TOWARDS A NEW LAW OF GRAVITATION: From Newton's verbal formula to a new formula for the law of universal gravitation.

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Abstract. The evolution of the law of universal gravitation is shown from the verbal formula proposed by Newton in 1687 to the present day. The law discovered by Newton $F_N \propto mM/r^2$ was not the exact law of gravitation. Newton only indicated the proportional dependence of force on masses and did not attribute any numerical value to the gravitational force. In an unfinished form, Newton's law $F_N \propto mM/r^2$ existed for almost 200 years (!) until the constant G appeared in it. But even in its modern formulation, Newton's law $F = GMm/r^2$ gives only a part of the force of universal gravitation. Newton's law does not work at large distances. Newton's law is not applicable on the scale of the Universe. The formula for Newton's law shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. The formula for the law describes gravitation only to one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies. For more than 300 years, it has not been possible to derive a formula for the law of gravitation that would describe the gravitation to all bodies in the Universe. An obstacle on this path is the unsolved gravitational problem of N bodies. Here we provide a solution to this problem. To obtain a complete law of universal gravitation, a reduction of the differential problem of N bodies to an integral problem of a system of N bodies is used. An approach is used in which, instead of individual N bodies, the system of N bodies is studied as a whole object. The solution of the integral problem for the system of N bodies leads to a new law of gravitational force: $F_{Cos}=(mc^2)\sqrt{\Lambda}$. This is the part of the gravitational force that Newton's law "does not see". This is the part of the gravitational force with which bodies simultaneously gravitate to all other bodies. This is the part of the gravitational force that gives a new formula for the law of universal gravitation. Based on the additivity of gravitational forces, a new formula for the law of universal gravitation was obtained in the form $Fu = GMm/r^2 + (mc^2)\sqrt{\Lambda}$. The new formula for the law of universal gravitation provides a complete and consistent description of the gravitational interaction in the Universe. As a result, many problems of cosmology and astrophysics are solved, including the problem of the cosmological constant Λ and the problem of dark matter.

Keywords: Newton's law; N-body problem; law of universal gravitation; parameters of the observable universe; dark matter; galaxy rotation curve; cosmological constant Λ; Pioneer anomaly.

1. Introduction

Newton formulated his law of gravity in 1687 in verbal form. It was not an exact law of gravity. Newton only indicated the proportional dependence of force on masses and did not attribute any numerical value to the gravitational force. The symbolic form of the original law proposed by Newton is as follows:

$$F_N \propto \frac{mM}{r^2} \qquad (1)$$

Where: F_N is the gravitational force, m, M are the masses of bodies, r is the distance, ∝ is the proportionality sign.

As we can see, Newton did not present the world with an exact formula for the law of gravitation. In this unfinished form, Newton's law existed for almost 200 years. It turned into the exact equality $F_N = GmM/r^2$ only after the gravitational constant G was introduced into it [1 - 6]. This happened in several stages (Fig. 1).

J. A. M. Pereira in [1] showed the history of the appearance of the constant G. In 1803, S. D. Poisson presented the formula for the law of universal gravitation $F = f\,mM/r^2$ with the coefficient f, which later became the constant G [1, 2]. In 1873, A. Cornu and J. B. Baille also used the symbol (f) for the coupling constant in Newton's law of gravitation (1 - 4). The law of universal gravitation in the form familiar to us $F = GmM/r^2$ was presented by A. K¨onig, F. Richarz, [5, 6], J. H. Poynting [1, 7].

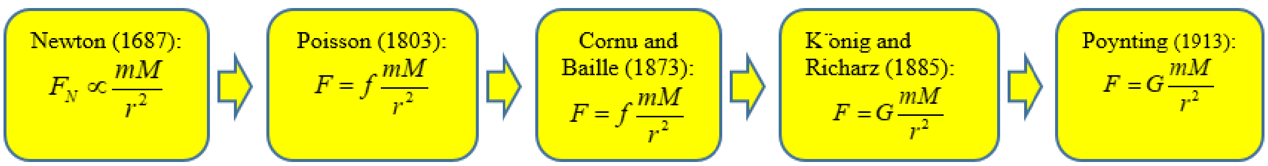

Fig.1 Step-by-step refinement of Newton's law of universal gravitation.

