Title: Simulating Free Fall Motion Using Excel A User-Friendly Alternative For Public Accessibility

Authors: Yi Sheng Ng, Zi-Jing Chiah

Abstract: This paper explores the use of Microsoft Excel as a simple and accessible platform for simulating free-fall motion, providing an alternative to programming-intensive tools like MATLAB and C++. The aim is to demonstrate Excel's potential for numerical simulations, particularly for users with limited programming experience. The first section introduces the basics of Excel, covering data entry and plotting techniques, which are essential for setting up and visualizing simulations. This tutorial approach makes the process accessible to a wider audience, including students and educators.The study derives the governing equations of motion by rewriting Newton's second law, F=ma, into differential equation form. These equations are solved iteratively using Excel's built-in formulas and functions. The paper presents three case studies to illustrate the methodology: (1) free fall without air resistance, (2) free fall with air resistance, and (3) free fall in a viscous fluid. For each case, numerical solutions are computed step-by-step, and the results are displayed through graphs generated directly in Excel.The simulated results show excellent agreement with theoretical predictions, validating Excel's accuracy as a tool for physics simulations. The analysis reveals that viscous resistance in fluids significantly slows down motion compared to air resistance, as demonstrated by the velocity profiles for each case. This finding highlights the importance of considering the medium's properties when analyzing free-fall motion.By leveraging Excel's intuitive interface and accessibility, this study provides a practical approach for conducting numerical simulations without requiring advanced programming knowledge. The results demonstrate that Excel is not only capable of handling complex numerical problems but also offers an easier learning curve for beginners. This makes it a valuable tool for educational purposes and for individuals or institutions with limited access to specialized software. The inclusion of detailed tutorials ensures that users can replicate and build upon the simulations presented in this work, making physics simulations more accessible to the public.

Title

Excel Spreadsheet & Simulation of Falling Motion

Objectives

Learn how to:

- create a data table and enter data,
- perform calculations on the data,
- format cells,
- plot data on a graph.

Background theory

Experiment 1a.

1. Data Entry

   Open a new file in Excel. You will see a blank page (called spreadsheet) with a lot of little boxes (cells) as illustrated in Figure 1. The default filename will be set to **Book1** as shown in the upper left hand corner. Change to any preference but logic and understood file name when you safe the excel file. Label the column accordingly, e.g. column A: Distance (m) and column B: Time (s). Enter the experimental data directly into the spreadsheet as illustrated in Figure 1.

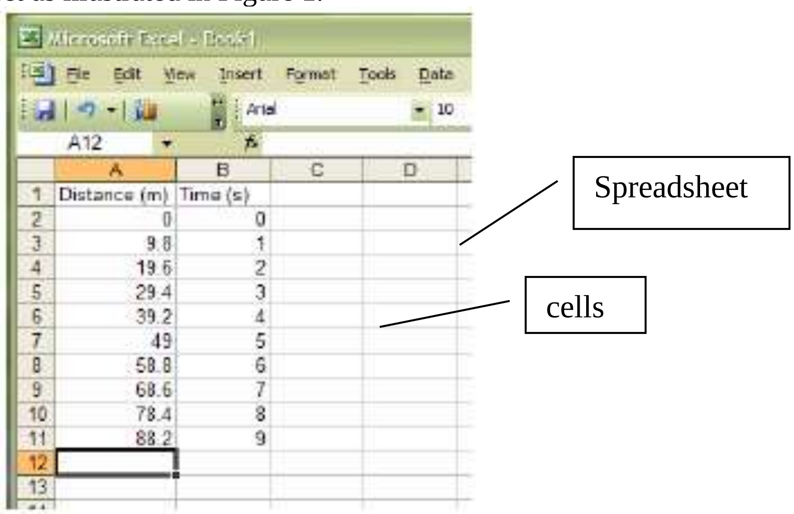


Figure 1. Example of spreadsheet

2. Calculations

   You may perform calculations in Excel. The table below lists the common arithmetic operations.

Operation	Symbol	Example
Addition	+	=2+10 (ans:12)
Multiplication	*	=2*10 (ans:20)
Subtraction	-	=10-2 (ans:8)
Division	/	=10/2 (ans:5)
Power	^	=10^2 (ans:100)

   For example, calculate velocity from distance and time. Label a third column, velocity. Enter a function by the formula, velocity=distance/time. In excel, you would put "**=A2/B2**" in the column C2 as shown in Figure 2. You may check the function you enter at the formula bar.

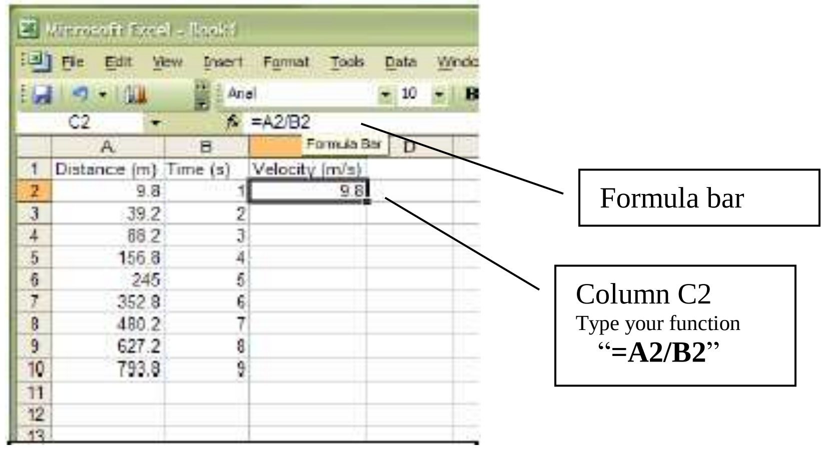


Figure 2. Calculation of velocity

Complete by created similar function in remaining cells C3 to C10. By putting your cursor over the lower right hand corner of C2, a small black cross will appear. Drag the cross at the corner of the box down until C10. The cells passed will become highlighted (refer Fig. 3). Double check the formula by click on the column, e.g. you will see “**=A10/B10**” in the formula bar when you click on column C10.

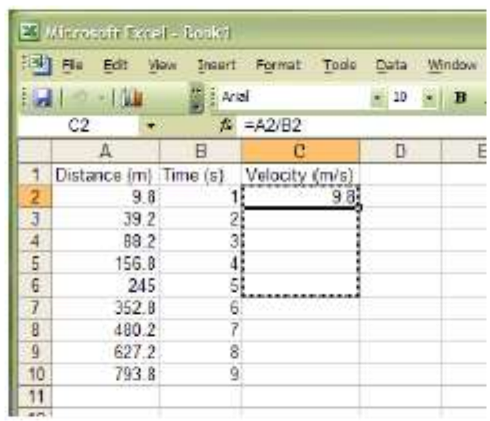


Figure 3. Formulate in excel

3. Plotting

For the most general graph plotting, select the cells that contain the data you want to use for your chart. Click **Chart Wizard**  (or from worksheet menu bar, select **Insert**> **Chart Wizard**). And follow the instructions in the Chart Wizard.

