Tunneling Time and Hartman effect: A Multivalued Perspective on Quantum Cosmological Tunneling Interpretation

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Abstract:

The present article is dedicated to Robert N. Boyd, PhD, with whom we have discussed several exotic subjects in physics, including interstellar travel, med beds for future medicine, and the Pleiadeian council. While we appreciate and admire his vast experience and involvement in several high-profile experiments, we respectfully disagree with his use of the Rodin coil with a special design to shrink traveling time needed to traverse galaxies through the concept of folded space. We previously argued for a connection between the Navier-Stokes and Schrödinger equations, then used standard tunneling time theory [1][2]. Here, we propose an alternative interpretation of the Hartman effect in tunneling, suggesting that it represents the multivaluedness of solutions to the Schrödinger equation. This implies that an electron or entity can exist in two places simultaneously, explaining how an entity can seemingly appear on the other side of a tunnel almost instantaneously upon initiating a quantum tunneling experiment. While counter-intuitive, this interpretation aligns with Schrödinger's initial ideas. This phenomenon could be detected through near-field effects, such as a spin supercurrent detector in low-temperature physics experiments.

Introduction

Quantum tunneling, a phenomenon where particles pass through potential barriers seemingly impenetrable in classical physics, has long fascinated physicists [1]. The concept of "tunneling time," how long a particle takes to traverse the barrier, has been a subject of much debate. While various theoretical frameworks exist to describe tunneling time, experimental verification remains challenging. The Hartman effect, where tunneling time appears independent of barrier width beyond a certain point, further complicates the picture. Standard interpretations often invoke complex mathematical formalisms and can lead to seemingly paradoxical conclusions, such as superluminal tunneling.

This article proposes a novel interpretation of the Hartman effect, connecting it to the multivalued nature of solutions to the Schrödinger equation. Instead of focusing on the time taken to traverse the barrier, we suggest that tunneling reflects the inherent ability of a quantum entity to occupy multiple states or locations simultaneously.

What is Hartman effect and tunnelling time?

Quantum tunneling, a bizarre yet fundamental phenomenon in quantum mechanics, allows particles to pass through potential barriers even when they lack the energy to do so classically. Imagine a ball rolling towards a wall; classically, if it doesn't have enough energy to go over the wall, it will bounce back. In the quantum world, however, there's a non-zero probability that the ball will simply *appear* on the other side of the wall, as if it had tunneled through it. This "tunneling" is crucial to various processes, from nuclear fusion in stars to scanning tunneling microscopy.

A key question arises: how long does this tunneling process take? This is where the concepts of "tunneling time" and the "Hartman effect" come into play.

Determining the time a particle spends tunneling has proven surprisingly complex and controversial. Several theoretical approaches exist, each with its own definition of tunneling time, leading to a lack of universally accepted framework. Some definitions focus on the time it takes for the particle's wavefunction to penetrate the barrier, while others consider the time it takes for the particle to appear on the other side.

One might naively expect that tunneling time should increase with the width of the barrier. After all, it seems logical that it would take longer to tunnel through a thicker wall. However, experiments and theoretical calculations have revealed a counterintuitive result: the Hartman effect.

The Hartman Effect: A Head-Scratcher

The Hartman effect, named after physicist Thomas E. Hartman, describes the surprising observation that, beyond a certain barrier width, the tunneling time appears to become *independent* of the barrier width. In other words, increasing the thickness of the wall doesn't necessarily increase the time it takes for the particle to tunnel through it. This saturation of tunneling time has been experimentally verified and is a robust phenomenon.

This effect raises several intriguing questions. Does it imply that particles can tunnel faster than light? This would seemingly violate Einstein's theory of relativity. However, it's crucial to understand that tunneling time doesn't represent the time it takes for a particle to physically traverse the barrier. Instead, it's related to the time it takes for the probability amplitude to build up on the other side of the barrier.

Interpretations and Implications

The Hartman effect has sparked much debate and several interpretations. One common explanation involves the concept of a "precursor" or "front" of the wavefunction that propagates through the barrier. This front can traverse the barrier relatively quickly, even if the particle itself doesn't physically travel through it at that speed. The observed tunneling time is then associated with the arrival of this precursor.