But even the refined Newton's law $F = GMm/r^2$ gives only a part of the force of universal gravitation. Newton's law shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. The formula for Newton's law $F = GMm/r^2$ "*does not see*" a significant part of the gravitational force. It describes gravitation only to one local source of attraction "M" and does not take into account that bodies simultaneously gravitate to all other bodies. For more than 300 years, it has not been possible to derive a formula for the law of gravitation that should describe gravitation to all N bodies in the Universe. The incompleteness of Newton's law of universal gravitation is indicated by significant discrepancies between observations and the predictions of the law. This applies to the shift in the perihelion of planets, to the rotation curves of galaxies, to the Pioneer anomaly [8 - 12]. The revealed gravitational anomalies in the dynamics of stars show that at large distances Newton's law is not fulfilled to a significant extent and has significant discrepancies with observations [10, 11].

The term "*law of universal gravitation*" implies taking into account all acting gravitational forces. Newton's law $F = GMm/r^2$ "does not see" all acting gravitational forces. A law of gravitation is needed that should show not only the force of gravitational interaction of two bodies, but also describe the attraction to all N bodies in the Universe. Such a law of gravitation should be sought beyond the framework of Newtonian dynamics.

2. On the necessity of transition from differential problem of N bodies to integral problem of system of N bodies.

The refined Newton's law $F = GMm/r^2$ does not reflect the realities of gravity in the Universe. Newton's law does not work at large distances. The formula of the law includes two point masses "M" and "m". This law is applicable for gravitational interaction of two bodies. It describes attraction only to one local source of attraction. The real picture of gravity is different. Bodies simultaneously gravitate to all other bodies, and not only to one local source of attraction. There are a huge number of bodies in the Universe. But their parameters are not included in the formula $F = GMm/r^2$.

Newton's law gives a good approximation for the gravitational problem of two bodies. Even if it were possible to solve the problem of 3, 4, 5, etc. bodies, the description of gravity will still be incomplete until the gravitational action of the entire Universe is taken into account.

It is known that the N-body problem has no analytical solution [13]. As applied to the Universe, the value of N is very large. Even numerical methods for solving large N are powerless. The absence of an analytical solution to the N-body problem has always been an argument to justify the impossibility of obtaining an equation for the gravitational force that takes into account the gravitational force of a huge number of bodies in the Universe. The N-body problem became an insurmountable obstacle to the discovery of the law of gravitational force for all N bodies in the Universe.

Newton was also convinced of the impossibility of obtaining an exact law of gravity that takes into account the gravitational action of many bodies. This is indicated by the text from Newton's manuscript of 1684 [14]: "*...the planets neither move exactly in ellipses nor revolve twice in the same orbit. Each time a planet revolves it traces a fresh orbit, as in the motion of the Moon, and each orbit depends on the combined motions of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind*."

The N-body gravitational problem is given in a differential formulation. It is formulated for trajectories. Its direct application to Newton's law requires an additive representation of the total force as a sum of N forces. This runs into the insurmountable problem of "*bad infinity*" ("die Schlecht-Unendliche") [15]. The differential formulation of the N-body problem played a destructive role in solving the problems of gravitation.

3. The integral problem of N bodies as a reduction of the differential problem of N bodies to the problem of two bodies.

To find the gravitational force that takes into account the action of N bodies, there is no need to solve the differential problem of N bodies. It is proposed to obtain the equation of the gravitational force for a system of N bodies instead of searching for a direct solution to the differential problem of N bodies. To do this, it is necessary to move on to solving the integral problem for the system of N bodies. As a result, the differential problem of N bodies admits a reduction to the problem of two bodies, in which the central body is represented by a system consisting of (N-1) bodies.