(a) Selecting data and Chart Type

Select the data for plot as shown in Figure 4.

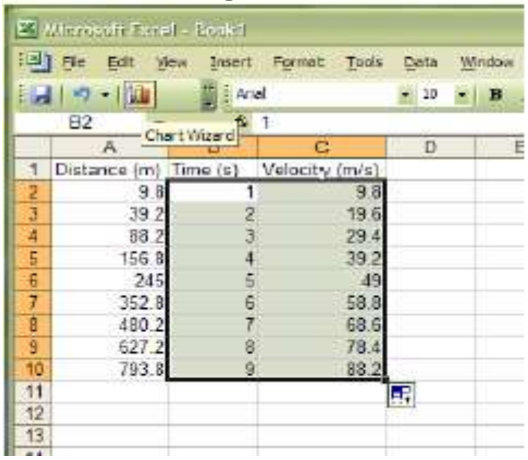


Figure 4. Select the data for ploting

Click on the shortcut  on excel, a Chart Wizard window will pop up. Select **XY (Scatter)** in the list of chart types on the left as illustrated in Figure 5. Click **Next** to proceed next step.

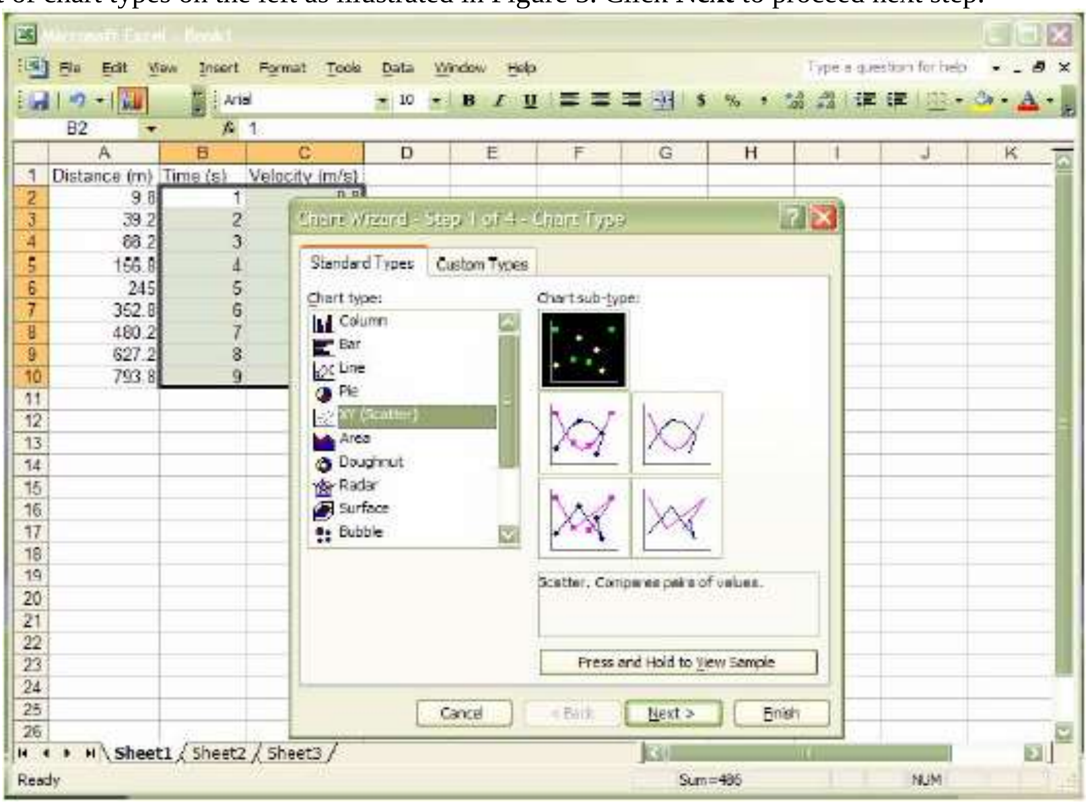


Figure 5. Select the chart type

(b) Defining axes:

The Chart Wizard has two tabs, **Data Range** and **Series.** Click on the **Series** tab as illustrated in Figure 6.

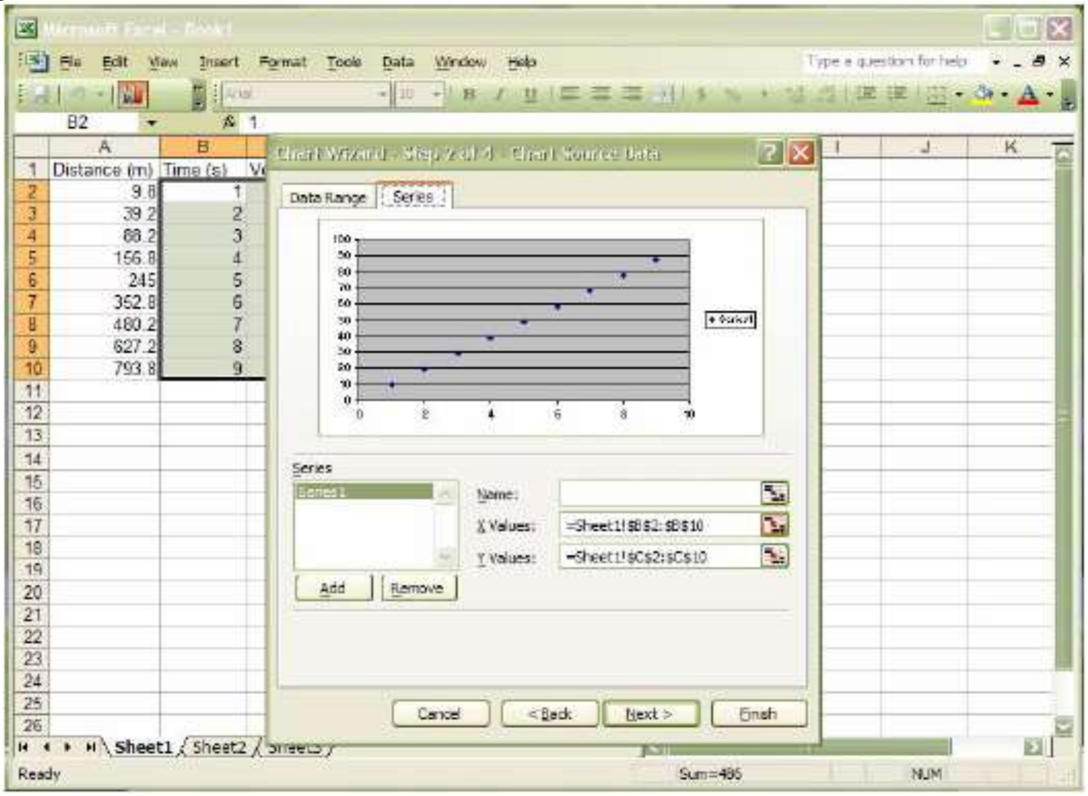


Figure 6. Data Range and Series in excel

The **X values** and **Y values** represent your X and Y data respectively. E.g. '**=Sheet1!$A$2:$A$11**". The letter indicates the column and rows of your data. **Sheet1** represent wordsheet1, the X values are in **A2 to A11**. You can change the X data by edit the letter or click the right shortcut beside the **X values** column. Select the data for X values and click on the button on the right side of the Source Data window as shown in Figure 7. Similar procedures for Y values, and click **Next** to proceed next step.