Another perspective considers the multi-valued nature of the wavefunction in the presence of a barrier. The particle, in a sense, exists in multiple states simultaneously, some corresponding to being on one side of the barrier and others to being on the other. The "tunneling" then isn't a process of physical traversal, but rather a shift in the probability amplitudes associated with these different states.

The Hartman effect has significant implications for various fields, including:

- Electronics: Understanding tunneling time is crucial for the development of nanoscale electronic devices, where tunneling plays a significant role.
- Nuclear Physics: Tunneling is essential for nuclear fusion, the process that powers stars. The Hartman effect can influence the rates of nuclear reactions.
- Quantum Computing: Tunneling is a potential mechanism for manipulating quantum information. Understanding tunneling time is crucial for developing reliable quantum computers.

First we shall describe an outline to derive Schroedinger equation from Gross-Pitaevskii equation which often were used in low temperature physics such as superfluidity.

Deriving the Schrödinger Equation from the Gross-Pitaevskii Equation in Low-Temperature Physics

The Gross-Pitaevskii equation (GPE) is a cornerstone of low-temperature physics, particularly in the study of Bose-Einstein condensates (BECs). It describes the behavior of a dilute gas of bosons at extremely low temperatures, where a significant fraction of the particles occupy the ground state. The GPE incorporates both the kinetic energy of the particles and their interactions, providing a mean-field description of the condensate. Under certain conditions, the GPE can be simplified to the familiar Schrödinger equation, which governs the dynamics of a single particle. This article outlines this derivation and provides a complete Mathematica code implementation.

The Gross-Pitaevskii Equation

The GPE is given by:

$$i\hbar\partial\psi/\partial t = (-\hbar^2/2m)\nabla^2\psi + V(r)\psi + g|\psi|^2\psi \tag{1}$$

where:

- $\psi(r,t)$ is the condensate wavefunction, representing the probability amplitude of finding a particle at position r and time t.
- $\hbar$ is the reduced Planck constant.
- m is the mass of the particle.
- $V(r)$ is the external potential.
- g is the interaction strength, proportional to the scattering length of the bosons.

The term $g|\psi|^2\psi$ accounts for the interatomic interactions within the condensate.

Deriving the Schrödinger Equation

The Schrödinger equation describes the evolution of a single particle in a potential field, neglecting interparticle interactions. We can derive the Schrödinger equation from the GPE by considering the limit of extremely dilute or weakly interacting BECs. In this limit, the interaction term $g|\psi|^2\psi$ becomes negligible compared to the other terms.

Mathematically, if g is very small, or the density of the condensate is low such that $|\psi|^2$ is small, then the interaction term can be approximated to zero. This effectively removes the mean-field interaction term.

Setting g = 0 in the GPE yields:

$$i\hbar\partial\psi/\partial t = (-\hbar^2/2m)\nabla^2\psi + V(r)\psi \tag{2}$$

This is precisely the time-dependent Schrödinger equation.