The system of N bodies in the integral gravitational problem is considered as the Universe with known parameters. As a result, the problem of obtaining the equation of force from the action of N bodies has a solution even for $N \to \infty$. The solution scheme is as follows:

1. N individual bodies are considered as a single object - as a system of N bodies.
2. The reduction of the problem of N bodies to the problem of two bodies is realized.
3. The N-body system is the Universe with known parameters.
4. Selecting an integral parameter for the N-body system.

5. Obtaining an acceleration formula for the central body represented by the N-body system.
6. Obtaining an equation for a new law of gravity.
7. Obtaining a new formula for the law of universal gravitation.

4. The law of cosmological force.

The transition from the differential problem of N bodies to the integral problem of the system of N bodies provides a solution to the problem of gravitational interaction of N bodies. In the integral problem of N bodies, a holistic object is considered - the Universe, consisting of N bodies. Thus, the N-body problem is reduced to the two-body problem, in which the central body is represented by a system consisting of (N-1) bodies. It remains to solve the problem of choosing a parameter that uniquely characterizes the system of N bodies. For the Universe, we can accept the condition: (N-1) ≈ N. The Universe, as a holistic object, has such an integral parameter - this is the cosmological constant Λ. This constant is an integral characteristic of the system of all bodies in the Universe. The next step is to find the acceleration for the central body, represented by the system of N bodies. The constant Λ allows us to represent the cosmological acceleration Ao, which is caused by all bodies in the Universe, in the form:

$$\mathbf{A_0 = c^2\sqrt{\Lambda}} \qquad (2)$$

Accordingly, the gravitational force with which the system of N bodies acts on a test body of mass "m" is expressed by the formula $F_{Cos} = mA_0 = (mc^2)\sqrt{\Lambda}$. Fig. 2 shows the law of cosmological force.

$$F_{Cos} = mc^2\sqrt{\Lambda}$$

Fig. 2. The law of cosmological force. Where: $\mathbf{F_{Cos}}$ is the cosmological force, **m** is the mass of the body, **c** is the speed of light in vacuum, **Λ** is the cosmological constant.

The unknown cosmological force is due to the gravitational effect of all bodies in the Universe on a test body of mass "m". Since all bodies in the Universe are distributed in space, the formula for the law of cosmological force does not include the inverse square law. This new law of gravitation reveals an unknown cosmological force that acts on any body in the Universe. This is the part of the gravitational force that Newton's law "does not see". This is the part of the gravitational force with which bodies simultaneously gravitate toward all other bodies. This is the part of the gravitational force that makes the law of universal gravitation complete.

Instead of the gravitational constant G, the law of cosmological force contains the cosmological constant Λ. The new law of gravity shows that any body of mass m experiences a cosmological force proportional to the mass of the body and the cosmological constant Λ. The law of the cosmological force operates beyond the applicability of Newton's law of gravity. It applies to the gravitational interaction in the Universe. The cosmological force has a linear dependence on the mass of the body and does not obey the inverse square law.

On small scales, the additional cosmological force is much smaller than the Newtonian force. For example, on Earth, F_{Cos} is ~10^10 times lessthan the Newtonian force. As we can see, within the solar system, Newton's law of gravity has a highaccuracy. At small distances, the main part of the universal gravitational force is the Newtonian force.

On the scale of the Universe, the cosmological force is enormous. At large distances, it exceeds the Newtonian force. At large distances, the main part of the force of universal gravitation is the cosmological force F_{Cos}.

The study of the equation of the new law of the cosmological force shows that the value of the cosmological force in the limit is equal to the Planck force:

$$\lim_{m \to M_U} F_{Cos} = \lim_{m \to M_U} mc^2 \sqrt{\Lambda} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \qquad (3)$$

The theoretical limit of the cosmological force at m → M_U reaches the enormous value c^4/G = 1.21027 x 10^{44} N.

5. New formula for the law of universal gravitation.

The completed law of universal gravitation can be represented as a combination of the force of gravitational interaction of two bodies and the cosmological force. Based on the additivity of gravitational forces, the new formula for the law of universal gravitation (Fig. 3) combines Newton's law $F = GMm/r^2$ and the law of cosmological force F_{Cos}=(mc^2)√Λ.

$$F_U = G\frac{mM}{r^2} + mc^2\sqrt{\Lambda}$$

Fig. 3. New formula of the law of universal gravitation.

