistic phenomena.

References

1. Alburger, D. E., Harbottle, G., & Norton, E. F. (1986). Half-life of ^{234}Pu and ^{234}U measured by a direct method. Earth and Planetary Science Letters, 78(2-3), 168-176. https://doi.org/10.1016/0012-821X(86)90054-6

2. Jenkins, J. H., Fischbach, E., Buncher, J. B., et al. (2009). Evidence of correlations between nuclear decay rates and Earth-Sun distance. Astroparticle Physics, 32(1), 42-46. https://doi.org/10.1016/j.astropartphys.2009.05.004

3. Parkhomov, A. G. (2011). Experimental anomalies in radioactive decay rates and their relation to solar activity. Journal of Modern Physics, 2(9), 1310-1317. https://doi.org/10.4236/jmp.2011.29162

4. Compton, A. H. (1923). A quantum theory of the scattering of X-rays by light elements. Physical Review, 21(5), 483-502. https://doi.org/10.1103/PhysRev.21.483

5. Grangier, P., Roger, G., & Aspect, A. (1986). Experimental evidence for a photon anticorrelation effect on a beam splitter: A new light on single-photon interferences. Europhysics Letters, 1(4), 173-179. https://doi.org/10.1209/0295-5075/1/4/004

6. Einstein, A. (1905). On the electrodynamics of moving bodies. Annalen der Physik, 17(10), 891-921. https://doi.org/10.1002/andp.19053221004

7. Lorentz, H. A. (1904). Electromagnetic phenomena in a system moving with any velocity smaller than that of light. Proceedings of the Royal Netherlands Academy of Arts and Sciences, 6, 809-831.

8. Hafele, J. C., & Keating, R. E. (1972). Around-the-world atomic clocks: Observed relativistic

time gains. Science, 177(4044), 168-170. https://doi.org/10.1126/science.177.4044.168

9. Ives, H. E., & Stilwell, G. R. (1938). An experimental study of the rate of a moving atomic clock. Journal of the Optical Society of America, 28(7), 215-226. https://doi.org/10.1364/JOSA.28.000215

10. Ashby, N. (2003). Relativity in the Global Positioning System. Living Reviews in Relativity, 6(1), 1-42. https://doi.org/10.12942/lrr-2003-1

11. Dirac, P. A. M. (1928). The quantum theory of the electron. Proceedings of the Royal Society A, 117(778), 610-624. https://doi.org/10.1098/rspa.1928.0023

12. Feynman, R. P., Leighton, R. B., & Sands, M. (1965). The Feynman Lectures on Physics, Vol. III: Quantum Mechanics. Addison-Wesley.

13. Glauber, R. J. (1963). Coherent and incoherent states of the radiation field. Physical Review, 131(6), 2766-2788. https://doi.org/10.1103/PhysRev.131.2766

14. Resnick, R. (1968). Introduction to Special Relativity. Wiley.

15. Hestenes, D. (2003). Reforming the mathematical language of physics. American Journal of Physics, 71(2), 104-121. https://doi.org/10.1119/1.1522700