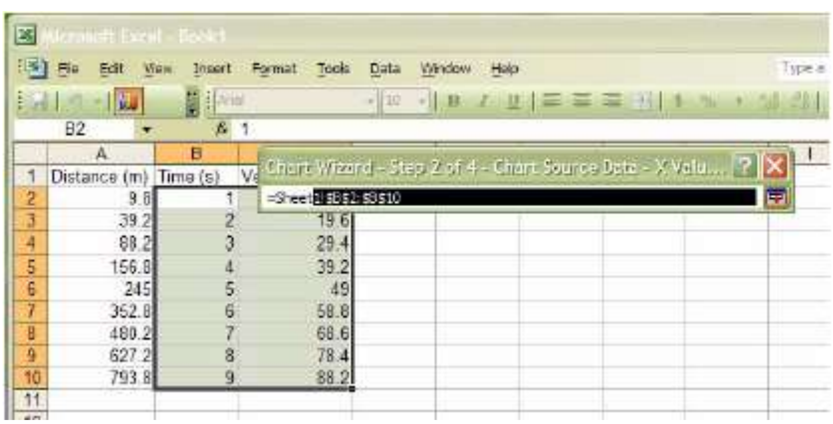


Figure 7. Chart source data

(c) Labeling chart

As illustrated in Figure 8, you can label and customize **Titles, Axes, Gridline, Legend** and **Data Labels** in **Chart options.** Fill up all information needed in graph and click **Next** for final step.

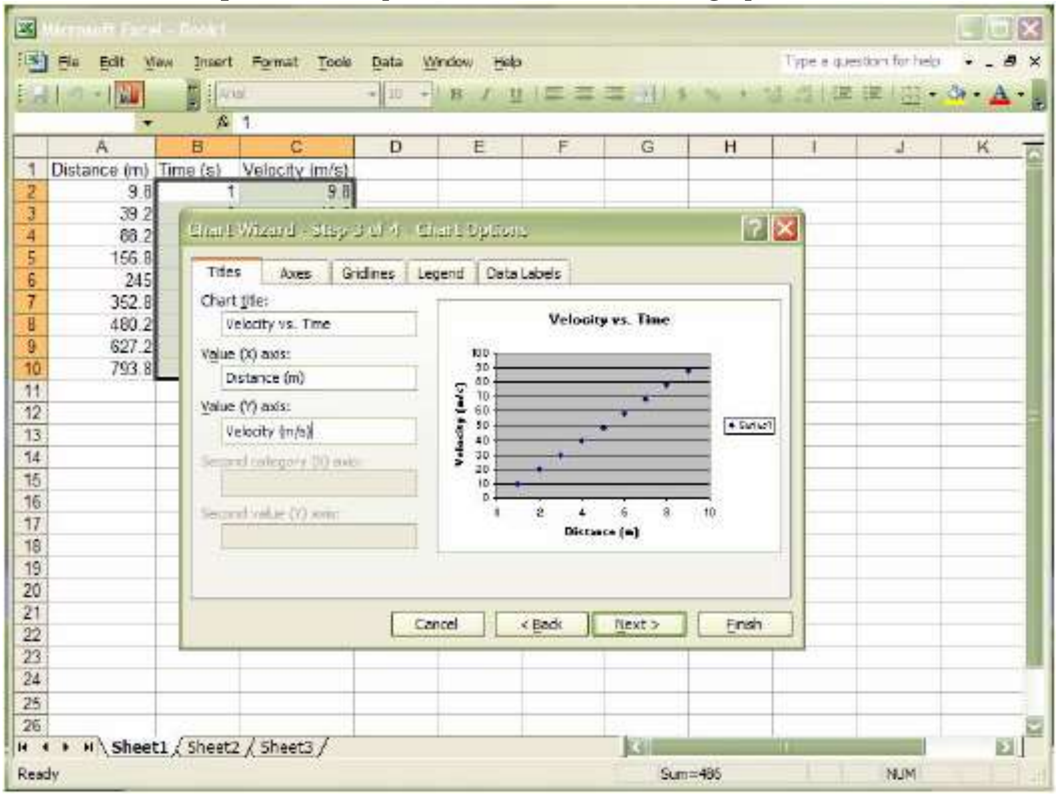


Figure 8. Labeling the chart

(d) Finishing

The final step is **chart location**; select the location of the created chart (default as **object in Sheet 1**) or in separate sheet (as **chart in Chart 1**). Click **Finish** and done the graph plotting.

4. Putting a fit line on your graph:
   Curve fitting is the process of constructing a curve, or mathematical function that has the **best fit** to a series of data points, possibly subject to constraints.

   (a) Create and choose type of fitting

   Right click on any data point in your graph, and select **Add Trendline** from the list that appears as shown in Figure 9 (or from worksheet menu bar, select **Chart**> **Add Trendline**). Choose a type of fitting suitable for your data as illustrated in Figure 10. (Choose linear if you want to make a straight line)