The force of universal gravitation is represented by the vector sum of two forces: the Newtonian gravitational force F_N and the cosmological gravitational force F_{Cos}:

$$\vec{F}_U = \vec{F}_N + \vec{F}_{Cos} \qquad (4)$$

The value of the resulting force of universal gravitation is in the range of values from F_U = GmM/r^2 - (mc^2)√Λ to F_U = GmM/r^2 + (mc^2)√Λ (Fig. 4).

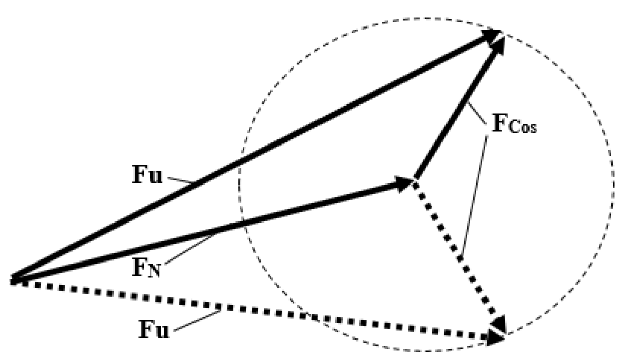

Fig. 4. The force of universal gravitation F_U as a vector sum of two forces: F_N and F_{Cos}.

Fig. 5 shows the long path from Newton's law of universal gravitation to the completed law of universal gravitation.

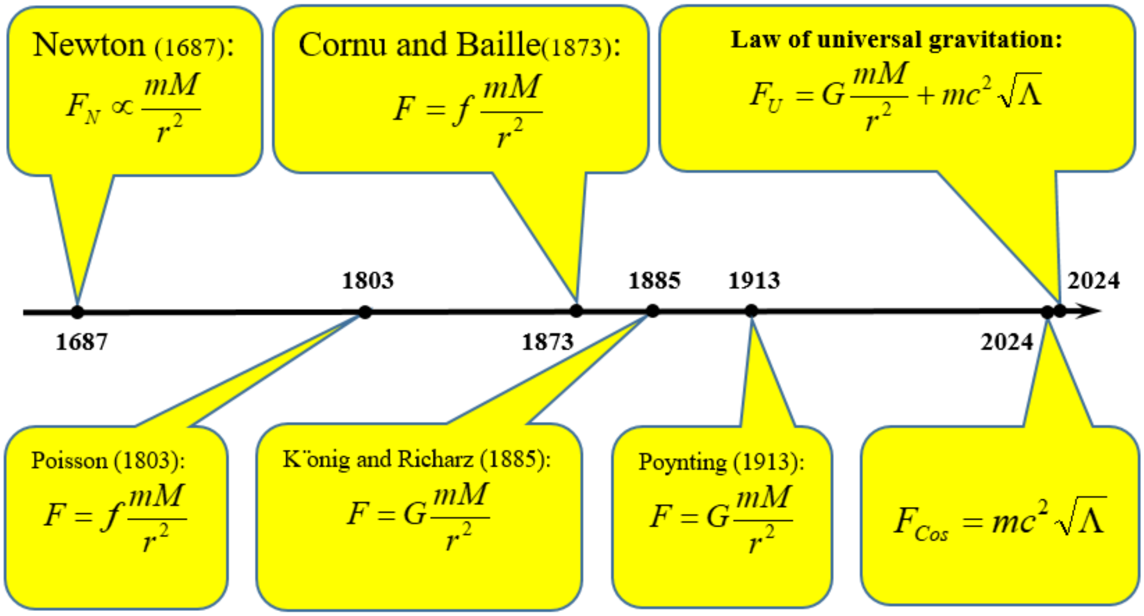

Fig. 5. Evolution of the law of universal gravitation from Newton's verbal formula to a new formula for the law of universal gravitation.

