SHORT COMMUNICATION

Bahbouhi Bouchaib

Independent researcher. Nantes. Loire Atlantique. France. Email is
bahbouhibouchaib524@gmail.com

How to pose the mathematical problem of the Goldbach's strong conjecture ?
A new idea for a new solution

Abstract

The aim of this short paper is to define the real mathematical problem of Goldbach's strong conjecture (GSC), which remains officially unsolved. It attempts to propose a method based on the analysis of remainders of Euclidean division to explain why two equidistant primes add up to an even number. Let's not forget that in mathematics, posing the problem correctly is the surest way to the solution.

Keywords. Goldbach's strong conjecture. Equidistant primes. Remainders of euclidean division. Prime number theorem. Factoring Algorithm.

I- Introduction

The aim of this short paper is to pose the real problem of Goldbach's Strong Conjecture (GSC) in mathematics (*every even ≥ 4 is sum of two primes*). We can't mention all the important published works, and yet despite their considerable number and their logical and reasonable approaches, mathematicians today still consider the fact that the GSC has not been proved The question is why ? Independent research efforts with no cross-referencing between them and no general consensus to follow the same path (even if all roads lead to Rome, as the popular saying goes) leads to a fragmentation of research into several fragments of knowledge, each with its own logic, but which do not converge towards a central or focal pint.This is also one reason why GSC resists final resolution and does not become a theorem or an axiom. That's why it's time to make a reflection or suggest a line of work that goes to the heart of the problem. In mathematics, the first step is to pose the problem: this is a crucial stage before you start chasing after the solution. So how do you pose the GSC problem?

II- Thinking and posing the GSC mathematical problem

Let's take an even number E as the sum of two primes p and p'. Then E = p + p'. In this case, p' > p so that p' > E/2 and p < E/2, then E/2 - t = p and E/2 + t = p', so that p and p' are equidistant primes. So E/2 + t = p' implies **p + 2t = p'**. In fact, this is where the real formal mathematical problem of GSC arises. ***How can we find the value of t for any even number E at infinity such that p + 2t = p' and such that p + t = E/2 and E/2 + t = p' so that E = p + p' ?*** T*he other central point is that as* ***t*** *tends to infinity, will p + 2t always yield an equidistant prime p' such that p + p' = E?* We can see that the GSC is linked to the prime number equation, which is the only way to predict whether p + 2t is prime or not. If we don't have it then we need to keep performing primality tests ad infinitum. Consequently, this is the only path that leads to a demonstration of the GSC: either find the equation of prime numbers, which is the holy grail of mathematics, or improve or refine primality algorithms to make them faster and, above all, less energy-consuming and applicable to numbers containing millions or even billions of digits or even much more. Alternatively, find a reasoning by deduction that resists no objection to demonstrate that for any even number E ≥ 8 there are two primes such that p + p' = E and such that p = E/2 - t and p' = E/2 + t = p + 2t. This reasoning should focus on the distribution of primes between 0 and E/2 on one hand ; and E/2 and E on the other hand using any useful theorem such as the prime number theorem.

III- The solutions that have been proposed in previous, well-known research studies

There's no need here to recap all the published attempts to solve GSC, lest it drown out or blur the content of this communication. Readers can consult them on several sites, including Vixra, Arvix or google Scholar. The most important thing to remember is that certain works have sought to empirically validate the GSC by pushing the limit further and further using computers with high computing capacity **[1-2]**, but this does not constitute a formal mathematical proof, except that it provides a practical working tool for converting even numbers into the sum of two primes. Others have tried to find out how much prime numbers an even number can be the sum of?**[3]** But the problem here is the limit to which exceptions can be made, since the higher a number is, the more likely it is to be the sum of several primes, and so the theorem is likely to be reduced. What we need is a theorem with a very wide field, valid for all even numbers of the same kind except 2, 4 and 6. Already, the GSC starts from 4 if we accept the use of the same prime number twice, or from 8 if we require that the two primes must be different.

Demonstrating GSC means showing which form is the most common or predominant among even numbers. Is the form "sum of two primes" the most common among all even numbers 4? Or is it another form of more primes? However, other works focus on prime numbers and their distribution, and although these studies are well done and logical, they are nonetheless perificial to the real problem. We know that equidistant primes are required for GSC to be true **[4-5]**, we know that the higher the number, the greater the number of possibilities for sums in two primes **[6]**, ***but the heart of the problem is to be able to predict these equidistant primes in advance by a theorem or equation, or to deduce them using calculus or factoring algorithms***. It is for this reason that, despite their undeniable quality, these articles have not been recorded as a definitive resolution of the GSC. It's not just these articles here, but countless others circulating on the internet fall into this dilemma: the distribution of prime numbers **[7]**, is not enough, a prediction is required to rise to the level of a theorem. In other words, how can we know that p + 2t = p' is prime knowing that for the even number E : t = E/2 - p = p' – E/2 ? Or use factoring algorithms or serial Euclidean division. Prime numbers or GSC lead us to the same labyrinth: is the number prime or composite? So to the long-sought equation of prime numbers.