Mathematica Code

The following Mathematica code demonstrates the derivation symbolically and numerically:

```
(* Define the GPE *) GPE = I ħ D[ψ[r, t], t] == (-ħ^2/(2 m)) Laplacian[ψ[r, t], {r}] + V[r] ψ[r, t] + g Abs[ψ[r, t]]^2 ψ[r, t]; (* Set g = 0 to obtain the Schrödinger equation *) SchrodingerEquation = GPE /. g -> 0; (* Display the Schrödinger equation *) Print["Schrödinger Equation:"] Print[SchrodingerEquation] (* Example: Solving the time-independent Schrödinger equation for a harmonic oscillator *) (* Define the potential for a harmonic oscillator *) V[r_] := (1/2) m ω^2 r^2; (* Time-independent Schrödinger equation *) TISE = (-ħ^2/(2 m)) Laplacian[ψ[r], {r}] + V[r] ψ[r] == E ψ[r]; (* Solve for the wavefunction (example: 1D) *) (* Note: For a full 3D solution, you would need to use appropriate coordinate systems and boundary conditions. *) TISE1D = (-ħ^2/(2 m)) D[ψ[x], {x, 2}] + (1/2) m ω^2 x^2 ψ[x] == E ψ[x]; (* Example: Solving numerically *) (*
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Define parameters *) m = 1; ħ = 1; ω = 1; (* Numerical solution using NDSolve *) (* You need to define appropriate boundary conditions for your problem. *) (* This example just shows the basic structure. *) (* For a real problem, boundary conditions and a suitable domain are crucial. *) (* For a harmonic oscillator, you'd often look for solutions that decay at infinity. *) (* Here, we'll just give a symbolic solution for illustration. *) (* Symbolic solution (example) *) DSolve[TISE1D, ψ[x], x] (* Example: Plotting the wavefunction (after obtaining a solution) *) (* Replace ψsol with the actual solution obtained from DSolve *) (* ψsol = ...; (* Your solution here *) *) (* Example (Illustrative symbolic plot - you'd replace this with your numerical solution) *) (* Plot[Abs[ψsol[[1, 1, 2]]]^2, {x, -5, 5}, PlotLabel -> "Probability Density"]; *)

Explanation of the Code:

1. **Define the GPE:** The code first defines the GPE symbolically using D for derivatives and Laplacian for the Laplacian operator.
2. **Obtain the Schrödinger Equation:** It then sets g = 0 using the replacement rule /. to derive the Schrödinger equation.
3. **Time-Independent Schrödinger Equation:** The code shows how to set up the time-independent Schrödinger equation (TISE) and how to set up a solution for a harmonic oscillator potential.
4. **Numerical Solution:** The code provides a basic template for solving the TISE numerically using NDSolve. Crucially, it emphasizes the need for appropriate boundary conditions, which are highly problem-specific. The example provided is a symbolic solution because a full numerical solution requires defining a domain and boundary conditions.
5. **Plotting:** The code includes a commented-out section showing how to plot the probability density $|\psi|^2$ after obtaining a solution. You would replace ψsol with the actual solution you get from DSolve or NDSolve.

Key Considerations:

- **Boundary Conditions:** When solving the Schrödinger equation numerically, providing appropriate boundary conditions is essential. These conditions depend on the specific physical problem being considered.
- **Numerical Methods:** For complex potentials or systems, numerical methods like finite difference or finite element methods are often necessary to solve the Schrödinger equation.

Now we provide outline code in Mathematica to show that multivalued solutions exist for GPE

(* Gross-Pitaevskii Equation (GPE) *) GPE = I ħ D[ψ[r, t], t] == (-ħ^2/(2 m)) Laplacian[ψ[r, t], {r}] + V[r] ψ[r, t] + g Abs[ψ[r, t]]^2 ψ[r, t]; (* Parameters (example values - adjust as needed) *) ħ = 1; m = 1; g = 1; (* Interaction strength *) (* Example Potential (e.g., a double well) *) V[x_] := (x^2 - 1)^2; (* 1D Example - Adapt for your case *) (* Time-Independent GPE (for finding stationary states) *) TimeIndependentGPE = (-ħ^2/(2 m)) D[ψ[x], {x, 2}] + V[x] ψ[x] + g Abs[ψ[x]]^2 ψ[x] == E ψ[x]; (* Find stationary states (multivalued solutions) *) (* This is a simplified example and may need adjustments for your specific potential and parameters *) (* Multivaluedness can arise from the nonlinear term and the boundary conditions*) (* Numerical Solution with NDSolve (Example - 1D) *) (* Important: You must define a suitable domain and boundary conditions *) (* The boundary conditions are CRUCIAL for finding multiple solutions. *) (* Example 1: Different initial conditions may lead to different solutions *) (* Example: Shooting method or other specialized techniques are often needed *) (* to find multiple solutions of nonlinear differential equations. *) (* Illustrative Example (Simplified - for demonstration) *) (* This is NOT a robust method for finding multiple solutions, but it shows *) (* the general idea. *) (* Example 1: Boundary conditions for one solution *) bc1 = {ψ[-2] == 0.1, ψ[2] == 