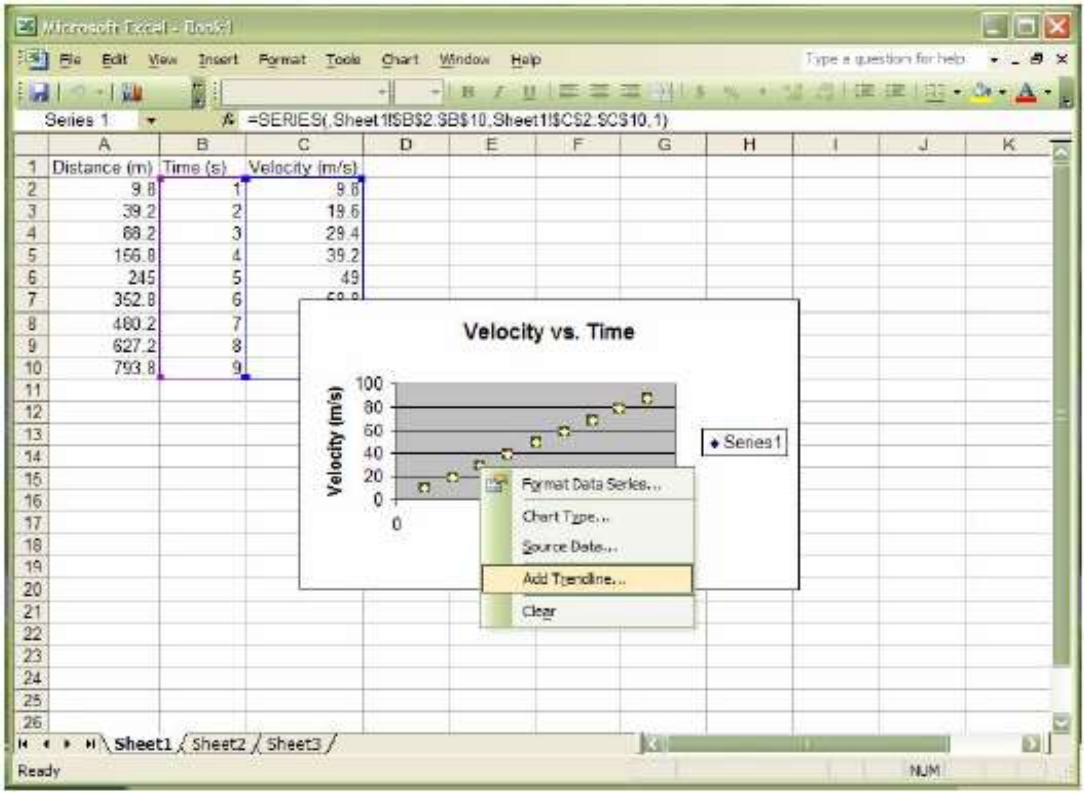


Figure 9.Adding trendline

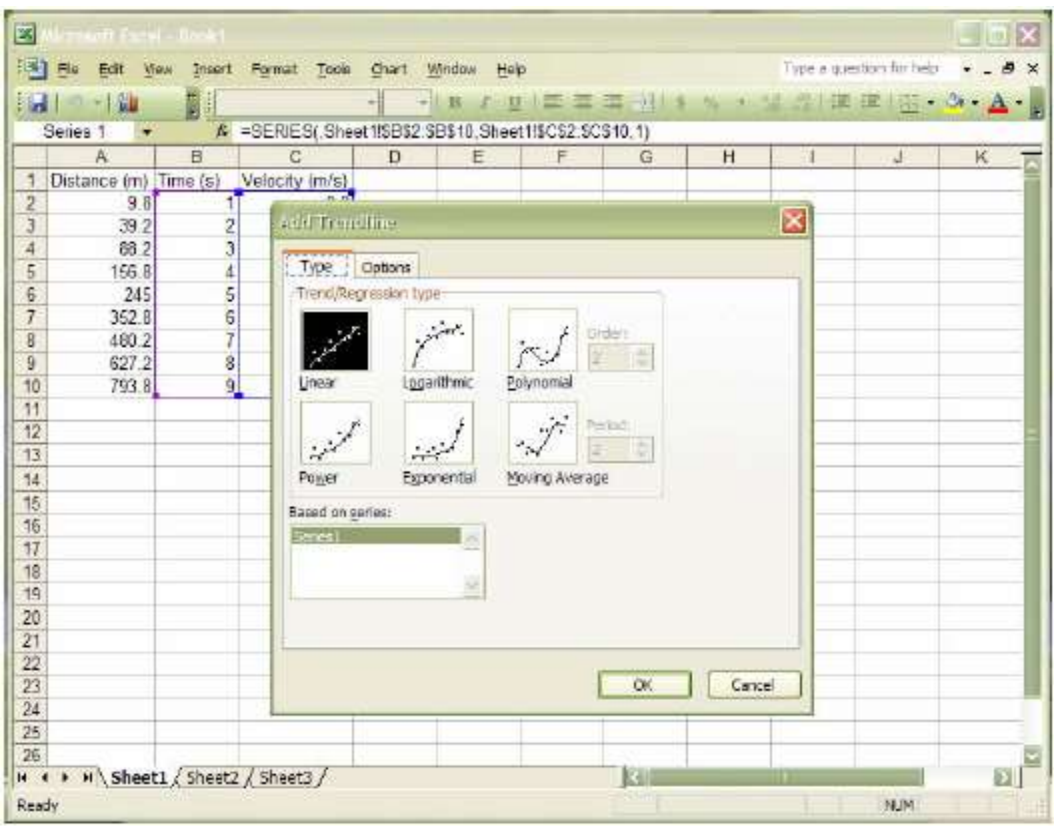


Figure 10.Choose the type of fitting

(b) Display the fitting equation and error

Click on the **options** tab to customize the display of fitting equation and error. Check the boxes to **display the equation** and **R-squared value** on the chart as shown in Figure 11. **R squared** is a statistical measure of how well a regression line approximates real data points. The box of intercept and set to 0 to force the intercept through 0. Click **Ok** and you should see the equation and R-squared value as illustrated in Figure 12. Delete the unwanted legend and arrange the graph in more appropriate form.