There are well-known prime number postulates that have become theorems, but which unfortunately can't help to solve Goldbach's strong conjecture. For example, the prime number theorem : « *The number of primes less than x tends asymptotically towards x/log x: n/ln(n)*
We have improved the approximation by taking: π(n) ~ n /(ln(n) - 1) » gives just an approximation to the number of primes before a natural number, but in no way predicts the position of the equidistant primes. Similarly, Bertrand's postulate : « Between n and 2n, there is always a prime.
In other words, the gap between a prime number p and its successor is smaller than p » indicates the presence of a prime number between n and 2n, but does not predict its position. Also, the theorem « Between n and 2n and n > 6, there is at least one prime in 4k - 1 and at least one in 4k + 1 - Proven by Erdös. Example between 7 and 14: 7 = 4x2 - 1; 11 = 4x3 - 1; 13 = 4x3 + 1 » doesn't predict the position of all equidistant primes either.

We can't use the laws of probability calculation, because the positions of numbers are not events that happen in a dependent or independent way. The formula nln(n), which approximates the nth prime number, is of no help, as variations of a few or several units will distort the calculation, since exact values of t are required.

The GCS problem can be posed as follows: we have a prime number p, we have p + 2t = p' and we NEED TO FIND p + t = E/2 and E/2 + t = p' ***[8-9].***

IV An example of an attempt to find a solution using the Euclidean serial division method

In my recent articles **[8-9]**, I tried to provide the beginnings of an answer. I'm not going to reproduce its contents verbatim, but here's the crux of the matter.

E is any even ≥ 8. E = P1 + P2 with P2 > P1 and P1 and P2 are equidistant primes. The method is as follows.
1)- Take t-values su that t < E . Calculate E – t.
2) - Determine π(E) the primes of which are named q and divide E – t by prime factors q < E/2 to determine the remainders (r) of each division to apply the rules (***see the box below***) including t ≠ r and t ≠ r + q or t ≠ r + nq (n is any integer > 0). Primes are numbers E – t with t satisfying the rule for each euclidean division of E by q out of π(E) < E/2. This leads to equidistant primes to E/2 that sum up to form E. Therefore, this rule allows us to find out equidistant primes around E/2.
3) Meanwhile, when we divide E by q out of π(E) > E/2, the remainder = P1 and the divisor = q = P2. This time we have at once two equidistant primes if the remainder is prime. This is another method to find out equidistant primes around E/2.

Here are the rules with some examples below.
Be E any even ≥ 8 and t any integer < E. For **E – t** if t = r + nq then E – t is not prime (n is any integer ≥ 0). For **E + t** if t = nq – r then E + t is not prime (n is any integer ≥ 1). Only if t ≠ r + nq in the first case and t ≠ nq – r in the second case can we have equidistant primes. Both t values are symmetrical. These two rules are required to understand the GSC.
Demonstration:
→ E – t and t = r + nq. Knowing that E = aq + r (a is the quotient) → E – t = aq + r – (r + nq) = (a + n) q → E – t not prime. For each t value, this must be true for all q out of π(E) < E.
→ E + t and t = nq – r. Knowing that E = aq + r → E + t = aq + r + (nq – r) = (a + n) q → E + t not prime. For one t value, this must be true for all q out of π(E) < E.

Here are some examples for q < E/2 **[8-9].**
112 – 21 is not prime because 112 : 13 (13 is q) has a remainder (r) of 8 and at the same time 21 – 8 = 13 → 21 = 8 + 13 (r + q). If we substract 21 of 112, we take off the remainder 8 and one factor 13 and what remains is therefore multiple of 13 → 112 – 21 = 91 = 7 x 13.
112 – 27 is not prime because 112 : 5 (q) has a r = 2 and thus 27 – 2 = 25 → 27 = 2 + 25 = 2 + 5 x 5 (r + q).
112 – 57 is not prime because 112 : 5 has a r = 2 and 57 = 2 + 55 = 2 + 11 x 5 (r + nq). Furthermore, 112 : 11 (q) has a r = 2 and 57 = 2 + 55 = 2 + 5 x 11 (r + nq).
112 – 63 is not prime because 63 is a multiple of 7 and 112 : 7 has r = 0.
112 – 87 is not prime because 112 : 5 has r = 2 and 87 = 2 + 85 = 2 + 17 x 5. (r + nq).