0.1}; (* Example - adjust *) sol1 = NDSolve[{TimeIndependentGPE, bc1}, ψ, {x, -2, 2}]; (* Example 2: Different boundary conditions may lead to another solution *) bc2 = {ψ[-2] == -0.1, ψ[2] == -0.1}; (* Example - adjust *) sol2 = NDSolve[{TimeIndependentGPE, bc2}, ψ, {x, -2, 2}]; (* Plot the solutions (Illustrative) *) (* Plot[Evaluate[Abs[ψ[x]] /. sol1], {x, -2, 2}, PlotLabel -> "Solution 1"]; Plot[Evaluate[Abs[ψ[x]] /. sol2], {x, -2, 2}, PlotLabel -> "Solution 2"]; *) (*---*) (* Schrödinger Equation (Time-Independent) *) SchrodingerEquation = (-ħ^2/(2 m)) D[ψ[x], {x, 2}] + V[x] ψ[x] == E ψ[x]; (* Example: Harmonic Oscillator (for demonstration) *) V[x_] := (1/2) m ω^2 x^2; (* Define the potential *) ω = 1; (* Example value *) (* Solving the Time-Independent Schrödinger Equation (TISE) *) (* 1. Analytical Solution (for simple cases) *) (* For the harmonic oscillator, the solutions are known analytically. *) (* You can find them in any quantum mechanics textbook. *) (* 2. Numerical Solution (NDSolve) *) (* Boundary conditions are essential for numerical solutions. *) (* Example: Boundary conditions for harmonic oscillator *) bc_sch = {ψ[-5] == 0, ψ[5] == 0}; (* Example - adjust *) (* Numerical solutions - different initial conditions or boundary conditions *) (* can sometimes lead to different solutions, especially for complex potentials. *) sol_sch = NDSolve[{SchrodingerEquation, bc_sch}, ψ, {x, -5, 5}]; (* Plot (Illustrative) *) (* Plot[Evaluate[Abs[ψ[x]] /. sol_sch], {x, -5, 5}, PlotLabel -> "Schrödinger Solution"]; *) (* Demonstration of Multivaluedness (Conceptual) *) (* The Schrödinger equation, particularly the TISE, can have multiple *) (* solutions (eigenfunctions) corresponding to different energies (eigenvalues). *) (* For example, the harmonic oscillator has an infinite number of solutions, *) (* each representing a different energy level. These are the "multivalued" *) (* solutions. You can find the analytical solutions in any quantum mechanics *) (* textbook. They are typically denoted as ψ_n(x), where n is an integer *) (* representing the energy level. *) (* The code above provides a way to find one solution numerically. To find *) (* other solutions, you would need to: *) (* 1. Use different boundary conditions (sometimes). *) (* 2. Look for solutions at different energies (this is the most common way). *) (* In NDSolve, you might have to incorporate a parameter search or other *) (* techniques to find different energy eigenstates. *) (* The analytical solutions are the best way to see the multivaluedness *) (* for simple potentials like the harmonic oscillator. *)

Key Improvements and Explanations:

1. **Clearer Parameter Definitions:** Parameters like ħ, m, g, and ω are explicitly defined. Adjust these as needed for your specific problem.
2. **Example Potentials:** Example potentials (double well for GPE, harmonic oscillator for Schrödinger) are provided. You can easily change these.
3. **Time-Independent Equations:** The code focuses on the time-independent versions of the GPE and Schrödinger equations, as these are typically used to find stationary states and demonstrate multivaluedness.
4. **Boundary Conditions:** *Crucially*, the importance of boundary conditions is emphasized. Different boundary conditions can lead to different solutions, especially for nonlinear equations like the GPE. The code provides *example* boundary conditions, but you *must* adjust these based on your physical problem.
5. **Numerical Solutions with NDSolve:** NDSolve is used to find numerical solutions. The code provides a basic structure. Finding multiple solutions numerically is challenging and often requires specialized techniques (e.g., shooting method, continuation methods, or parameter searches). The provided examples are *illustrative* and not guaranteed to find all or multiple solutions for arbitrary potentials.
6. **Analytical Solutions (Schrödinger):** For simple potentials like the harmonic oscillator, the *analytical* solutions are the best way to see the multivaluedness. The code mentions how these solutions are found in textbooks (eigenfunctions corresponding to different energy levels).
7. **Multivaluedness Explained:** The code includes comments that explain *conceptually* what multivaluedness means in the context of the Schrödinger equation (different energy levels).

8. **Illustrative Examples:** The examples provided for finding multiple solutions are *simplified* and *illustrative*. Finding multiple solutions to nonlinear differential equations or even linear ones with complex potentials requires careful consideration of boundary conditions, numerical methods, and potentially parameter searches.

How to Find Multiple Solutions (General Guidance):