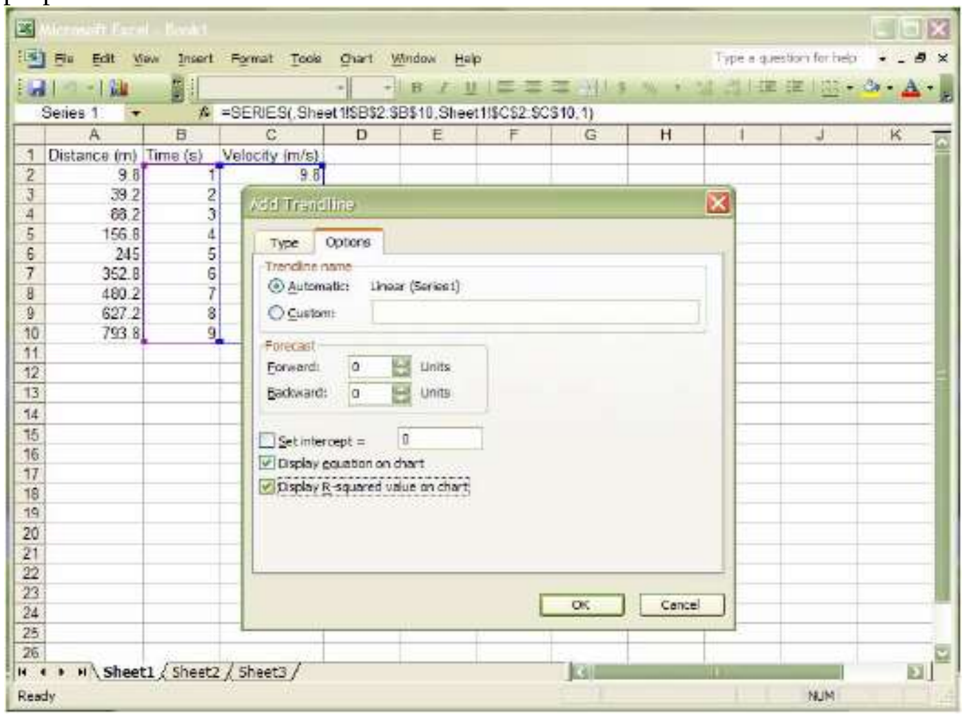


Figure 11.Display fitting equation and R-square

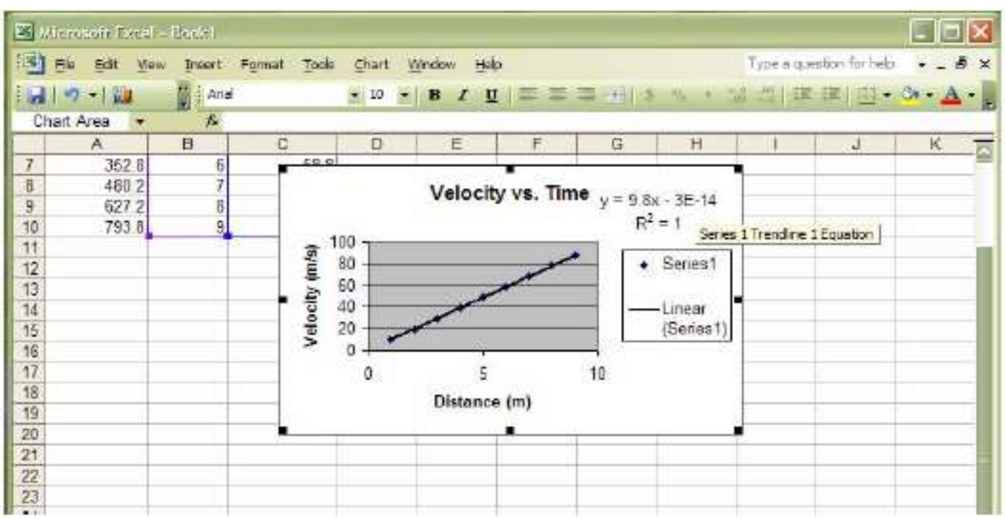


Figure 12. Partially complete graph

Experiment 1b.

Newton's Second Law is usually written as $F = ma$. It is more useful if expressed as a differential equation:

$$m\frac{dv}{dt} = F_{net}$$

where F_{net} is the net force acting on an object of mass m. We usually integrate the above equation with respect to t to get a solution for v, provided F_{net} is simple enough. This can be done only for the simple cases, for example, if F_{net} = constant. But if F_{net} depends on the velocity of the object itself, that is, $F_{net} = F_{net}(v)$ (as in the case of an object falling with air resistance), it is generally difficult to integrate the equation analytically by hand. Computer programming provides easier alternative to solve the problem.

Rewrite the differential equation as

$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{F_{net}(v)}{m}.$$

where F_{net} is velocity dependent, $\Delta v = v_{n+1} - v_n$ and $\Delta t = t_{n+1} - t_n$ to discretize v and t. In other words, we represent v and t as a series of numbers (v_0, v_1, v_2, …, v_n, ….) and (t_0, t_1, t_2, …, t_n, …). The solution of the differential equation corresponds to the pairs of values (v_0, t_0), (v_1, t_1), (v_2, t_2), …, (v_n, t_n). In general, we know the values of v_0 and the times t_0, t_1, t_2 … where v_0 is known as the initial velocity. From v_0, we would like to find the rest of the velocity values v_1, …, v_n by solving the discretized equation.

In discretized form, the differential equation becomes

$$v_n - v_{n-1} = \frac{F_{net}(v_{n-1})}{m}\Delta t, \qquad n = 1,\ 2,\ 3, \ldots$$

or

$$v_n = v_{n-1} + \frac{F_{net}(v_{n-1})}{m}\Delta t, \qquad n = 1,\ 2,\ 3, \ldots$$

We use this discrete equation to solve v by the method of iteration. For n = 1, we get

$$v_1 = v_0 + \frac{F_{net}(v_0)}{m}\Delta t$$

Hence, we get the value of v_1 if we know the value of v_0, Δt and $\frac{F_{net}(v_0)}{m}$. For n = 2, we get

$$v_2 = v_1 + \frac{F_{net}(v_1)}{m}\Delta t$$

Since we know the values on the right-hand-side, we can get a value for v_2. In other words, we can rewrite v_2 in terms of the known quantities:

$$v_2 = v_1 + \frac{F_{net}(v_1)}{m}\Delta t = (v_0 + \frac{F_{net}(v_0)}{m}\Delta t) + \frac{F_{net}(v_1)}{m}\Delta t$$

where

$$F_{net}(v_1) = F_{net}\left(v_0 + \frac{F_{net}(v_0)}{m}\Delta t\right)$$

Similarly, we can get the value of v_n by iterating the discretized equation n times.

In this lab, we simulated the velocity of a falling object for two cases.

Case 1: Object falling without air resistance

This is a trivial case which can be solved by hand easily. Nevertheless, we use computer program to make sure that it is working correctly. The equation of motion is

$$\frac{dv}{dt} = g$$

where $F_{net} = mg$. The discretized equation is

$$v_n - v_{n-1} = g\Delta t \ .$$

Case 2: Object falling with air resistance

The equation of motion is

$$m\frac{dv}{dt} = mg - kv$$

where k is the strength of the air resistance. Rewriting it as

$$\frac{dv}{dt} = g - \frac{k}{m}v \ ,$$

the discretized equation is

$$v_n - v_{n-1} = (g - \frac{k}{m}v_{n-1})\Delta t \ .$$

The analytical solution for v (with v(0) = 0) is

$$v = \frac{mg}{k}\left(1 - e^{\frac{-kt}{m}}\right)$$

Case 3: Object falling in viscous liquid

For an object falling in viscous liquids, the equation of motion is

$$m\frac{dv}{dt} = mg - bv^2$$

The bv^2 term means that the resistance force is higher compared to the case where the air resistance force is kv. Rewriting it as

$$\frac{dv}{dt} = g - \frac{b}{m}v^2 \ ,$$

the discretized equation is

$$v_n - v_{n-1} = (g - \frac{b}{m}v_{n-1}^2)\Delta t \ .$$

Materials

Computer with Microsoft Excel

Methodology

Experiment 1a

Part A: Linear Fitting

1. Enter the data in Table 1 into Sheet1 of your Excel file.
2. Make a scatter plot using your data in Table 1 and label your graph appropriately. Place your chart as object in Sheet 1.
3. Add a linear trendline to your plot.
4. From the trendline, state the equation's parameters and the R-squared value.
5. Calculate the percentage difference between the value of the initial velocity *u* obtained from your answer in step 4 and the given value of initial velocity is $u = 1$ m.s^{-1}. (Calculate using $\frac{|u_{Excel} - u|}{u} \times 100\%$)
6. Repeat step 5 for the acceleration *a*.
7. Create a third column next to *v*. Label this column v_calc (v calculated).
8. Fill this column with values of *v* from the equation $v = u + at$ using the given values for *u* and *a*.