6. Conclusion.

The law of universal gravitation has come a long way from Newton's verbal formula to a complete formula for the law of universal gravitation. The intermediate formula $F = GMm/r^2$ is quite accurate on the scale of the solar system. But it is not applicable on the scale of the Universe. The formula for Newton's law $F = GMm/r^2$ shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. The formula for the law describes gravitation only to one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies. The differential problem of N bodies created the illusion of the impossibility of obtaining an exact law of gravitational force that describes gravitation to all bodies in the Universe. It is shown here that in order to find the gravitational force that takes into account the action of all bodies in the Universe, there is no need to solve the differential N-body problem. The total gravitational force can be obtained by reducing the N-body problem to a two-body problem, in which the central body is represented by a system of (N-1) bodies.

The transition from the differential N-body problem to the integral problem for the N-body system leads to a new law of gravitational force: F_{Cos}=(mc^2)√Λ. The complete law of universal gravitation combines Newton's law $F = GMm/r^2$ and the law of cosmological force F_{Cos}=(mc^2)√Λ. Newton's law is included as a component in the complete law of universal gravitation. The new formula for the law of universal gravitation gives a complete and consistent description of gravitational interaction in the Universe. As a result, many problems of cosmology are solved, including the problem of the cosmological constant Λ and the problem of dark matter [16].

7. Conclusions

1. The law discovered by Newton FN ∝ mM/r^2 was not the exact law of gravity. Newton only pointed out the proportionality and did not attribute any numerical value to the gravitational force.

2. Newton's law $F = GMm/r^2$, refined almost 200 years later, does not give the full force of universal gravitation. The law shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. It describes the attraction to only one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies.

3. The unsolved N-body problem gave rise to the illusion of the impossibility of obtaining an exact law of gravitational force that describes the attraction to all N bodies in the Universe.

4. The complexity of solving the N-body problem as applied to gravity is too exaggerated. To find the gravitational force that takes into account the action of N bodies, there is no need to solve the N-body differential problem. To do this, we must move on to solving the integral problem, in which N individual bodies are replaced by one system consisting of N bodies. The differential problem of N bodies can be reduced to the problem of two bodies, in which the central body is represented by a system of bodies consisting of (N-1) bodies of the Universe.

5. The integral parameter of the system of N bodies as applied to the Universe is the cosmological constant Λ. This constant is a characteristic of the system that includes all the bodies of the Universe.

6. The integral problem for the system of N bodies leads to a new law of gravitational force: $F_{Cos}=mc^2\sqrt{\Lambda}$. This is the part of the gravitational force that Newton's law "does not see". This is the part of the gravitational force with which bodies simultaneously gravitate toward all other bodies. This is the part of the gravitational force that makes the law of universal gravitation complete.

7. The law of the cosmological force $F_{Cos}=mc^2\sqrt{\Lambda}$ shows that any body of mass m is acted upon by a cosmological force proportional to the mass of the body and the cosmological constant Λ. The cosmological force is linearly dependent on the mass of the body and does not obey the inverse square law.

8. Based on the additivity of gravitational forces, the complete law of universal gravitation combines Newton's law $F = GMm/r^2$ and the law of the cosmological force $F_{Cos}=mc^2\sqrt{\Lambda}$. The formula for the complete law of universal gravitation $F_U = GMm/r^2+mc^2\sqrt{\Lambda}$ includes two constants: the gravitational constant G and the cosmological constant Λ.

9. On small scales, the additional cosmological force is much less than the Newtonian force. On small scales, the Newtonian force makes up the bulk of the force of universal gravitation. On large scales (galaxies...) the cosmological force exceeds the Newtonian force. On large scales the main part of the universal gravitational force is the cosmological force F_{Cos}.

10. The complete law of universal gravitation $F_U = GMm/r^2+mc^2\sqrt{\Lambda}$ gives a complete and consistent description of the gravitational interaction in the Universe. As a result, many problems of cosmology are solved, including the problem of the cosmological constant Λ and the problem of dark matter.

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