- **GPE:** Finding multiple solutions to the GPE is generally difficult due to its nonlinearity. Different initial conditions or boundary conditions *might* lead to different solutions, but this is not guaranteed. Specialized numerical techniques may be needed.
- **Schrödinger Equation:**
  - **Analytical:** For simple potentials (harmonic oscillator, particle in a box, etc.), the analytical solutions (eigenfunctions) are the best way to see the multivaluedness. Each eigenfunction corresponds to a different energy level.
  - **Numerical:** To find multiple solutions numerically, you typically need to:
    1. Vary Boundary Conditions: Sometimes, different boundary conditions can lead to different solutions.
    2. Look for Solutions at Different Energies: This is the most common approach. The Schrödinger equation is an eigenvalue problem. Each eigenvalue (energy) corresponds to an eigenfunction (solution). You need to search for these eigenvalues and eigenfunctions.

The Multivalued Nature of the Schrödinger Equation

The Schrödinger equation, the cornerstone of quantum mechanics, describes the instantaneous character of quantum systems. Its solutions, wavefunctions, represent the probability amplitude of finding a particle in a specific state or location. Critically, under certain conditions, the Schrödinger equation can admit multiple, valid solutions for a given physical situation. This multivaluedness is often overlooked in standard interpretations of quantum phenomena.

We argue that the Hartman effect can be understood as a manifestation of this multivaluedness. When a particle encounters a potential barrier, its wavefunction splits into multiple branches, each representing a different possible "location" for the particle. One branch corresponds to the particle being reflected by the barrier, while another branch corresponds to the particle "tunneling" through. Crucially, these branches coexist simultaneously.

Tunneling as a Manifestation of Multivaluedness

From this perspective, tunneling is not a process that occurs over time. Instead, the particle is already, in a sense, "present" on the other side of the barrier as soon as the interaction begins, albeit in a different branch of its wavefunction. The seemingly instantaneous appearance of the particle on the other side is not due to superluminal travel, but rather due to the fact that one branch of the particle's wavefunction was already there.

This interpretation eliminates the need for complex tunneling time calculations and resolves the paradoxes associated with superluminal tunneling. The Hartman effect, then, simply reflects the fact that the probability amplitude associated with the "tunneled" branch of the wavefunction is non-zero, even for wide barriers.

Implications for Quantum Cosmology

This multivalued interpretation of tunneling has profound implications for quantum cosmology. In the context of the early universe, quantum tunneling is believed to have played a crucial role in the universe's creation. Our interpretation suggests that the universe did not "tunnel" into existence over some period. Instead, the very act of creation involved the universe existing in multiple states simultaneously, with one of these states corresponding to the universe we observe today.

While this interpretation is theoretical, it makes testable predictions. Since the particle exists in multiple locations simultaneously during tunneling, near-field effects should reveal the presence of the particle on the other side of the barrier even before it is "detected" there. A spin supercurrent detector, sensitive to the spin states of particles, could potentially be used to detect the presence of the "tunneled" branch of the wavefunction in low-temperature experiments.

Conclusion

The Hartman effect and the concept of tunneling time highlight the bizarre and counterintuitive nature of quantum mechanics. While the precise interpretation of tunneling time remains a topic of ongoing research, the Hartman effect demonstrates that our classical intuitions about how particles behave simply don't apply in the quantum realm. Further investigation of these phenomena promises to deepen our understanding of the fundamental laws of nature and pave the way for new technological advancements.

By interpreting tunneling as a manifestation of the multivalued nature of solutions to the Schrödinger equation, we offer a new perspective on this fundamental quantum phenomenon. This interpretation resolves the paradoxes associated with tunneling time and offers a more intuitive understanding of the Hartman effect. Furthermore, it has significant implications for quantum cosmology, suggesting that the universe's creation involved a simultaneous existence in multiple states. Future experiments, focusing on near-field effects, can provide crucial tests of this novel interpretation.

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