Part B: Nonlinear Fitting

1. Enter the data in Table 2 into Sheet1 of your Excel file.
2. Make a scatter plot using your data in Table 2 and label your graph appropriately. Place your chart as object in Sheet 1.
3. Add a polynomial trendline of order 2 to your plot.
4. From the trendline, state the equation's parameters and the R-squared value.
5. Calculate the percentage difference between the value of the initial displacement x_0 obtained from your answer in step 4 and the given initial displacement of the object is $x_0 = 0.5$ m. (Calculate using $\frac{|x_{0\,Excel} - x_0|}{x_0} \times 100\%$)
6. Repeat step 5 for the initial velocity *u* and acceleration *a*.
7. Create a third column next to *x*. Label this column x_calc (x calculated).
8. Fill this column with values of *x* from the equation $x = x_0 + ut + \frac{1}{2}at^2$ using the given values for x_0, *u* and *a*. ($x_0 = 0.5$ m, $u = 1$ m.s^{-1} and $a = 2.1$ m.s^{-2})

Data

Experiment 1a.

Time, t (s)	Velocity, v (ms^{-1})
0.0	1.0
0.5	2.2
1.0	2.8
1.5	4.0
2.0	5.4
2.5	6.5
3.0	7.0
3.5	8.8
4.0	9.5
4.5	10.0
5.0	11.1

Table 1: First Data set

Time, t (s)	Displacement, x(m)
0	0.5
0.5	1.0
1.0	3.1
1.5	4.0
2.0	6.3
2.5	10.5
3.0	13.3
3.5	15.6
4.0	19.3
4.5	25.8
5.0	32.0

Table 2: Second Data set

Experiment 1b.

time(s)	velocity(m/s)
0	0
0.01	0.0981
0.02	0.1962
0.03	0.2943
0.04	0.3924
0.05	0.4905
0.06	0.5886
0.07	0.6867
0.08	0.7848
0.09	0.8829
0.1	0.981
0.11	1.0791
0.12	1.1772
0.13	1.2753
0.14	1.3734
0.15	1.4715
0.16	1.5696
0.17	1.6677
0.18	1.7658
0.19	1.8639

Table 3: Numerical data for free fall with zero air resistance

time(s)	velocity(m/s)	Analytical
0.00	0.0000	0.0000
0.02	0.1962	0.1778
0.04	0.3532	0.3234
0.06	0.4787	0.4426
0.08	0.5792	0.5402
0.10	0.6595	0.6201
0.12	0.7238	0.6855
0.14	0.7753	0.7391
0.16	0.8164	0.7829
0.18	0.8493	0.8188
0.20	0.8757	0.8482
0.22	0.8967	0.8723
0.24	0.9136	0.8920
0.26	0.9271	0.9081
0.28	0.9379	0.9213
0.30	0.9465	0.9322
0.32	0.9534	0.9410
0.34	0.9589	0.9483
0.36	0.9633	0.9542
0.38	0.9669	0.9591
0.40	0.9697	0.9630
0.42	0.9720	0.9663
0.44	0.9738	0.9690
0.46	0.9752	0.9711
0.48	0.9764	0.9729
0.50	0.9773	0.9744

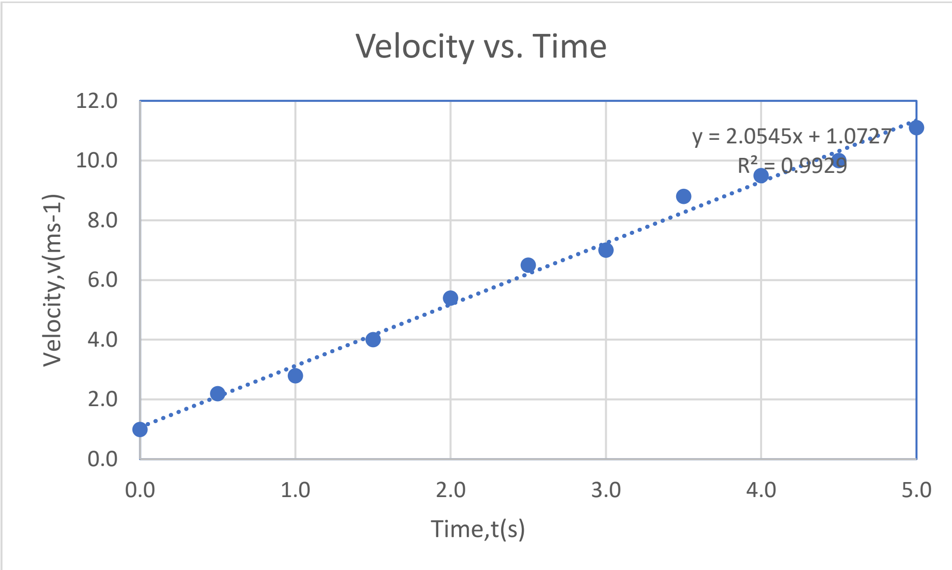
Table 4: Numerical and Analytical data for free fall with non-zero air resistance

time(s)	velocity(m/s)
0.00	0.0000
0.02	0.1962
0.04	0.3770
0.06	0.5163
0.08	0.6059
0.10	0.6553
0.12	0.6797
0.14	0.6911
0.16	0.6963
0.18	0.6985
0.20	0.6996
0.22	0.7000
0.24	0.7002
0.26	0.7003
0.28	0.7003
0.30	0.7003
0.32	0.7004
0.34	0.7004
0.36	0.7004
0.38	0.7004
0.40	0.7004
0.42	0.7004
0.44	0.7004
0.46	0.7004
0.48	0.7004
0.50	0.7004