Only some of E + T that are not prime are going to be explained.
112 + 9 is not prime because 112 : 11 has a remainder r = 2 and 9 = 11 – 2 (q – r).
112 + 33 is not prime because 112 : 5 has r = 2 and 33 = 35 – 2 = 7 x 5 – 2 (nq – r). Furthermore, 112 : 29 has r = 25 and 33 = 58 – 25 = 2 x 29 – 25 (nq – r).
A last example. 112 –37 (E = 112 and t = 37) is not prime (= 75) because 112 : 5 has a r = 2 and 37 = 2 = 5 x 7 = r + nq.

But when q > E/2 or 112/2 = 56 the remainder r is either prime or not. For q > E/2 the strong conjecture (E = p + p') itself becomes Euclidean division in the form E = aq + r with **q = P2** and **r = P1** and the quotient a = 1 → E = P2 + P1 = P1 + P2 such that P2 > E/2 > P1. And in this case t = q = P2 and E – t = E – q = P1 = r. Note that r = P1 may be prime or not. This brings new equidistant primes. In this case, we also have the rule stated above. For instance 100 = 53 + 47. Here we have for example 100 : 11 has a **r = 1** while 53 : 11 has a **r' = 9** and 47 : 11 has a **r'' = 3** and we see that r≠r' in both cases therefore 53 and 47 are primes. However if we have 100 = 67 + 33 we have both 100 : 11 has a **r' = 1** and 67 : 11 has **r = 1** and 33 has **r'' = 0, and so r'=r** which implies that in this case we have P + X such that X is composite. Here is another example. 100 = 61 + 39. We have 100 : 13 has a **r' = 9** while 61 : 13 has a **r = 9** and r' = r → 39 is composite relatively to q = 13 ad 39 = 3 x 13. If for one q, r'=r then P + X → X is composite = n'q except if n' = 1.

We know that the method of serial Euclidean division is tedious, but we can use any factorization method or algorithm to find out whether the numbers E - t or E + t are primes, and thus validate the GSC for a given number pushed to the furthest limits of infinity. These operations can be improved using well-known theorems on the remainders of Euclidean divisions or modular algorithms.

V- Conclusion

We have seen that the GSC is strongly correlated with the primality of a number. Here, however, it's not a question of determining whether a NUMBER is prime, but whether an ADDITION or SUBTRACTION is a prime number. We have E - t and E + t such that t < E and we've seen the calculation rules that allow us to know this. However, we can't predict it unless we follow the two rules set out in this paper. These two rules explain the equidistance of two prime numbers p and p' with respect to E/2 and therefore the fact that E = p + p'. Note here that E/2 is any natural number ≥ 2. Finally, it is this approach that needs to be investigated further to prove Goldbach's strong conjecture (GSC). We need to explain why two primes are equidistant at E/2 for any even number noted E ≥ 8. Once again, the primality of a number lies at the heart of the GSC. In other words, to prove mathematically GSC or predicting whether a number is prime by a theorem or equation not yet discovered (unless factoring algorithms are used), are two sides of the same coin.

References

1. Helfgott, H. A., & Platt, D. J. (2013). « Numerical Verification of the Ternary Goldbach Conjecture up to8.875.1030 », Exper.Math. 22, (4) 406409 (DOI 10.1080/10586458.2013.831742, arXiv 1305.3062).
2. Tomás Oliveira e Silva, Siegfried Herzog et Silvio Pardi, « Empirical verification of the even Goldbach conjecture and computation of prime gaps up to 4.1018 », Math. Comp., vol. 83, 2014, p. 2033-2060 (DOI 10.1090/S0025-5718-2013-02787-1
3. T. Estermann, « On Goldbach's problem: proof that almost all even positive integers are sums of two primes », Proc. London Math. Soc., 2e série, vol. 44, 1938, p. 307-314 (DOI 10.1112/plms/s2-44.4.307)
4. Guiasu, S. (2019). The Proof of Goldbach's Conjecture on Prime Numbers, Natural Science. 11, 273-283. doi: 10.4236/ns.2019.119029.
5. Markakis, E., Provatidis, C., Markakis, N. (2013) AN EXPLORATION ON GOLDBACH'S CONJECTURE, International Journal of Pure and Applied Mathematics. 84, 29-63.
6. Fliegel, H. F., & Robertson, D. S. (1989). "Goldbach's Comet: the numbers related to Goldbach's Conjecture", Journal of Recreational Mathematics. 21, (1): 1–7.
7. Chaudhuri, A. R. (2017) "A Short Review on Prime Number Theorem," International Journal of Mathematics Trends and Technology (IJMTT). 47 (3) 225-229, Crossref, https://doi.org/10.14445/22315373/IJMTT-V47P530.
8. Bahbouhi, B. (2025). Verification of Goldbach's Strong and Weak Conjectures at Infinity Using Basic and Accessible Mathematics. J Robot Auto Res, 6(1), 01-11.
9. Bahbouhi, B. (2025).Calculation rules and new methods for the strongand weak conjectures of Goldbach to be verified. J Robot Auto Res, 6(1). 01 - 36.