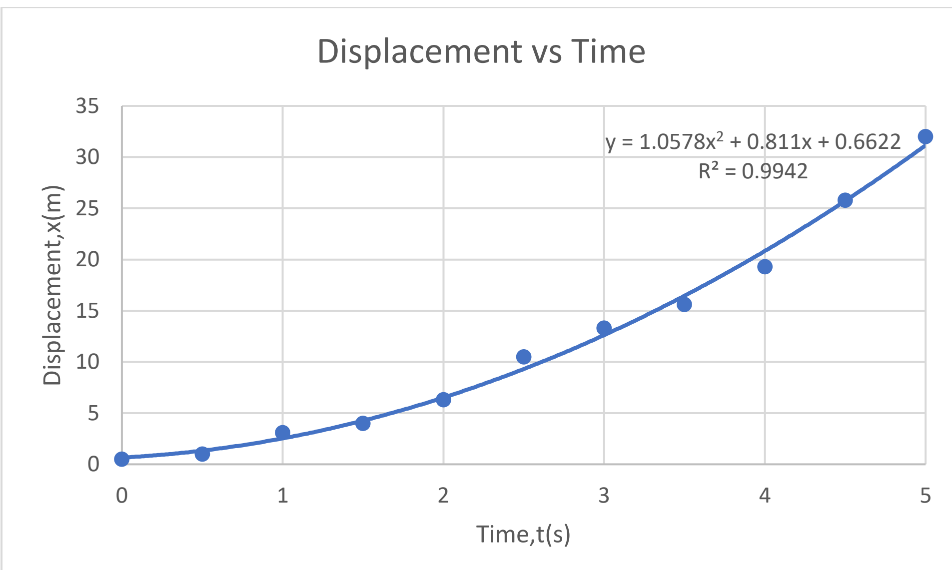
Table 5: Numerical data for free fall in viscous liquid.

Results

Experiment 1a

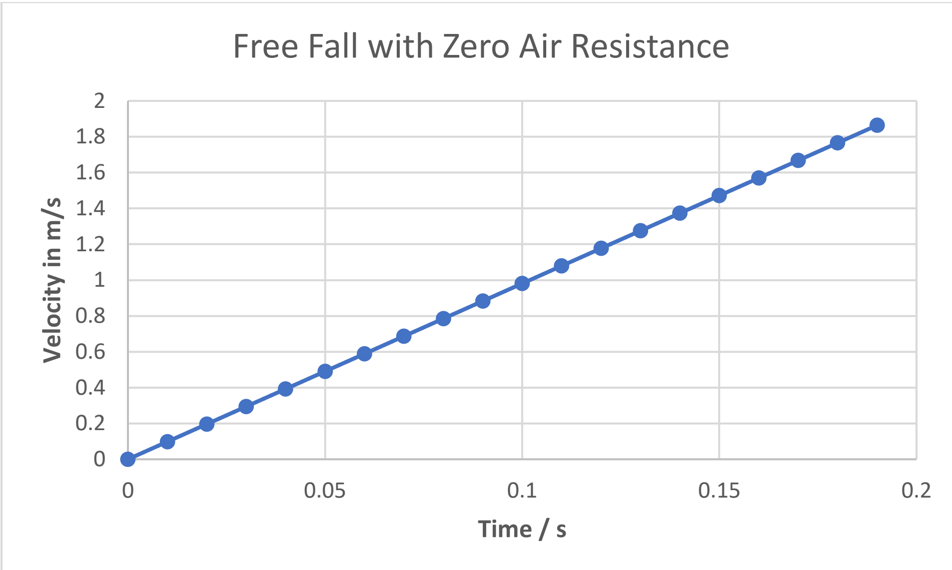


Graph 1: Velocity vs Time graph based on Table 1

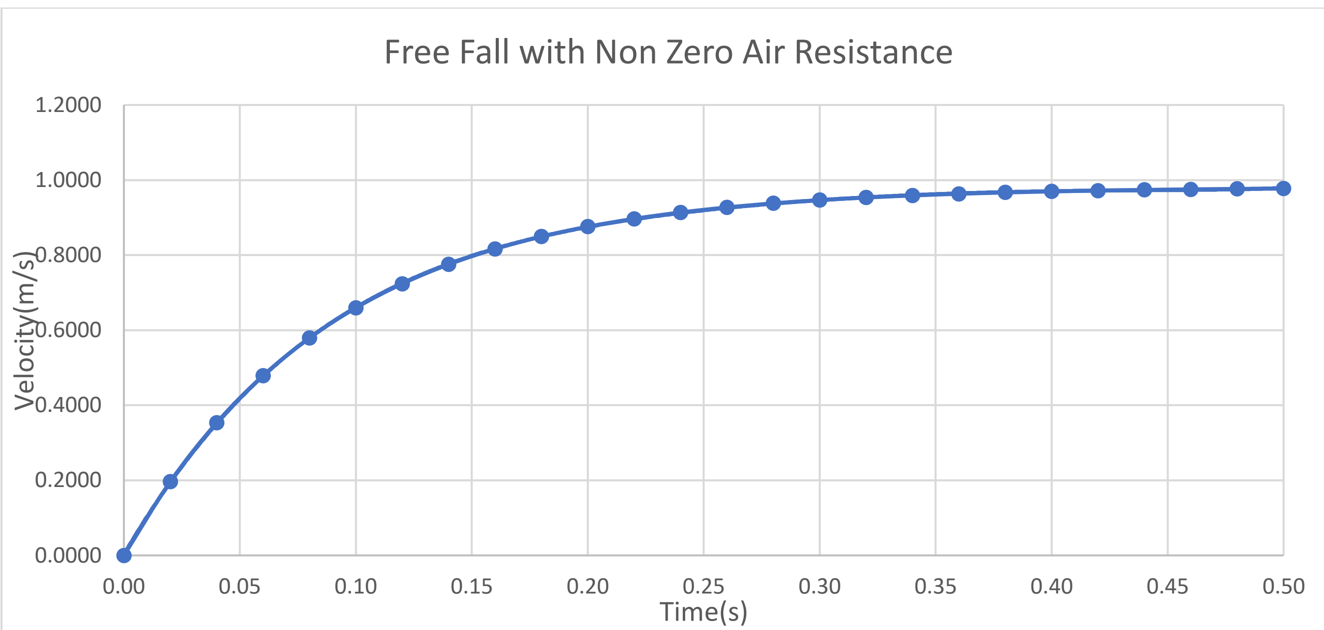


Graph 2: Displacement vs Time graph based on Table 2

Experiment 1b

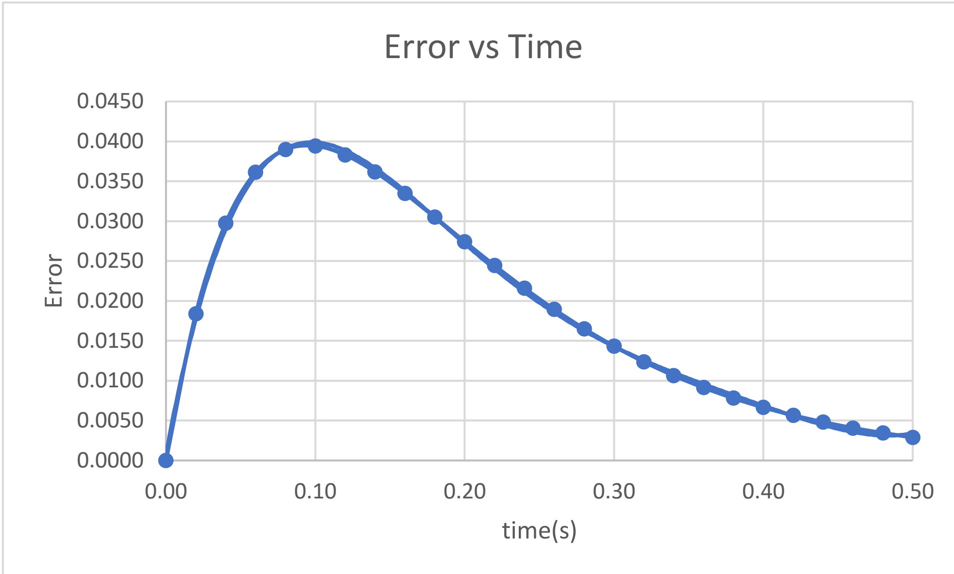


Graph 3: velocity vs time graph based on Table 3

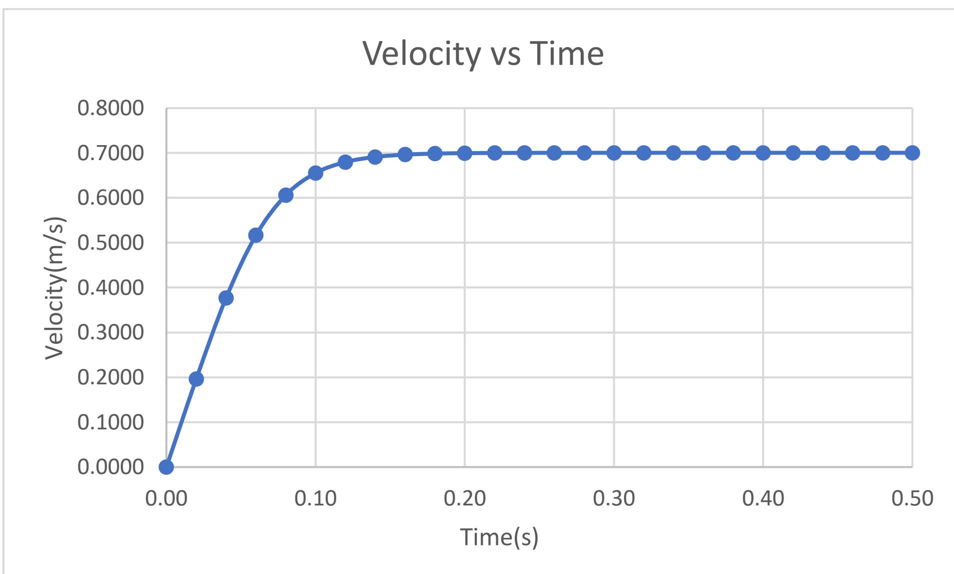


Graph 4: velocity vs time graph based on table 4

Terminal velocity obtained from:
Numerical method: 0.977 m/s
Analytical method: 0.974 m/s



Graph 5: Error (absolute difference between analytical and numerical value) graph



Graph 6: velocity vs time graph for free fall in viscous liquid

Terminal velocity = 0.700 m/s

Discussion

Experiment 1a

1. Curve fitting is the process of constructing a line or curve which has the best fit to a set of data which minimize the deviation of the line from each data point and usually uses a technique called least-squares regression. The accuracy of the fit is determined by the R-squared value which will be explained in (3).

2. Linear least-squares regression is a method to minimize the vertical offset of the points from the line by minimizing the sum of the square of the offsets

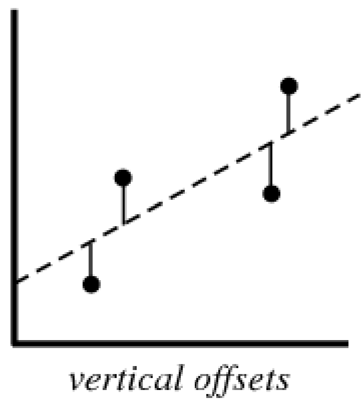


Figure 13: Vertical offsets

3. R-squared value is a statistical measure of how close the data are to the fitted regression line. In other words, it determines the accuracy of a fit. The formula is

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

Where SS_{res} is the sum of the squares of the residual, which is given by:

$$SS_{reg} = \sum_i (f_i - \bar{y})^2$$

and SS_{tot} is:

$$SS_{tot} = \sum_i (y_i - \bar{y})^2$$

Where y_i, $\bar{y}$, and f_i are data set value, mean of the observed data and the predicted value associated with each data point.

4. Curve fitting for nonlinear equation required the use of nonlinear regression which finds a function which minimizes the offset between the points on the curve and data points.

Experiment 1b

1. Numerical analysis is a technique to provide a numerical solution which uses formula or algorithm to obtain an approximation numerical solution of an equation. Analytical analysis is a technique which provide solution to an equation in terms of known functions/polynomials and constants.

2.Pros of using analytical solution:

i) Can be solved easily if equations involved have known general solution

ii) Does not required iterative method which can be tedious if it is done by hand

iii) Provides a general idea of the shape of the solution as it provides a symbolic.

iv) Provides exact solution.

Cons of using analytical solution:

i) Certain equations can be difficult to solve, and some do not have solution.

ii) Can be time consuming if the equations are very complicated.

Pros of using numerical solution:

i) Gives numerical solution to equations which do not have solution or is very difficult to solve.

ii) Can be done quickly by a computer to provide approximate solution when only approximations are required.

Cons of using numerical solution:

i) Solutions are not exact.

ii) Solution can deviate hugely from the true value if not enough signification figures are taken into consideration.

iii) Requires computer most of the time.

Conclusion

Microsoft Excel is able to generate graph based on a table, fit line/curve based on the data and output the R-squared value. Freefall without air resistance produces a linear graph with no terminal velocity, whereas free fall with air resistance and in a viscous liquid produce data which is best fitted by a nonlinear curve with terminal velocity of 0.970m/s and 0.700 m/s respectively, which shows viscous velocity provides a larger resistance to the motion.

References

1. https://www.researchgate.net/post/Measure_of_the_quality_of_a_curve-fit
2. https://www.linkedin.com/pulse/regression-analysis-how-do-i-interpret-r-squared-assess-gaurhari-dass
3. http://mathworld.wolfram.com/LeastSquaresFitting.html
4. https://math.stackexchange.com/questions/1435002/why-are-numerical-solutions-preferred-to-analytical-solutions
5. https://math.stackexchange.com/questions/935405/what-s-the-difference-between-analytical-and-numerical-approaches-to-problems
6. https://en.wikipedia.org/wiki/Nonlinear_regression